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REDUCED DESCRIPTION OF LIGHT IMPURITIES IN HYDROGEN CORONAL PLASMAS

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RESUMEN

Se aplica la teoría de impurezas radiativas a plasmas de reactores termonucleares, así como a nubes interestelares e intergalácticas, atmósferas estelares y prominencias solares. Las impurezas radiativas son capaces de cambiar la dinámica del plasma. Permiten la aparición de nuevos tipos de equilibrio, de ondas lineales y no-lineales y cambian significativamente los criterios de estabilidad. Para una descripción completa de las impurezas, uno necesita un gran conjunto de ecuaciones que describan a cada uno de los estados de ionización. Se muestra que es posible describir impurezas ligeras, con exactitud aceptable, con sólo dos o tres estados representativos de ionización. Se muestra que la descripción de cinco estados puede reducirse a uno de tres estados de ionización. Se propone un modelo reducido para impurezas ligeras o para impurezas pesadas altamente ionizadas en plasmas coronales. Esto permite obtener soluciones analíticas en muchos casos, reduciendo los tiempos de cómputo. Se deriva un conjunto de ecuaciones simples para determinar la dinámica de la distribución de impurezas en los diferentes estados de ionización.

ABSTRACT

The theory of radiative impurities in hydrogen plasmas is applied to edge and divertor plasmas in thermonuclear devices, as well as to interstellar and intergalactic clouds, stellar atmospheres and solar prominences. Radiative impurities are able to qualitatively change the plasma dynamics. The lead to the appearance of new types of equilibria, linear and nonlinear waves, and change significantly the stability criteria. For full impurity descriptions one needs an extremely cumbersome set of equations describing each ionization state separately. It is shown that it is possible to describe light impurities with acceptable accuracy by using only two or three representative ionization states. It is shown that a five ionization state description may be reduced to a three ionization state one. Reduced models for light impurities and strongly ionized heavy impurities in coronal hydrogen plasmas are proposed. These hold out the possibility of getting analytical results in many cases, as well as to significantly reducing computation times. A simple set of equations determining dynamics of impurity distributions over ionization states is derived.

Key Words: **PLASMAS — RADIATION MECHANISMS: THERMAL**

It is well known that radiative impurities are able to change the plasma dynamics qualitatively and lead to the appearance of new types of equilibria, linear and nonlinear waves, and change significantly the stability criteria. For full impurity descriptions one needs an extremely cumbersome set of equations describing each

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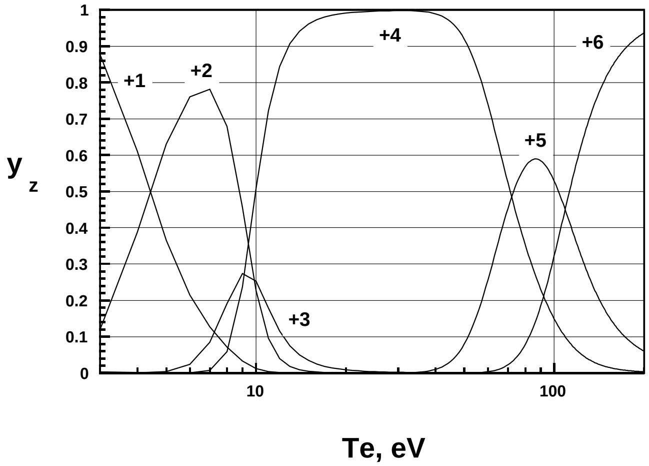

Fig. 1. Relative concentrations of carbon ionization states versus electron temperature. Characters near the curves show the ion charge.

ionization state separately. Thus, for carbon, which is the main impurity in tokamak plasmas, seven ionization states must be described, from neutral to fully ionized ions. In order to build up analytical models, as well as to minimize computation time, some reduced descriptions of impurity ionization states are in development (Ivanov, Kukushkin, & Lisitsa 1987; Arutunov, Krasheninnikov, & Prokhorov 1991; Morozov, Herrera, & Gervids 1998). Relatively heavy impurities with $Z \geq 20$ exist in coronal plasmas as collections of a large number of neighboring ionization states, if the electron temperature is not too high, $T_e \leq 100$ eV, so the impurity is not too deeply ionized, and correspondingly, the principal quantum number $n \geq 3$. Properties of these states change slowly with the transition from one state to another.

Ivanov et al. (1987) and Arutunov et al. (1991) showed that the heavy impurity distribution over ionization states may be approximated by a Gaussian function under these conditions. Under the same temperature conditions, light impurities with $Z < 10$ and $n \leq 2$ exist in coronal plasmas in the form of two or three neighboring ionization states with significantly different properties. Under these conditions the impurity distribution over ionization states includes only two or three most representative ionization states (Galushkin, Gervids, & Kogan 1971). The carbon distribution over ionization states is shown in Figure 1 for a coronal equilibrium. One can see that only for the electron temperature range 7 eV $< T_e <$ 10 eV do four ionization states exist simultaneously.

