Laser Review

Generation and Transformation of Partially Coherent Laser Beams

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The recent progresses in the generation and transformation of partially coherent beams are reviewed. The analytical propagation formulae for the cross-spectral density of the partially coherent anisotropic Gaussian-Schell model (GSM) beams through ABCD optical systems are provided. These formulae provide a powerful tool for analyzing and calculating the transformation of partially coherent GSM beams, such as focusing, spectral shift, and fractional Fourier transform etc. A new kind of resonator with spatial-temporal phase modulation is proposed and analyzed. This kind of resonator can produce partially coherent flattop-alike beams under properly chosen parameters. The changes of the coherence of light in optical resonator are investigated. The generation of partially coherent laser beam directly from spatial-temporal phase modulated optical resonator is investigated experimentally. The results show that intra-cavity phase modulation is an effective way to generate partially coherent beams. The two-slit interference experiment confirms that the output beam is partially coherent.

Key Words: Partially coherent beam, Tensor ABCD law, Spatial-temporal phase modulation

1. Introduction

Partially coherent light exists widely and has some specific applications in practice. Completely coherent and completely incoherent beams, in the strict sense, don't exist. Collett-Wolf source is a typical source emitting partially coherent Gaussian-Schell model (GSM) beams.[1] GSM beams and later the twisted GSM beams have attracted particular interest, because such beams can not only be analyzed theoretically,[2-11] but they can also be constructed in the laboratory.[12] The Wigner distribution function has been widely used to treat the propagation and imaging of GSM beams.[5-10]

On the other hand, it is well known that the transformation of completely coherent Gaussian beams with circular symmetry through axially symmetric optical system satisfies the famous Kogelnik *abcd*-law.[13] This law was generalized in several directions, so as to make it applicable for more complicated beams, such as nonorthogonal Gaussian beams,[14] non-Gaussian beams,[15] ultrashort pulsed Gaussian beams,[16] and general astigmatic beams in both spatial and spatial-frequency domain.[17,18] One may naturally ask, is it possible to obtain a generalized *ABCD* law for partially coherent GSM beams? We shall show in section 2 that the answer is yes.

Laser beams usually have high degree of coherence. In some applications, however, the high coherence is not desired, because it is easy to cause speckles that are harmful to the uniformity of beam intensity distribution. The uniformity is very important to many applications, such as inertial confinement fusion,[19,20] laser material processing,[21] laser cladding[22] etc.. There are several approaches to improve the uniformity including array optical elements,[20] binary optical elements,[23,24] random phase plate,[19,25,26] random polarization control plate,[27] superposition of several beams,[28] etc.. Among these methods, random phase plate is very effective in transforming coherent laser beam into partially coherent beam. Another approach is to use special resonator structure to make the output beam to be super-Gaussian beam or alike,[29,30] such as graded reflectivity mirrors,[31] graded phase mirrors[32] and adaptive mirrors[33] etc..

In sections 3 and 4, we are going to present a different way to obtain partially coherent beams directly from optical resonator. This method is achieved by inserting a spatial-temporal phase modulator into a conventional resonator. The theoretical and experimental results show that intra-cavity phase modulation can change the output intensity distribution and degree of coherence effectively. Under some conditions, partially coherent laser beam with intensity distribution similar to Gaussian Schell-model beam can be obtained.

2. Tensor *ABCD* law for partially coherent beams

The cross-spectral density of the most general partially coherent anisotropic GSM beam is expressed as[4]

