2005 年 8 月
第 32 卷 第 4 期

西安电子科技大学学报(自然科学版)
JOURNAL OF XIDIAN UNIVERSITY

Aug. 2005
Vol. 32 No. 4

带状随机非线性系统的状态估计

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摘要:讨论了系统噪声和测量噪声为非零均值并且互为相关情形下的广义随机非线性系统,利用广义逆矩阵和矩阵的奇异值分解,给出了奇异矩阵为带状的广义随机非线性系统的奇异值标准形式,得到了该系统状态的最优预测和滤波递推方程.

关键词:广义逆矩阵;广义系统;奇异值分解;状态估计
中图分类号:O211.6 **文献标识码**:A **文章编号**:1001-2400(2005)04-0634-05

State estimation of band stochastic nonlinear systems

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Abstract: A band stochastic nonlinear system with the nonzero mean and correlated white noise is discussed. A singular values standard form for band stochastic nonlinear systems with the band singular matrix is given by generalized inverse matrix and singular value decomposition of the matrix. The optimal recursive estimates for predicting and filtering of this system's states are obtained.

Key Words: generalized matrix; generalized system; singular value decomposition; state estimation

H. H. Rosenbrock 在文献[1]中提出了广义系统问题,尽管没有因果性,然而其鲜明的实际背景引起了人们的广泛关注[2~4]. 随着对广义系统研究的不断深入,出现了许多新成果[5~9]. 但是几乎所有的讨论都集中在奇异矩阵为方阵和系统为线性的情况,为此笔者讨论了系统噪声和测量噪声为非零均值及具有相关噪声的带状随机非线性系统,利用线性系统近似代替非线性系统,将非零均值白噪声、系统噪声及测量噪声互为相关的情况通过线性化的过程化为零均值且系统噪声及测量噪声不相关的情形. 利用广义逆矩阵和矩阵的奇异值分解理论[10]及离散化方法,给出了具有相关噪声干扰的带状随机非线性系统的最优递推方程,得到的结果更具有一般性.

1 带状随机非线性系统的奇异值标准形式

考虑下列带状连续随机非线性系统:

$$\boldsymbol{M}\dot{\boldsymbol{x}}(t)=f(\boldsymbol{x}(t),\boldsymbol{u}(t),t)+G(\boldsymbol{x}(t),\boldsymbol{u}(t),t)\,\boldsymbol{v}(t)\quad,\quad t\in[0,T]\quad,\tag{1}$$

$$\boldsymbol{y}(t)=h(\boldsymbol{x}(t),\boldsymbol{u}(t),t)+\boldsymbol{e}(t)\quad,\quad t\in[0,T]\quad,\tag{2}$$

$$\boldsymbol{x}(0)=\boldsymbol{x}_0\quad,\tag{3}$$

其中 $\boldsymbol{M}\in R^{n\times m}$ 是常数奇异矩阵,rank $\boldsymbol{M}=r<\min\{n,m\}$,$\boldsymbol{x}(t)\in R^m$ 是状态向量,$\boldsymbol{u}(t)\in U\subset R^l$ 是控制向量,$\boldsymbol{v}(t)\in R^m$ 是系统噪声,$\boldsymbol{y}(t)\in R^m$ 是测量向量,$\boldsymbol{e}(t)\in R^m$ 是测量噪声,$f:R^m\times U\times[0,T]\to R^n$,$h:R^m\times U\times[0,T]\to R^m$ 是非线性函数,$G:R^m\times U\times[0,T]\to R^{n\times m}$,$\boldsymbol{v}(t)$ 与 $\boldsymbol{e}(t)$ 是相关的,具有如下统计特性:

$$E\{\boldsymbol{v}(t)\}=m_v(t),\quad \mathrm{cov}\{\boldsymbol{v}(t),\boldsymbol{v}(\tau)\}=\boldsymbol{Q}(t)\,\delta(t-\tau)\quad,\tag{4}$$

$$E\{\boldsymbol{e}(t)\}=m_e(t),\quad \mathrm{cov}\{\boldsymbol{e}(t),\boldsymbol{e}(\tau)\}=\boldsymbol{R}(t)\,\delta(t-\tau)\quad,\tag{5}$$

$$\mathrm{cov}\{\boldsymbol{v}(t),\boldsymbol{e}(\tau)\}=\boldsymbol{S}(t)\,\delta(t-\tau)\quad.\tag{6}$$

