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Gradient Wind Parabola

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SVERRE PETTERSSEN's interesting »Note on the Relation between the Actual Wind and the Geostrophic Wind» (J. of Met. 1950, p. 76) gave the stimulus for the following contribution. PETTERSSEN's paper disclosed a hyperbola, showing the relation between wind speed, curvature and geostrophic wind in terms of non-dimensional variables. In my lectures on dynamic meteorology I have used a slightly different method of representing the relation between gradient wind and geostrophic wind. This method can also be used for representing the actual horizontal frictionless wind, treated by PETTERSSEN.

In this case the acceleration components tangential and normal to the actual wind are

$$\text{(1)}\quad \begin{aligned} &\text{(a)} & \frac{dv}{dt} &= G \sin\psi & &= fV \sin\psi \\ &\text{(b)} & \frac{v^2}{R} &= G \cos\psi - fv & &= fV \cos\psi - fv \end{aligned}$$

where ψ is the angle from pressure gradient force $\vec{G}$ to the normal of the actual wind path towards the curvature centre. Furthermore $G = \frac{1}{\varrho}\frac{\partial p}{\partial n}$, $f = 2\,\Omega \sin\varphi =$ the Coriolis parameter. R denotes the curvature radius of the trajectory, negative for anticyclonal curvature. The geostrophic wind velocity V is introduced by means of (1b) letting $R = \infty$, $\psi = 0$, $v = V$:

$$G = fV \text{ or } V = \frac{G}{f} \tag{2}$$

The equations (1) are reduced to gradient wind equations when the angle $\psi = 0$.

Dividing the equation (1b) by Rf^2 and introducing the following non-dimensional, reduced velocities:

$$u = \frac{v}{Rf}, \quad U = \frac{V}{Rf} \cos \psi \tag{3}$$

we finally get

$$U = u + u^2 \tag{4}$$

This is the equation of the gradient wind parabola shown in the figure. The quantity $V \cos \psi$ is the component of the geostrophic wind on the direction of the actual wind. U and u are respectively the geostrophic wind component and the actual wind measured by inertial motion velocity Rf.

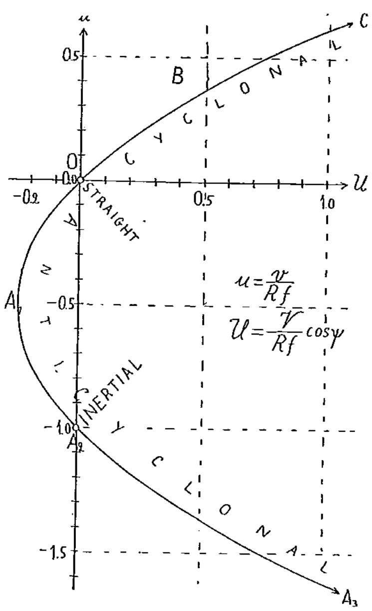

Fig. 13.

G, f (for positive latitudes), v and V are positive by definition. Thus u is always positive in cyclonic and negative in anticyclonic motion. Therefore the branch CO of the parabola belongs to the cyclonal motion and the resting branch $OA_1A_2A_3$ to the anticyclonal motion. Concerning U the sign of $\cos\psi$ is important. Because in cyclonic motion u always is positive, also U is positive in consequence of (4). Hence, it follows from (3) that only positive values of $\cos\psi$ are possible in cyclonic motion. In anticyclonic motion $\cos\psi$ may be negative also.

On the parabola there are five points of singular significance to the motion:

1°. The far part (C) on the cyclonal branch represents the predominantly cyclostrophic wind $U = u^2$.

2°. Point O, $u = 0$ and $U = 0$, will be reached in two ways: firstly, in the trivial case of gradientless ($V = 0$) calm ($v = 0$), secondly, in the important case of straight motion with $R = \infty$. In this case it follows from (1b) that $v = V \cos\psi$ or $v \leq V$. With $\psi = 0$ the case is reduced to the geostrophic wind.

3°. Point A_1 on the anticyclonal branch represents the minimum value, $-\frac{1}{4}$, of U. In this case the two solutions of (4) meet with $u = -\frac{1}{2}$.

4°. At point A_2 U again is $= 0$ and $u = -1$. The value $U = 0$ will be reached in two ways: Firstly, with $\psi = \pm 90°$ and $V \pm 0$. In this case the motion is accelerated or retarded according to (la). Secondly, $V = 0$ gives also $U = 0$ and means the inertial motion. Because of $u = -1$ the inertial velocity is $v = -Rf$.

5°. Going on the branch A_2A_3 to the far part A_3 we get again cyclostrophic motion, now with anticyclonal circulation.

In the synoptic work a small part of the parabola on both sides of the origo only, say the arc A_1OB, comes into use.