$$W(\boldsymbol{r}_1,\boldsymbol{r}_2)=G_0\exp\left\{-\frac{1}{4}\left[\boldsymbol{r}_1^{\mathrm T}\left(\sigma_{\mathrm I}^2\right)^{-1}\boldsymbol{r}_1+\boldsymbol{r}_2^{\mathrm T}\left(\sigma_{\mathrm I}^2\right)^{-1}\boldsymbol{r}_2\right]-\frac{1}{2}\left(\boldsymbol{r}_1-\boldsymbol{r}_2\right)^{\mathrm T}\left(\sigma_{\mathrm g}^2\right)^{-1}\left(\boldsymbol{r}_1-\boldsymbol{r}_2\right)\right.$$
$$\left.-\frac{ik}{2}\left(\boldsymbol{r}_1-\boldsymbol{r}_2\right)^{\mathrm T}\left(\boldsymbol{R}^{-1}+\mu\boldsymbol{J}\right)\left(\boldsymbol{r}_1+\boldsymbol{r}_2\right)\right\} \tag{1}$$

where G_0 is a constant, k is the wave number, $\boldsymbol{r}_1=(x_1, y_1)$ and $\boldsymbol{r}_2=(x_2, y_2)$ are position vectors of two arbitrary points in the transverse plane. $\sigma_{\mathrm I}^2$ is transverse spot width matrix, $\sigma_{\mathrm g}^2$ is transverse coherence width matrix. $\boldsymbol{R}^{-1}$ is wave front curvature matrix. $\sigma_{\mathrm I}^2$, $\sigma_{\mathrm g}^2$ and $\boldsymbol{R}^{-1}$ are all 2×2 matrices with transpose symmetry, given by:

$$\left(\sigma_{\mathrm{I}}^{2}\right)^{-1}=\begin{pmatrix}\sigma_{\mathrm{I}11}^{-2} & \sigma_{\mathrm{I}12}^{-2}\\ \sigma_{\mathrm{I}12}^{-2} & \sigma_{\mathrm{I}22}^{-2}\end{pmatrix},\quad \left(\sigma_{\mathrm{g}}^{2}\right)^{-1}=\begin{pmatrix}\sigma_{\mathrm{g}11}^{-2} & \sigma_{\mathrm{g}12}^{-2}\\ \sigma_{\mathrm{g}12}^{-2} & \sigma_{\mathrm{g}22}^{-2}\end{pmatrix},\quad \boldsymbol{R}^{-1}=\begin{pmatrix}R_{11}^{-1} & R_{12}^{-1}\\ R_{21}^{-1} & R_{22}^{-1}\end{pmatrix}, \tag{2}$$

$\boldsymbol{J}$ is a transpose anti-symmetric matrix given by: $\boldsymbol{J}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$.
μ is a scalar real-valued twist factor with inverse distance dimension. It is limited by $0 \le \mu^2 \le \left[k^2 \det\left(\sigma_g^2\right)\right]^{-1}$ due to the nonnegativity requirement on the cross-spectral density of the beam.[3,5,10] All the known families of coherent and partially coherent Gaussian beams are subsets of the beam described by Eq.(1). Eq. (1) can be rearranged into a more compact form:

$$W(\boldsymbol{r})=G_0\exp\left[-\frac{ik}{2}\boldsymbol{r}^{\mathrm{T}}\boldsymbol{M}^{-1}\boldsymbol{r}\right], \tag{3}$$

where $\boldsymbol{r}^{\mathrm{T}}=\left(\boldsymbol{r}_1^{\mathrm{T}}\quad \boldsymbol{r}_2^{\mathrm{T}}\right)=\left(x_1\quad y_1\quad x_2\quad y_2\right)$, $\boldsymbol{M}^{-1}$ is a 4×4 complex matrix given by:

$$\begin{aligned}\boldsymbol{M}^{-1}&=\begin{pmatrix}\boldsymbol{R}^{-1}-\frac{i}{2k}\left(\sigma_{\mathrm{I}}^{2}\right)^{-1}-\frac{i}{k}\left(\sigma_{\mathrm{g}}^{2}\right)^{-1} & \frac{i}{k}\left(\tau_{\mathrm{g}}^{2}\right)^{-1}+\mu\boldsymbol{J}\\ \frac{i}{k}\left(\sigma_{g}^{2}\right)^{-1}+\mu\boldsymbol{J}^{\mathrm{T}} & -\boldsymbol{R}^{-1}-\frac{i}{2k}\left(\sigma_{\mathrm{I}}^{2}\right)^{-1}-\frac{i}{k}\left(\sigma_{\mathrm{g}}^{2}\right)^{-1}\end{pmatrix}.\\ &=\begin{pmatrix}\boldsymbol{M}_{11}^{-1} & \boldsymbol{M}_{12}^{-1}\\ \left(\boldsymbol{M}_{12}^{-1}\right)^{T} & \left(-\boldsymbol{M}_{11}^{-1}\right)^{*}\end{pmatrix}\end{aligned} \tag{4}$$

$\boldsymbol{M}^{-1}$ is a transpose symmetric matrix called "*partially coherent complex curvature tensor*". In general, ten real physical parameters are needed to characterize a partially coherent GSM beam.

The propagation formula of cross-spectral density of partially coherent beam through the axially non-symmetric optical system can be derived as follows[34]:

$$W(\rho)=G_0\left[\det\left(\overline{\boldsymbol{A}}+\overline{\boldsymbol{B}}\boldsymbol{M}_{\mathrm{i}}^{-1}\right)\right]^{-1/2}\exp\left[-\frac{ik}{2}\rho^{\mathrm{T}}\boldsymbol{M}_{\mathrm{o}}^{-1}\rho\right], \tag{5}$$

where $\boldsymbol{M}_{\mathrm{i}}^{-1}$ and $\boldsymbol{M}_{\mathrm{o}}^{-1}$ denote the partially coherent complex curvature tensor in the input and the output plane, respectively. They satisfy the following formula:

$$\boldsymbol{M}_{\mathrm{o}}^{-1}=\left(\overline{\boldsymbol{C}}+\overline{\boldsymbol{D}}\boldsymbol{M}_{\mathrm{i}}^{-1}\right)\left(\overline{\boldsymbol{A}}+\overline{\boldsymbol{B}}\boldsymbol{M}_{\mathrm{i}}^{-1}\right)^{-1}. \tag{6}$$

where $\overline{\boldsymbol{A}}=\begin{pmatrix}\boldsymbol{A} & 0\\ 0 & \boldsymbol{A}\end{pmatrix}$, $\overline{\boldsymbol{B}}=\begin{pmatrix}\boldsymbol{B} & 0\\ 0 & -\boldsymbol{B}\end{pmatrix}$, $\overline{\boldsymbol{C}}=\begin{pmatrix}\boldsymbol{C} & 0\\ 0 & -\boldsymbol{C}\end{pmatrix}$,

$\overline{\boldsymbol{D}}=\begin{pmatrix}\boldsymbol{D} & 0\\ 0 & \boldsymbol{D}\end{pmatrix}$. $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are all 2×2 sub-matrices of the 4×4 ray transfer matrix describing the axially non-symmetric optical system. Eq. (6) may be called the tensor *ABCD* law for general partially coherent beams.

By using the tensor *ABCD* law given above, we can study the focusing properties of twisted anisotropic GSM beam.[35] Assume the focusing lens is astigmatic located in the plane $z = 0$, the output plane is located at z. The focal length of the astigmatic lens in the direction of x and y are f_x and f_y respectively. The on-axis intensity distributions of focused twisted anisotropic GSM beam with different transverse coherence width matrix element $\sigma_{\mathrm{g}11}^{2}$ are shown in Fig. 1. The initial parameters

used in the calculation are $\left(\sigma_{\mathrm{I}}^{2}\right)^{-1}=\begin{pmatrix}1 & 0.2\\ 0.2 & 0.5\end{pmatrix}(mm)^{-2}$,
$f_x = 20$ mm, $f_y = 30$ mm, $\boldsymbol{R}^{-1} = 0$, $\mu = 0\ \mathrm{mm}^{-1}$, $\sigma_{\mathrm{g}12}^{2} = 1\ \mathrm{mm}^2$, $\sigma_{\mathrm{g}22}^{2} = 0.33\ \mathrm{mm}^2$, $\lambda = 632.8$ nm. From Fig. 1, we can find that the