收稿日期:2004-08-26
基金项目:国家自然科学基金资助项目(69972036;70371031)
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假设 :(i)系统(1) ~ (3) 是正则的 ;(ii)系统(1) ~ (3) 除 $u(t)$ 外的所有向量都是正态向量, $Q(t) \geqslant 0$, $R(t) > 0$, $t \in [0, T]$;(iii) 容许初值 $x(0)$ 的均值和方差分别为 $\bar{x}(0)$ 和 P_0, 且 $E\{x(0)\, v^{\mathrm{T}}(t)\} = 0$, $E\{x(0)\, e^{\mathrm{T}}(t)\} = 0$, $t \in [0, T]$;(iv)系统是强可控的,即系统的脉冲模和指数模都是可控的. 目标函数

$$J(\hat{x}(t)) = \min_{u(t)} \left\{ \int_0^T ([\Phi(x(t), u(t), t]^{\mathrm{T}} \Phi(x(t), u(t), t))\, \mathrm{d}t + [\Psi(x(T))]^{\mathrm{T}} \Psi(x(T)) \right\} ,$$

其中 $\Phi : R^m \times U \times [0, T] \to R^n$, $\Psi : R^m \to R^n$.

将 $f(\cdot)$ 在状态估计值 $\hat{x}(t)$ 及相应的控制 $\tilde{u}(t)$ 处进行 Taylor 展开,并只保留线性部分,式(1)可化为

$$M\dot{x}(t) = F_1(t)\, x(t) + F_2(t)\, u(t) + \phi(t) + G(t)\, v(t) , \tag{7}$$

其中
$$F_1(t) = \left.\frac{\partial f}{\partial x}\right|_{(\hat{x}(t), \tilde{u}(t), t)} , \quad F_2(t) = \left.\frac{\partial f}{\partial u}\right|_{(\hat{x}(t), \tilde{u}(t), t)} , \quad G(t) = G(\hat{x}(t), \tilde{u}(t), t) ,$$
$$\phi(t) = f(\hat{x}(t), \tilde{u}(t), t) - F_1(t)\, \hat{x}(t) - F_2(t)\, \tilde{u}(t) .$$

类似地式(2) 可化为
$$y(t) = H(t)\, x(t) + \varphi(t) + e(t) , \tag{8}$$

其中 $H(t) = \left.\dfrac{\partial h}{\partial x}\right|_{(\hat{x}(t), \tilde{u}(t), t)}$, $\varphi(t) = h(\hat{x}(t), u(t), t) - H(t)\, \hat{x}(t)$.

令 $A(t) \in R^{n\times m}$ 为一个待定的矩阵,利用式(4) ~ (6) 及式(8) ,式(7) 可化为

$$M\dot{x}(t) = F_1^*(t)\, x(t) + F_2(t)\, u(t) + \phi^*(t) + G(t)\, v^*(t) , \tag{9}$$

其中 $F^*(t) = F_1(t) - A(t)\, H(t)$, $\phi^*(t) = \phi(t) + G(t)\, m_v(t) + A(t)\, [y(t) - \varphi(t) - m_e(t)]$, $v^*(t) = v(t) - m_v(t) - A(t)\, [e(t) - m_e(t)]$.

相应地式(8) 可化为
$$y(t) = H(t)\, x(t) + \varphi^*(t) + e^*(t) , \tag{10}$$

其中 $\varphi^*(t) = \varphi(t) + m_e(t)$, $e^*(t) = e(t) - m_e(t)$.

