Geophysica (2006), 42(1-2), 3-15

Fracture of Ice Cover under Thermal Stresses

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(Received: September 2005; Accepted: March 2006)

Abstract

In the paper we suggest an asymptotic approach for the description of multiple brittle fracture of the ice cover of fresh water basins at an abrupt changing of the ice surface temperature (at a thermal shock) or at occuring the temperature difference between the central and peripheral regions of the ice cover (in plane). A method for solving the problems of thermal fracture under both types of loading was suggested. The method implies solving the heat transfer problem for the whole ice cover region, while the crack initiation and growth is considered within the separate effective blocks-beams formed by parallel growing cracks. An energetic method is used for modeling the crack growth conditions. The estimates of the heat flux and bearing capacity variations caused by the crack influence were obtained. An influence of the contact boundary conditions at the shore line on the fracture process features was also analyzed.

Key words: fresh water ice, fracture, thermal crack, thermal shock

1. Introduction

Low temperatures in winter create thermal stresses in the ice cover of fresh water basins. Ice has a rather high coefficient of thermal expansion ($\alpha \sim (50\text{-}80)10^{-6}\ {}^{o}C^{-1}$) in the temperature range $0^{o}C$ - $-30^{o}C$ (*Bogorodski and Gavrilo* 1980). Hence, the thermal stresses can cause ice cover fracture by formation of cracks-faults nucleated at the cooling surface.

Cracks of thermal origin in ice cover have been known for a long time. Thermal cracks belong to the factors forming the mechanical structure of sea ice cover and its stress state (*Evans* 1971, *Bazant* 1992). The role of thermal stresses for ice cover state in fresh-water basins is more important as compared to sea ice cover, since some characteristic types of loads in sea ice cover such as loads caused by the ice drift (or by the wind) are absent (or less intensive) in fresh-water basins.

Structures of thermal cracks in ice cover include crack systems formed at different line scales and scales of temporal parameters of actions. Often these scales essentially differ such that the processes occurring at each of the scales are weakly interacting. This feature enables to distinguish their scenarios. In the paper we consider some basic scenarios of the formation of the thermal crack systems at the scales of ice thickness and ice basin. The attention is also paid to the role of the structure of the crack system in the processes of heat transfer through the ice cover.

In the paper first we consider a 2D-model problem on a limit equilibrium of a crack under ice cover bending in a vertical plane under different boundary conditions (the model problem demonstrates a variant of ice cover fracture by single cracks). Then a problem on local multiple fracture of ice cover by surface cracks is considered and the appropriate exchange processes are analyzed. Finally a problem of multiple fracture of ice cover at the scale of the ice basin is considered accounting for the temperature difference between the central and circumferential parts of the basin.

2. *Statement of the problem*

Let us consider a model of a plane section of the ice cover. Assume that the ice cover is rather thin such that the temperature distribution through its thickness is linear and stationary. The temperature at the bottom of the ice is at the freezing point, while at the upper surface it is close to the air temperature. We separate two main situations of thermomechanical quasistatic damage of the ice cover in the bounded fresh water reservoirs: 1. Ice cover is fully joined with the shore line; 2. Ice cover is free floating. Note, that the processes of the leveling of the temperature gradient through thickness of the thin ice cover (of thickness 0.5-0.7 m) in the winter period are ahead of the stress relaxation processes. Hence, we can assume that the temperature and stress distributions are close to linear for such quasisteady regimes. Quasistatic thermal stresses in the ice cover plane are calculated in a quasielastic approximation such that the relaxation processes are accounted for by a coefficient $k < 1$ (*Pehovich* 1983)

$$\sigma_y \approx -kE\varepsilon(\Delta T)/(1-\mu) \tag{1}$$

where E, μ are the Young's modulus and Poisson ratio of ice ($E \sim 5\cdot 10^9$ Pa, $\mu \sim 0.3$), $\varepsilon(\Delta T)$ is the deformation caused by the temperature variation (ΔT), k is the coefficient taking account of the stress relaxation. Thermal deformation depends on the aforementioned boundary conditions on shore. For instance, if the dispacements are absent at the boundaries and, hence, thermoelastic stresses compensate the thermal deformations, we can write