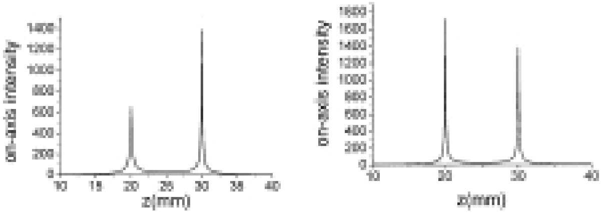

Fig. 1 The on-axis intensity profile of GSM beam focused by a thin lens.

heights of the two peaks are closely related with the initial transverse coherence width in the two directions. That means a beam or a part of a beam with higher coherence can be focused more tightly. The tensor method can also be used in treating propagation,[36-38] spectral shift,[39,40] and fractional Fourier transform of partially coherent GSM beams.[41-44]

3. Mode of resonators with spatial-temporal phase modulation

Fox-Li method has been used widely to study the mode of laser resonators, in which the light field is treated as strictly monochromatic, or completely coherent.[45] In order to get coherence properties, Wolf and Agarwal developed a new theory of laser resonator mode based on an integral equation that expresses a steady-state condition for the cross-spectral density of the field of any spectral composition.[46] Under the condition of paraxial approximation, the propagation formula of the cross-spectral density is given by[47]

$$W(\vec{r},\vec{r}',\nu)=\frac{1}{\lambda_0^2 z^2}\iint W_0(\vec{\rho},\vec{\rho}',\nu)e^{\frac{ik}{2z}\left[(\vec{r}-\vec{\rho})^2-(\vec{r}'-\vec{\rho}')^2\right]}d^2\vec{\rho}d^2\vec{\rho}' \tag{7}$$

where z is the propagation distance, λ_0 is wave length, $\vec{r}$, $\vec{r}'$ and $\vec{\rho}$, $\vec{\rho}'$ are transverse coordinates on incident plane and observation plane respectively, ν is the light frequency. The cross-spectral density is defined by:

$$W(\vec{r},\vec{r}',\nu)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\langle V(\vec{r},t)V^{*}(\vec{r}',t+\tau)\right\rangle e^{i2\pi\nu\tau}d\tau, \tag{8}$$

where $V(\vec{r}, t)$ is the complex amplitude of the optical field, the angular brackets denotes time average and the asterisk denotes the complex conjugate.

For the sake of simplicity, we treat the resonator as two-dimensional plane parallel resonator, i.e., the widths of the two mirrors are infinity in the y direction and $2a$ in x direction, and the length of cavity is L, as shown in Fig. 2.

When the Fresnel number $F = a^2/\lambda L$ of the resonator is much less than $(L/a)^2$, the eigen equation of the normalized cross-spectral density can be expressed by[46,48]:

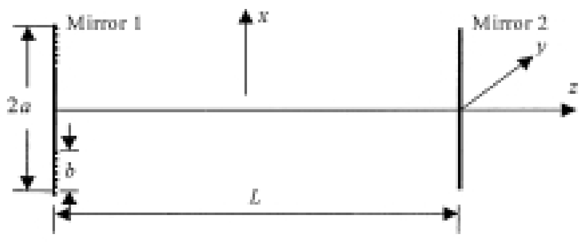

Fig. 2 Schematic diagram of 2-D plane parallel resonator with phase modulation mirror.