这时就有
$$E\{v^*(t)\} = 0 , \quad E\{e^*(t)\} = 0 , \tag{11}$$
$$\mathrm{cov}(v^*(t), v^*(\tau)) = Q^*(t)\, \delta(t-\tau) , \tag{12}$$
$$\mathrm{cov}(e^*(t), e^*(\tau)) = R^*(t)\, \delta(t-\tau) , \tag{13}$$
$$\mathrm{cov}(v^*(t), e^*(\tau)) = [S(t) - A(t)\, R(t)]\delta(t-\tau) . \tag{14}$$

于是由式(11)和(14)可知,只要选择 $A(t) = S(t)\, R^{-1}(t)$, $v^*(t)$ 和 $e^*(t)$ 就成为互不相关的零均值高斯白噪声,为了讨论问题方便起见,不妨略去系统(9) 和(10) 中的平凡项 $\phi^*(t)$ 和 $\varphi^*(t)$, 从而系统(1) ~ (3)化为带状随机线性系统

$$M\dot{x}(t) = F_1^*(t)\, x(t) + F_2(t)\, u(t) + G(t)\, v^*(t) , \quad t \in [0, T] , \tag{15}$$
$$y(t) = H(t)\, x(t) + e^*(t) , \quad t \in [0, T] , \tag{16}$$
$$x(0) = x_0 . \tag{17}$$

由于 $\mathrm{rank}\, M = r$,根据矩阵奇异值分解,存在正交矩阵 $U \in R^{n\times n}$ 和 $V \in R^{m\times m}$, 使得

$$UMV = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} , \tag{18}$$

其中 $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_r)$, σ_i , $i = 1, 2, \cdots, r$, 为矩阵 M 的奇异值, 令 $x(t) = V(x_1^{\mathrm{T}}(t), x_2^{\mathrm{T}}(t))^{\mathrm{T}}$, 将式(18) 代入式(15) ,并以 U 左乘式(15)的两端得

$$\Sigma\dot{x}_1(t) = A_{11}(t)\, x_1(t) + A_{12}(t)\, x_2(t) + B_1(t)\, u(t) + \Gamma_1(t)\, v^*(t) , \tag{19}$$
$$0 = A_{21}(t)\, x_1(t) + A_{22}(t)\, x_2(t) + B_2(t)\, u(t) + \Gamma_2(t)\, v^*(t) , \tag{20}$$
$$y(t) = C_1(t)\, x_1(t) + C_2(t)\, x_2(t) + e^*(t) , \tag{21}$$

式中 $x_1(t) \in R^r$, $x_2(t) \in R^{m-r}$, 且 $UF_1^*(t)\, V = \begin{bmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{bmatrix}$, $UF_2(t) = \begin{bmatrix} B_1(t) \\ B_2(t) \end{bmatrix}$, $UG(t) = \begin{bmatrix} \Gamma_1(t) \\ \Gamma_2(t) \end{bmatrix}$, $H(t)\, V = [C_1(t) \quad C_2(t)]$, 称式(19) ~ (21) 和式(17) 为系统(15) ~ (17) 的奇异值标准形式.

2　带状随机非线性系统的状态估计

先寻找等价离散观测过程,令采样时间间隔为 Δt ,

$$y(t)=\frac{1}{\Delta t}\int_{t_i}^{t_{i+1}}y(t)\,\mathrm{d}t=\frac{1}{\Delta t}\int_{t_i}^{t_{i+1}}[C_1(t)x_1(t)+C_2(t)x_2(t)+e^*(t)]\,\mathrm{d}t\approx C_1(t_i)x_1(t_i)+C_2(t_i)x_2(t_i)+e_d(t_i)\triangleq y(t_i)\quad, \tag{22}$$

其中 $e_d(t_i)=\frac{1}{\Delta t}\int_{\Delta t}e^*(t)\,\mathrm{d}t$,显然 $\{e_d(t_i)\}$ 为零均值高斯白噪声序列,并且

$$R_d(t_i)=E\{e_d(t_i)e_d^{\mathrm{T}}(t_i)\}=\frac{1}{\Delta t}\iint_{\Delta t}E\{e^*(t)e^*(\tau)\}\,\mathrm{d}t\,\mathrm{d}\tau=\frac{1}{(\Delta t)^2}\iint_{\Delta t}R^*(t)\delta(t-\tau)\,\mathrm{d}t\,\mathrm{d}\tau=\frac{R^*(t_i)}{\Delta t}\quad.$$