$$\varepsilon(\Delta T) \sim \alpha\Delta T \tag{2}$$

Assume the linear temperature distribution through the ice cover thickness

$$\Delta T \approx T(x) = -T_u\left[1-(x/h)\right] \tag{3}$$

where x is the vertical coordinate referenced from the upper ice cover surface, T_u is the temperature of the upper ice surface, and h is the ice cover thickness. Then we obtain

$$\sigma_y \approx -kE\alpha T_u\left[1-(x/h)\right]/(1-\mu) \tag{4}$$

In case of free boundaries and uniform temperature fields we obtain

$$\varepsilon(\Delta T) \sim \alpha\left[\Delta T(x) - \overline{\Delta T}\right] \tag{5}$$

where $\overline{\Delta T} \sim 1/2T_u$ is the average (through the ice thickness) deviation of the temperature from its equilibrium value. By incorporating Eq. (5) we obtain

$$\sigma_y \approx kE\alpha T_u\left[1/2-(x/h)\right]/(1-\mu) \tag{6}$$

Note that in case of the free floating ice cover maximal tensile stresses at cooling the upper surface of the ice cover down to the temperature T_u is two times lower as compared to the case of the ice cover joined to the shore line. Moreover, the stress changes the sign along the ice thickness such that the compressive stresses act at the bottom side of the ice cover. Hence, the character of quasistatic thermal fracture of the ice cover can be varied by varying the boundary conditions at the reservoir contour.

To evaluate the conditions of the limit equilibrium of a crack growing from the upper surface of the ice cover we consider the strip with an edge transverse crack under the action of the stresses σ_y linearly varying through the strip thickness (*Hellan* 1984). In the first case the crack grows under the action of tension and bending, while in the second case the bending stresses are only acting. By superposition of the solutions we obtain for the fixed and free boundaries the formulae for the stress intensity factor at the crack tip, respectively

$$K_I = T_u\frac{kE\alpha}{12(1-\mu)}\left(h^{1/2}f_1\left(\frac{\ell}{h}\right)-(\pi\ell)^{1/2}f_2\left(\frac{\ell}{h}\right)\right);\;\; K_I = T_u\frac{kE\alpha}{12(1-\mu)}\left(h^{1/2}f_1\left(\frac{\ell}{h}\right)\right) \tag{7}$$

$$f_1\left(\frac{\ell}{h}\right)\approx 4.2\left(\left(1-\frac{\ell}{h}\right)^{-3}-\left(1-\frac{\ell}{h}\right)^{3}\right)^{1/2};\; f_2\left(\frac{\ell}{h}\right)\approx\frac{1.11+5(\ell/h)^4}{1-\ell/h}$$

where ℓ is the crack length (along the ice cover thickness), h is the ice thickness, and f_1, f_2 are the functions which approximate the numerical results given by *Rice* (1975).

We use the condition of the crack limit equilibrium in the form which is usual for the linear elastic fracture mechanics (LEFM)

$$K_I = K_{Ic} \tag{8}$$

Some estimates of the critical length of the thermal crack in the ice cover are given in Fig. 1. The dotted zone shows transition from usual strength criterion to the criterion of LEFM (the inherent flaw is related to the transition). In both cases the initiated edge crack grows through the whole ice thickness. This circumstance is important for the fracture mechanics of the ice cover. It means that the water can fill the thermal quasistationary through cracks in the ice cover. The water freezing preserves the thermal deformations in the whole ice cover. Hence, large compressive stresses occur in the ice cover plane at the subsequent variation of the temperature regime and ice expansion. These stresses can cause ice cover ridging or damage of structures being in the ice cover.

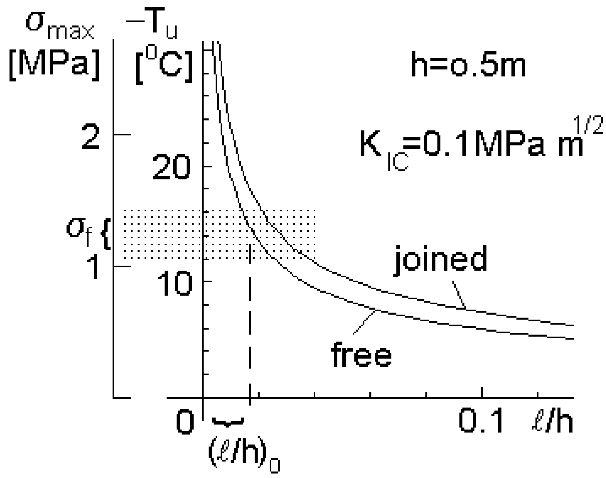

Fig. 1. Thermal cracks in the ice cover; (ℓ / h_0) - the relative value of the inherent flaw.