$$b_{1,m+1}W_{1,m+1}\left(x_1,x_1^{'},\nu\right)=\frac{1}{\lambda_0 L}\int_{-a}^{a}\int_{-a}^{a}W_{2,m}\left(x_2,x_2^{'},\nu\right)\exp\left\{\frac{ik}{2L}\left[\left(x_2-x_1\right)^2-\left(x_2^{'}-x_1^{'}\right)^2\right]\right\}dx_2dx_2^{'}, \quad (9)$$

$$b_{2,m}W_{2m}\left(x_2,x_2^{'},\nu\right)=\frac{1}{\lambda_0 L}\int_{-a}^{a}\int_{-a}^{a}W_{1,m}\left(x_1,x_1^{'},\nu\right)\exp\left\{\frac{ik}{2L}\left[\left(x_2-x_1\right)^2-\left(x_2^{'}-x_1^{'}\right)^2\right]+\varphi\left(x_1\right)-\varphi\left(x_1^{'}\right)\right\}dx_1dx_1^{'} \quad (10)$$

where $W_{1,m+1}\left(x_1,x_1^{'},\nu\right)$ denotes the normalized cross-spectral density of the light cross the mirror 1 after completion of m round trips start from mirror 1, $W_{2,m}\left(x_2,x_2^{'},\nu\right)$ denotes the normalized cross-spectral density cross the mirror 2 after completion of m - 1 round trips start from mirror 2. $x_1, x_1^{'}, x_2, x_2^{'}$ are coordinates on corresponding mirrors, m is the number of transits, m = 1, 2, 3, $\cdots$. $\varphi(x)$ is phase profile on mirror 1. The phase on mirror 2 is uniform. The m-th round trip loss is $\delta_m = 1 - b1_{m+1}b2_m$. The intensity profile can be obtained by: $I(x) = G(x) = W(x, x, \nu)$. The degree of coherence of the output beam can be easily calculated by:

$$\mu(x,x')=\frac{W(x,x')}{\sqrt{G(x)G(x')}}, \quad (11)$$

where x, x' are coordinates of two points on the output plane, and $G(x) = W(x, x)$.

In order to obtain partially coherent output beam, we assume the phase modulation function contains a time varying factor $\delta(t)$:

$$\varphi(x,t)=\begin{cases}\pi\sin\left[\dfrac{q\pi x}{a}+\pi\delta(t)\right], & |x|>a-b\\ 0, & |x|\le a-b\end{cases} \quad (12)$$

where $\delta(t)$ is a random quantity between 0 and 1 changing with time. Although the cross-spectral density $W(x, x')$ fluctuates with time, when it is averaged over a period of time Δt, the fluctuation will disappear, and the averaged intensity $\bar{I}(x)$ will reach a smooth profile. In Fig. 3, The term "n times average" means that the intensity is averaged over n round trips. It is equivalent to an average over a period of time Δt, which is equal to the time duration for the light propagating in the resonator for n round trips. The more detailed evolution of the mode under different parameters can be found in Ref.49. In the calculations, we assume the length of resonator to be L = 1000.0 mm, wavelength λ_0 = 10.6 μm, Fresnel number F = 2 and q = 10.

The spatial coherence of the light field is an increase function of round trips N in the resonators. Fig. 4 presents the evolution of the spatial coherence at the central frequency of the initial light.[49] From Fig. 4, we find that the light field changes from spatial incoherent light to partially coherent light with Gaussian Schell-model alike beam for the resonator of g = 1.0 and F = 8.0. If there is no degeneracy and only a single mode existing in resonator, the light field is completely spatially coherent at each frequency. If there is degeneracy and several transverse modes existing in resonator (in the case of large Fresnel number), the light field is spatially partially coherent.[46,49]

4. Generation of partially coherent laser beam

It is well known that the output mode structure of a laser is determined by the geometrical structure of optical resonator and pumping condition. If we want to change the mode structure, we can either use some special mirrors such as graded reflectivity or phase mirror[31,32] and adaptive mirror,[33] or inserting mode selecting elements into the resonator such as Fresnel Zone-Plate[50] and random phase modulator.[48]

Fig. 5 is the schematic diagram of our experimental setup. The active medium is Nd:YAG rod with size $\phi 6\times 104$ mm, which is pumped by two Krypton lamps working in continuous wave mode. M_1 and M_2 are two plane parallel mirrors separated by a distance of 510 mm. Between Mirror 1 and the YAG rod, there is a $LiNbO_3$ crystal with size of $7\times 7\times 25$ mm, which acts as a phase modulator. Two ends of $LiNbO_3$ crystal are coated with antireflection films at wavelength of 1.06 μm.