设 $y(t_i)$ 表示目前时刻 t_i 为止的全部观测值组成的向量,即 $y_i=y(t_i)=[y(t_i^{(1)}),y(t_i^{(2)}),\cdots,y(t_i^{(m)})]^{\mathrm{T}}$,相应地式(19)和(20)变为

$$\Sigma x(t_{i+1})=(\Sigma+A_{11}(t_i)\Delta t)x_1(t_i)+A_{12}(t_i)\Delta t\,x_2(t_i)+B_1(t_i)\Delta t\,u(t_i)+\Gamma_1(t_i)\Delta t\,v_d(t_i)\,, \tag{23}$$

$$0=A_{21}(t_i)x_1(t_i)+A_{22}(t_i)x_2(t_i)+B_2(t_i)u(t_i)+\Gamma_2(t_i)v_d(t_i)\quad, \tag{24}$$

其中 $v_d(t_i)=\frac{1}{\Delta t}\int_{\Delta t}v^*(t)\,\mathrm{d}t$,显然 $\{v_d(t_i)\}$ 是零均值高斯白噪声序列,并且 $Q_d(t_i)=Q^*(t_i)/\Delta t$.

① 当 $\mathrm{rank}\,A_{22}(t_i)=m-r$ 时,由文献[10]中的定理1及式(24)得

$$x_2(t_i)=-A_{22}^+(t_i)A_{21}(t_i)x_1(t_i)-A_{22}^+(t_i)B_2(t_i)u(t_i)-A_{22}^+(t_i)\Gamma_2(t_i)v_d(t_i)\,,t\in[0,T]\quad. \tag{25}$$

将式(25)代入式(22)和(23)得 r 维子系统

$$x_1(t_{i+1})=A_0(t_i)x_1(t_i)+B_0(t_i)u(t_i)+\Gamma_0(t_i)v_d(t_i)\,,\quad t_i\in[0,T]\quad, \tag{26}$$

$$y(t_i)=C_0(t_i)x_1(t_i)+D_0(t_i)u(t_i)+\bar{e}_d(t_i)\,,\quad t_i\in[0,T]\quad, \tag{27}$$

其中

$$A_0(t_i)=\Sigma^{-1}(\Sigma+A_{11}(t_i)-A_{12}(t_i)A_{22}^+(t_i)A_{21}(t_i))\Delta t\quad,$$

$$B_0(t_i)=\Sigma^{-1}(B_1(t_i)-A_{12}(t_i)A_{22}^+(t_i)B_2(t_i))\Delta t\quad,$$

$$\Gamma_0(t_i)=\Sigma^{-1}(\Gamma_1(t_i)-A_{12}(t_i)A_{22}^+(t_i)\Gamma_2(t_i))\Delta t\quad,$$

$$C_0(t_i)=C_1(t_i)-C_2(t_i)A_{22}^+(t_i)A_{21}(t_i)\quad,\quad D_0(t_i)=-C_2(t_i)A_{22}^+(t_i)B_2(t_i)\quad,$$

$$\bar{e}_d(t_i)=e_d(t_i)-F_0(t_i)v_d(t_i)\quad,\quad F_0(t_i)=C_2(t_i)A_{22}^+(t_i)\Gamma_2(t_i)\quad.$$

从而由文献[9]中的定理1容易得到下面的定理1.

定理1 带状随机非线性系统(1)~(3)的最优一步预测递推方程为

$$\hat{x}(t_{i+1}\mid t_i)=V[\hat{x}_1^{\mathrm{T}}(t_{i+1}\mid t_i),\hat{x}_2^{\mathrm{T}}(t_{i+1}\mid t_i)]^{\mathrm{T}}\quad,\quad t_i,t_{i+1}\in[0,T]\quad, \tag{28}$$

$$P(t_{i+1}\mid t_i)=V\begin{bmatrix}P_1(t_{i+1}\mid t_i) & P_{12}(t_{i+1}\mid t_i)\\ P_{21}(t_{i+1}\mid t_i) & P_2(t_{i+1}\mid t_i)\end{bmatrix}V^{\mathrm{T}}\quad,\quad t_i,t_{i+1}\in[0,T]\quad, \tag{29}$$