However, even in this temperature range the relative concentrations of the less representative ionization states do not exceed 6%. As shown in Rozhansky, Morozov, & Herrera (1999) to a first approximation the impurity velocity $\vec{u}_I$ is defined by the main plasma parameters and does not depend on the ionization state charge z for $z > 1$. Under this condition the reduced equation for the two most representative ionization states has been derived (Galushkin et al. 1971). More accurate results, which are valid for a wide range of the electron temperatures, may be obtained with the approximation of the three most representative ions (three ion approximation). In this approximation, states with charges $z-1$, z and $z+1$ are taken into account. Introducing relative ionization state concentrations $y_z = n_z/n_I$; $n_I = \sum_z n_z$, $y_{z-1} + y_z + y_{z+1} = 1$, one can get, excluding y_{z+1}:

$$\frac{\partial y_z}{\partial t} + \vec{u}_I \cdot \nabla y_z = n_e \left[R_{z+1} - \left(J_z + R_z + R_{z+1} \right) y_z + \left(J_{z-1} - R_{z+1} \right) y_{z-1} \right] \quad , \tag{1}$$

$$\frac{\partial y_{z-1}}{\partial t} + \vec{u}_I \cdot \nabla y_{z-1} = n_e \left[R_z y_z - J_{z-1} y_{z-1} \right] \quad . \tag{2}$$

However, even though we know that the most representative ions are ions with charges z and $z+1$, we know the values of z and $z+1$ for the case of the coronal equilibrium only. Moreover, z as well as y_z may change its value in a discrete way if the temperature changes are high. Thus, it is more convenient to replace these variables with continuous ones:

$$\begin{aligned} < z > &= (z-1) y_{z-1} + z y_z + (z+1) y_{z+1} \\ < z^2 > &= (z-1)^2 y_{z-1} + z^2 y_z + (z+1)^2 y_{z+1} \quad . \end{aligned} \tag{3}$$

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Substituting equation (3) into equations (1) and (2) one can get:

$$\left(\frac{\partial}{\partial t} + \vec{u}_I \nabla\right) < z >= -\nu_{<z>} \left[(< z > - z_*) + A_{<z^2>} \left(< z^2 > - z_*^{(2)}\right)\right] \quad , \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \vec{u}_I \nabla\right) < z^2 >= -\nu_{<z^2>} \left[\left(< z^2 > - z_*^{(2)}\right) + A_{<z>} (< z > - z_*)\right] \quad . \tag{5}$$

Here

$$\nu_{<z>} = n_e \left[z \left(J_{z-1} + 2R_z - R_{z+1} - 2J_z\right) + \frac{1}{2} \left(J_{z-1} + R_{z+1}\right)\right] \quad , \tag{6}$$

$$\nu_{<z^2>} = n_e \left[z \left(R_{z+1} - J_{z-1} - 2R_z + 2J_z\right) + \frac{1}{2} \left(J_{z-1} + R_{z+1} + 2R_z + 2J_z\right)\right] \quad , \tag{7}$$

$$A_{<z>} = \frac{4z^2 \left(J_{z-1} - 2J_z + 2R_z - R_{z+1}\right) - 4z \left(J_z + R_z\right) - J_{z-1} + R_{z+1}}{-2z \left(J_{z-1} - 2J_z + 2R_z - R_{z+1}\right) + J_{z-1} + 2J_z + 2R_z + R_{z+1}} \quad , \tag{8}$$

$$A_{<z^2>} = \frac{R_{z+1} - J_{z-1} + 2J_z - 2R_z}{J_{z-1} + R_{z+1} - 2z \left(R_{z+1} - J_{z-1} + 2J_z - 2R_z\right)} \quad , \tag{9}$$

where J_z and R_z are the ionization and recombination rates of an ion with the charge z correspondingly. These values have been calculated many times exactly by different authors using all ionization states. In the system derived one can use the exact equilibrium values to improve an accuracy of calculations. All parameters in the right hand side of equation (4) and equation (5) may be calculated approximately, using $< z >$ instead of z. The ionization state distribution may be obtained from equation (3):

$$y_z = 2z < z > - z^2 - < z^2 > + 1 \quad , \tag{10}$$

$$2y_{z-1} = < z^2 > - < z > (2z + 1) + z(z + 1) \quad . \tag{11}$$

The value z may be taken as the integer part of $\sqrt{< z^2 >}$. One can also estimate the small relative concentration y_{z-2}. Due to the exponential dependence of the ionization rate on temperature the relaxation time for y_{z-2} is significantly smaller than for y_{z-1}. Thus, it is possible to suppose that this value follows the value y_{z-1} adiabatically: $y_{z-2} = y_{z-1} R_{z-1}/J_{z-2}$.

Let us estimate different terms in equations (4) and (5). The values ν for $T_e \simeq 10-30$ eV may be estimated as $10^2 - 10^3$ s^{-1} (Galushkin et al. 1971). If the impurity velocity is of the order of the impurity sound speed 10^6 cm s^{-1}, the impurity "remembers" its initial state at a distance around $10^3 - 10^4$ cm. Thus, at least for laboratory plasmas the declination of impurity distributions over ionization states may be significant. It is important for radiative losses, depending strongly on the ionization state distribution as well as for plasma diagnostics. The reduced set of impurity equations derived in this work is useful to make analytical estimations as well as to reduce significantly computational times. The models obtained are valid also for a description of strongly ionized heavy impurities under thermonuclear temperatures.

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