Fig. 6 shows the output beam intensity distribution without (a) and with (b) phase modulation.[51] We can see from the figure that the intensity distribution is smoothed obviously. It should be pointed out that the dim hole in Fig. 6 (a) indicate a kind of mode structure. The output beam of a practical laser is usually not a pure transverse mode, but a mixture of several modes. The theoretical analysis about the dependency of the modulation depth

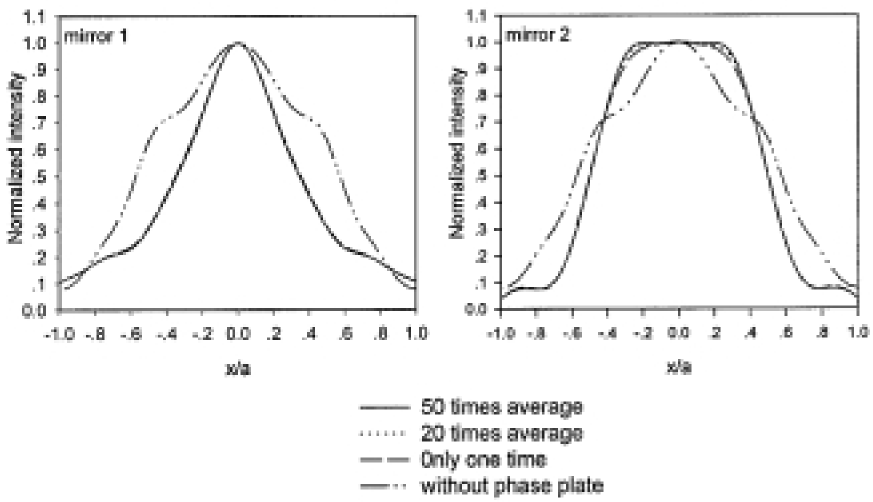

Fig. 3 Mode of resonators with time varying phase modulation mirror.

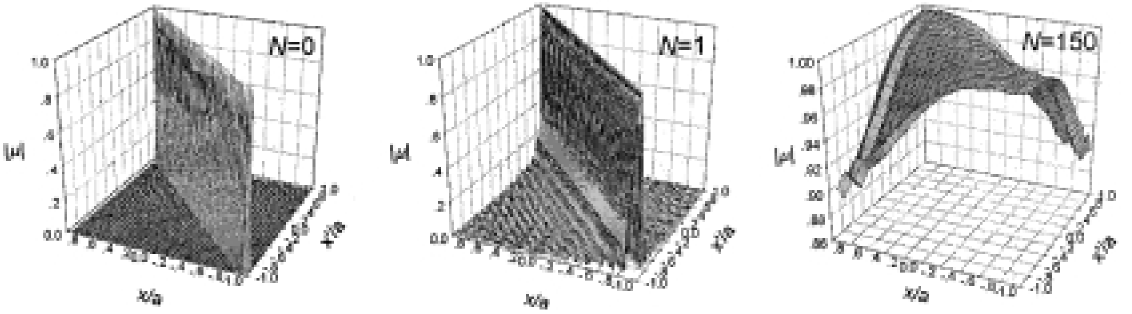

Fig. 4 The evolution of spatial coherence degree at the central frequensy ν_0.

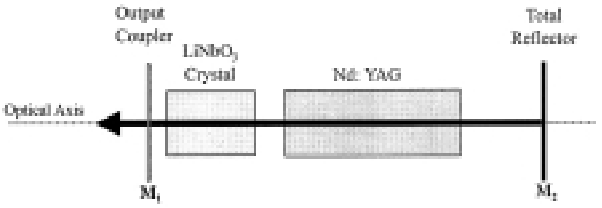

Fig. 5 Experimental Setup.