其中

$$\hat{x}_1(t_{i+1}\mid t_i)=A_0(t_i)\hat{x}_1(t_i\mid t_{i-1})+B_0(t_i)u(t_i)+K(t_i)[y(t_i)-C_0(t_i)\hat{x}_1(t_i\mid t_{i-1})-D_0(t_i)u(t_i)]\quad,$$

$$K(t_i)=[A_0(t_i)P_1(t_i\mid t_{i-1})C_0^{\mathrm{T}}(t_i)-\Gamma_0(t_i)Q_d(t_i)F_0^{\mathrm{T}}(t_i)]\cdot[C_0(t_i)P_1(t_i\mid t_{i-1})C_0^{\mathrm{T}}(t_i)+F_0(t_i)Q_d(t_i)F_0^{\mathrm{T}}(t_i)+R_d(t_i)]^+\quad,$$

$$P_1(t_{i+1}\mid t_i)=A_0(t_i)P_1(t_i\mid t_{i-1})A_0^{\mathrm{T}}(t_i)+\Gamma_0(t_i)Q_d(t_i)\Gamma_0^{\mathrm{T}}(t_i)-K(t_i)[C_0(t_i)P_1(t_i\mid t_{i-1})C_0^{\mathrm{T}}(t_i)+F_0(t_i)Q_d(t_i)F_0^{\mathrm{T}}(t_i)+R_d(t_i)]K^{\mathrm{T}}(t_i)\quad,$$

$$\hat{x}_2(t_{i+1}\mid t_i)=-A_{22}^+(t_i)A_{21}(t_i)\hat{x}_1(t_{i+1}\mid t_i)-A_{22}^+(t_i)B_2(t_i)u(t_i)\quad,$$

$$P_2(t_{i+1}\mid t_i)=A_{22}^+(t_i)A_{21}(t_i)P_1(t_{i+1}\mid t_i)A_{21}^{\mathrm{T}}(t_i)(A_{22}^+(t_i))^{\mathrm{T}}+A_{22}^+(t_i)\Gamma_2(t_i)Q_d(t_{i+1})\Gamma_2^{\mathrm{T}}(t_i)(A_{22}^+(t_i))^{\mathrm{T}}\quad,$$

$$P_{12}(t_{i+1}\mid t_i)=-P_1(t_{i+1}\mid t_i)A_{21}^{\mathrm{T}}(t_i)(A_{22}^+(t_i))^{\mathrm{T}}\quad,\quad P_{21}(t_{i+1}\mid t_i)=P_{12}^{\mathrm{T}}(t_{i+1}\mid t_i)\quad.$$

从而由文献[9]中的定理2及式(22)~(29)容易得到下面的定理2.

定理2 带状随机非线性系统(1)~(3)的最优一步滤波递推方程为

$$\hat{\boldsymbol{x}}(t_i \mid t_i) = \boldsymbol{V}[\hat{\boldsymbol{x}}_1^{\mathrm{T}}(t_i \mid t_i), \hat{\boldsymbol{x}}_2^{\mathrm{T}}(t_i \mid t_i)]^{\mathrm{T}} \quad , \quad t_i \in [0, T] \quad , \tag{30}$$

$$\boldsymbol{P}(t_i \mid t_i) = \boldsymbol{V}\begin{bmatrix} \boldsymbol{P}_1(t_i \mid t_i) & \boldsymbol{P}_{12}(t_i \mid t_i) \\ \boldsymbol{P}_{21}(t_i \mid t_i) & \boldsymbol{P}_2(t_i \mid t_i) \end{bmatrix}\boldsymbol{V}^{\mathrm{T}} \quad , \quad t_i \in [0, T] \quad , \tag{31}$$

其中$\hat{\boldsymbol{x}}_1(t_i \mid t_i) = \hat{\boldsymbol{x}}_1(t_i \mid t_{i-1}) + \overline{\boldsymbol{K}}(t_i)[\boldsymbol{y}(t_i) - \boldsymbol{C}_0(t_i)\hat{\boldsymbol{x}}_1(t_i \mid t_{i-1}) - \boldsymbol{D}_0(t_i)\boldsymbol{u}(t_i)]$,

$\overline{\boldsymbol{K}}(t_i) = \boldsymbol{P}_1(t_i \mid t_{i-1})\boldsymbol{C}_0^{\mathrm{T}}(t_i)[\boldsymbol{C}_0(t_i)\boldsymbol{P}_1(t_i \mid t_{i-1})\boldsymbol{C}_0^{\mathrm{T}}(t_i) + \boldsymbol{F}_0(t_i)\boldsymbol{Q}_d(t_i)\boldsymbol{F}_0^{\mathrm{T}}(t_i) + \boldsymbol{R}_d(t_i)]^{+}$,