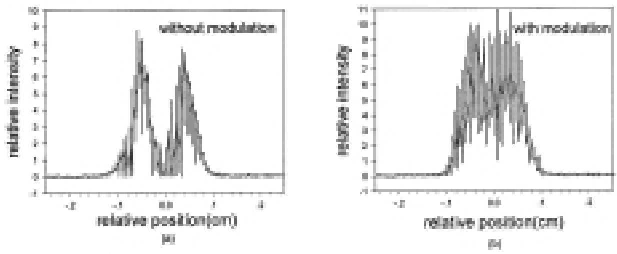

Fig. 6 Output beam intensity profile.

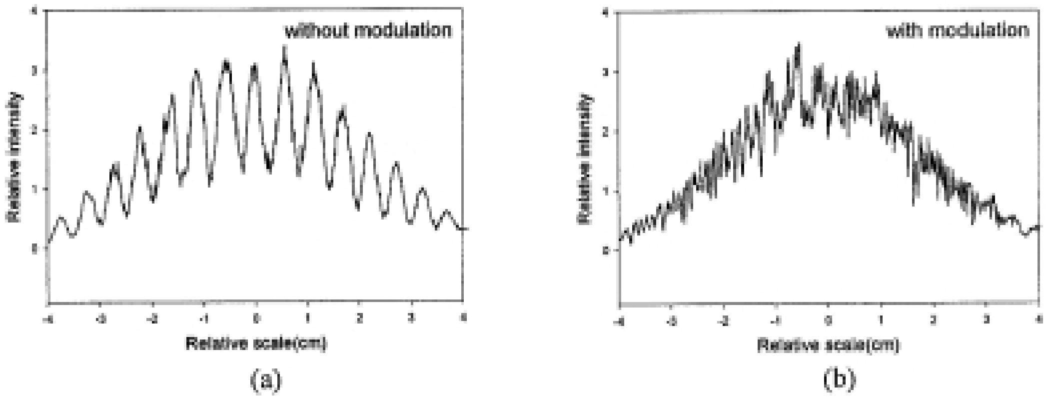

Fig. 7 Two-silt interference patterns.

on the coherence of light can be found in Ref.51.

In order to checkout the coherence property of the output beam, we resort to the Young's two-slit interference experiment. The contrast of the interference fringes is directly related to the coherence of the output beam. Fig. 7 (a) and (b) shows the interference patterns formed by two beams come out from two slits without and with phase modulation respectively. From Fig. 7 (a), we can see very clear interference pattern, which indicates that the output beam has high degree of coherence. While the modulation is applied, the interference pattern becomes very vague as shown in Fig. 7(b), which indicates that the output beam has low degree of coherence. The disappearance of the dim hole and the drop of coherence are two effects of the intra-cavity phase modulation. The differences between the method of intracavity phase modulation and the conventional mode selection with pinholes are clear.

5. Conclusions

We have introduced a 4×4 matrix $\boldsymbol{M}^{-1}$ to characterize the most complete general astigmatic partially coherent GSM beams. The partially coherent tensor *ABCD* law through axially non-symmetric optical system is derived. This propagation law is elegant in form, and can be used in various fields of the propagation and transformation of partially coherent GSM beams. More importantly, all the previous results for coherent Gaussian beams can be converted into partially coherent GSM beams, as the propagation law for them is exactly the same in form.

We also presented a new method to generate partially coherent laser beams directly from optical resonator with spatial-temporal phase modulation. The theoretical and experimental results show that, it is a very effective way to control the output modes through intra-cavity spatial-temporal phase modulation. When appropriate modulation parameters are chosen, partially coherent output beam with smooth intensity distribution similar to Gaussian Schell-model beam can be obtained.

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