$\boldsymbol{P}_1(t_i \mid t_i) = [\boldsymbol{I} - \overline{\boldsymbol{K}}(t_i)\boldsymbol{C}_0(t_i)]\boldsymbol{P}_1(t_i \mid t_{i-1})$,

$\hat{\boldsymbol{x}}_1(t_{i+1} \mid t_i) = \overline{\boldsymbol{A}}_0(t_i)\hat{\boldsymbol{x}}_1(t_i \mid t_i) + \boldsymbol{B}_0(t_i)\boldsymbol{u}(t_i) + \boldsymbol{U}(t_i)\boldsymbol{y}(t_i)$,

$\boldsymbol{P}_1(t_{i+1} \mid t_i) = \overline{\boldsymbol{A}}_0(t_i)\boldsymbol{P}_1(t_i \mid t_i)\overline{\boldsymbol{A}}_0^{\mathrm{T}}(t_i) + \boldsymbol{\Gamma}_0(t_i)\overline{\boldsymbol{Q}}(t_i)\boldsymbol{\Gamma}_0^{\mathrm{T}}(t_i)$,

$\hat{\boldsymbol{x}}_2(t_i \mid t_i) = -\boldsymbol{A}_{22}^{+}(t_i)\boldsymbol{A}_{21}(t_i)\hat{\boldsymbol{x}}_1(t_i \mid t_i) - \boldsymbol{A}_{22}^{+}(t_i)\boldsymbol{B}_2(t_i)\boldsymbol{u}(t_i) - \boldsymbol{A}_{22}^{+}(t_i)\boldsymbol{\Gamma}_2(t_i)\hat{\boldsymbol{v}}_d(t_i \mid t_i)$,

$\hat{\boldsymbol{v}}_d(t_i \mid t_i) = -\boldsymbol{Q}_d(t_i)\boldsymbol{F}_0^{\mathrm{T}}(t_i)[\boldsymbol{C}_0(t_i)\boldsymbol{P}_1(t_i \mid t_{i-1})\boldsymbol{C}_0^{\mathrm{T}}(t_i) + \boldsymbol{F}_0(t_i)\boldsymbol{Q}_d(t_i)\boldsymbol{F}_0^{\mathrm{T}}(t_i) + \boldsymbol{R}_d(t_i)]^{+} \cdot$
$[\boldsymbol{y}(t_i) - \boldsymbol{C}_0(t_i)\hat{\boldsymbol{x}}_1(t_i \mid t_{i-1}) - \boldsymbol{D}_0(t_i)\boldsymbol{u}(t_i)]$,

$\overline{\boldsymbol{P}}(t_i \mid t_i) = \boldsymbol{Q}_d(t_i) - \boldsymbol{Q}_d(t_i)\boldsymbol{F}_0^{\mathrm{T}}(t_i)[\boldsymbol{C}_0(t_i)\boldsymbol{P}_1(t_i \mid t_{i-1})\boldsymbol{C}_0^{\mathrm{T}}(t_i) + \boldsymbol{F}_0(t_i)\boldsymbol{Q}_d(t_i)\boldsymbol{F}_0^{\mathrm{T}}(t_i) +$
$\boldsymbol{R}_d(t_i)]^{+}\boldsymbol{F}_0(t_i)\boldsymbol{Q}_d(t_i)$,

$\boldsymbol{U}(t_i) = -\boldsymbol{\Gamma}_0(t_i)\boldsymbol{Q}_d(t_i)\boldsymbol{F}_0^{\mathrm{T}}(t_i)[\boldsymbol{F}_0(t_i)\boldsymbol{Q}_d(t_i)\boldsymbol{F}_0^{\mathrm{T}}(t_i) + \boldsymbol{R}_d(t_i)]^{+}$,

$\overline{\boldsymbol{A}}_0(t_i) = \boldsymbol{A}_0(t_i) - \boldsymbol{U}(t_i)\boldsymbol{C}_0(t_i)$,

$\boldsymbol{P}_{12}(t_i \mid t_i) = \boldsymbol{P}_{21}^{\mathrm{T}}(t_i \mid t_i) = -\boldsymbol{P}_1(t_i \mid t_i)\boldsymbol{A}_{21}^{\mathrm{T}}(t_i)(\boldsymbol{A}_{22}^{+}(t_i))^{\mathrm{T}} -$
$\overline{\boldsymbol{K}}(t_i)\boldsymbol{F}_0(t_i)\boldsymbol{Q}_d(t_i)\boldsymbol{\Gamma}_2^{\mathrm{T}}(t_i)(\boldsymbol{A}_{22}^{+}(t_i))^{\mathrm{T}}$,

$\overline{\boldsymbol{Q}}(t_i) = \boldsymbol{Q}_d(t_i) - \boldsymbol{Q}_d(t_i)\boldsymbol{F}_0^{\mathrm{T}}(t_i)[\boldsymbol{F}_0(t_i)\boldsymbol{Q}_d(t_i)\boldsymbol{F}_0^{\mathrm{T}}(t_i) + \boldsymbol{R}_d(t_i)]^{+}$.

② 当 $\operatorname{rank}\boldsymbol{A}_{22}(t_i) < m-r$ 时,根据假设(iv)及文献[10]中的引理及$\operatorname{rank}\begin{bmatrix} \boldsymbol{\Sigma} & \boldsymbol{A}_{12}(t_i)\Delta t & \boldsymbol{B}_1(t_i)\Delta t \\ 0 & \boldsymbol{A}_{22}(t_i) & \boldsymbol{B}_2(t_i) \end{bmatrix} = n$得 $\operatorname{rank}[\boldsymbol{A}_{22}(t_i) \quad \boldsymbol{B}_2(t_i)] = n-r$, 令 $\operatorname{rank}\boldsymbol{A}_{22}(t_i) = r_1$, 则存在可逆矩阵 $\boldsymbol{J} \in R^{(n-r)\times(n-r)}$, 使得 $\boldsymbol{J}\boldsymbol{A}_{22}(t_i) = \begin{bmatrix} \boldsymbol{\Delta}_1(t_i) & \boldsymbol{\Delta}_2(t_i) \\ 0 & 0 \end{bmatrix}$,其中$\boldsymbol{\Delta}_1(t_i) \in R^{r\times r}$ 是上三角矩阵且$\operatorname{rank}\boldsymbol{\Delta}_1(t_i) = r_1$,$\boldsymbol{\Delta}_2(t_i) \in R^{r_1\times(m-r-r_1)}$,令$\boldsymbol{D}(t_i) = \boldsymbol{J}\boldsymbol{B}_2(t_i) = [\boldsymbol{D}_1^{\mathrm{T}}(t_i) \quad \boldsymbol{D}_2^{\mathrm{T}}(t_i)]^{\mathrm{T}}$, 其中$\boldsymbol{D}_1(t_i) \in R^{r_1\times l}$, $\boldsymbol{D}_2(t_i) \in R^{(n-r-r_1)\times l}$, 则 $\operatorname{rank}\boldsymbol{D}_2(t_i) = n-r-r_1$. 令$\boldsymbol{K}_2(t_i) = \boldsymbol{D}_2^{+}(t_i)\begin{bmatrix} \boldsymbol{I}_{m-r-r_1} \\ 0 \end{bmatrix}$,其中 $\boldsymbol{I}_{m-r-r_1}$ 为$m-r-r_1$ 阶单位矩阵,则

$$\boldsymbol{J}[\boldsymbol{A}_{22}(t_i) \quad \boldsymbol{B}_2(t_i)]\begin{bmatrix} \boldsymbol{I}_1 & 0 \\ \boldsymbol{K}_1(t_i) & \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Delta}_1(t_i) & \boldsymbol{\Delta}_2(t_i) & \boldsymbol{D}_1(t_i) \\ 0 & 0 & \boldsymbol{D}_2(t_i) \end{bmatrix}\begin{bmatrix} \boldsymbol{I}_{11} & 0 & 0 \\ 0 & \boldsymbol{I}_{12} & 0 \\ 0 & \boldsymbol{K}_2(t_i) & \boldsymbol{I}_2 \end{bmatrix} ,$$

其中$\boldsymbol{I}_1 = \begin{bmatrix} \boldsymbol{I}_{11} & 0 \\ 0 & \boldsymbol{I}_{12} \end{bmatrix}$,$\boldsymbol{I}_2$,$\boldsymbol{I}_{11}$和$\boldsymbol{I}_{12}$分别为$l$阶,$r$阶和$m-r-r_1$ 阶单位矩阵,$\boldsymbol{K}_1(t_i) = [0 \quad \boldsymbol{K}_2(t_i)]$是$l\times(m-r)$阶矩阵,从而

$$\boldsymbol{J}[\boldsymbol{A}_{22}(t_i) + \boldsymbol{B}_2(t_i)\boldsymbol{K}_1(t_i) \quad \boldsymbol{B}_2(t_i)] = \begin{bmatrix} \boldsymbol{\Delta}_1(t_i) & \boldsymbol{\Delta}_2(t_i) + \boldsymbol{D}_1(t_i)\boldsymbol{K}_2(t_i) & \boldsymbol{D}_1(t_i) \\ 0 & \boldsymbol{D}_2(t_i)\boldsymbol{K}_2(t_i) & \boldsymbol{D}_2(t_i) \end{bmatrix} ,$$

由 $\boldsymbol{K}_2(t_i) = \boldsymbol{D}_2^{+}(t_i)\begin{bmatrix} \boldsymbol{I}_{m-r-r_1} \\ 0 \end{bmatrix}$及文献[10]中的定理 1 得$\boldsymbol{D}_2(t_i)\boldsymbol{K}_2(t_i) = \begin{bmatrix} \boldsymbol{I}_{m-r-r_1} \\ 0 \end{bmatrix}$, 从而

$$\boldsymbol{J}[\boldsymbol{A}_{22}(t_i) + \boldsymbol{B}_2(t_i)\boldsymbol{K}_1(t_i) \quad \boldsymbol{B}_2(t_i)] = [\boldsymbol{F}(t_i) \quad \boldsymbol{J}\boldsymbol{B}_2(t_i)] ,$$

其中

$$\boldsymbol{F}(t_i) = \boldsymbol{J}[\boldsymbol{A}_{22}(t_i) + \boldsymbol{B}_2(t_i)\boldsymbol{K}_1(t_i)] = \begin{bmatrix} \boldsymbol{\Delta}_1(t_i) & \boldsymbol{\Delta}_2(t_i) + \boldsymbol{D}_1(t_i)\boldsymbol{K}_2(t_i) \\ 0 & \boldsymbol{I}_{m-r-r_1} \\ 0 & 0 \end{bmatrix} ,$$

所以 $$\mathrm{rank}\,(A_{22}(t_i)+B_2(t_i)K_1(t_i))=m-r\quad.$$

令 $u(t_i)=K_1x_2(t_i)+\eta(t_i)$ 并代入式(23),(24),再令 $\tilde{A}_{12}(t_i)=A_{12}(t_i)+B_1(t_i)K_1(t_i)$, $\tilde{A}_{22}(t_i)=A_{22}(t_i)+B_2(t_i)K_1(t_i)$,得

$$\Sigma x_1(t_{i+1})=(\Sigma+A_{11}(t_i)\Delta t)x_1(t_i)+\tilde{A}_{12}(t_i)\Delta t\,x_2(t_i)+B_1(t_i)\Delta t\,\eta(t_i)+\Gamma_1(t_i)\Delta t\,v_d(t_i)\quad,$$
$$0=A_{21}(t_i)x_1(t_i)+\tilde{A}_{22}(t_i)x_2(t_i)+B_2(t_i)\eta(t_i)+\Gamma_2(t_i)v_d(t_i)\quad,$$
$$y(t_i)=C_1(t_i)x_1(t_i)+C_2(t_i)x_2(t_i)+e_d(t_i)\quad,$$

而且 $\mathrm{rank}\,\tilde{A}_{22}(t_i)=m-r$,这样就把②的情形转化为①的情况. 需要指出,由于 $\eta(t_i)$ 无法精确确定,因而实际得到的不是最优估计而是一种次优估计.

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(编辑:郭 华)

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(编辑:齐淑娟)