In this connection, note that the crack trajectory is influenced by the reservoir shape. In particular, the effect of fixing the ice cover boundaries is attained in the elongated reservoirs because of the resistance to the ice mass displacement along the reservoir axis. As a consequence thermal cracks in elongated reservoirs are oriented transverse to this axis. This effect is, in particular, observed in the Siberian rivers.

3. *The thermal shock loading of the ice cover*

Consider another asymptotics of thermal fracture of the ice cover related to the fast changing of the ice surface temperature caused by a thermal shock. Such situation can occur at the fast cooling of the surface ice at removing the snow cover or at the sharp variation of the air temperature over the reservoir.

We use the approach suggested by *Goldstein and Osipenko* (1997, 1998).

As a model problem we consider the problem on propagation of a tensile (mode I) crack from the boundary of a half-plane at the thermal shock.

Assume that the thermal shock causes initiation of a series of subparallel cracks (Fig. 2). The cracks follow the stress front advance deep in the body, however, the amount of growing cracks decreases with diminishing the stress level such that the final crack ensemble consists of the cracks of different lengths. Local thermal parameters and bearing capacity of the ice cover depend upon the characteristic relations between the crack sizes and distances between them in the ensemble. Let us estimate these characteristic relations.

We separate an elementary cell with a single crack of a subparallel cracks system. The side boundaries of the cell are located between the adjacent cracks (Fig. 2). By incorporating the compatibility conditions for deformations we can draw the neutral line where the transverse displacements are absent. Hence, to search for the conditions of the crack limit equilibrium we can use the scheme of a crack in a strip such that the loads act at the crack surfaces while the strip boundaries are fixed.

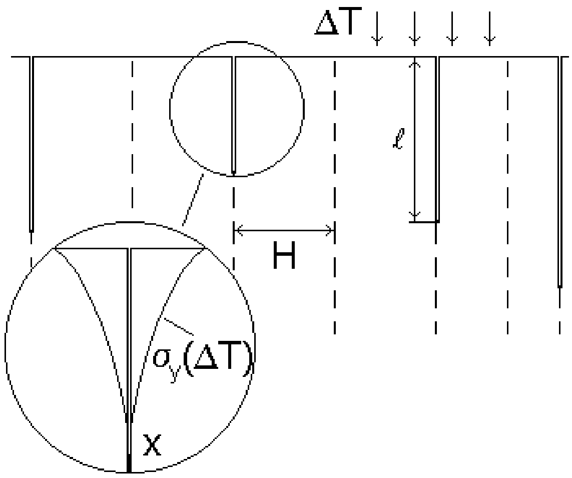

Fig. 2. Scheme of multiple fracture at thermal shock.

According to the compliance method (*Hellan* 1984) we can write for the symmetric loading

$$\left(dU/d\ell\right) = G = K_I^2(1-\mu^2)E^{-1}; \quad K_{II} = K_{III} = 0 \tag{9}$$

where U is the elastic energy released at the crack motion, K_I, K_{II}, K_{III} are the stress intensity factors at tension, transverse and longitudinal shear, respectively.

Modeling the separated strip by a beam we obtain for the elastic energy

$$U = \int_0^{\ell} \frac{\sigma_y^2(x,t)(1-\mu^2)H}{E} dx \tag{10}$$

where $\sigma_y(x, t)$ are the transverse stresses at the crack line, $2H$ is the width of the strip between two adjacent cracks, ℓ is the crack length.

Differentiating Eq. (10) with respect to ℓ and incorporating Eq. (9) we obtain

$$K_I = \sigma_y(\ell,t)\sqrt{H} \tag{11}$$

The thermal stresses along the beam axis are equal to

$$\sigma_y \approx 1/2EH\alpha(dT/dx)/(1+\mu) \tag{12}$$

where $T = T(x, t)$ is the temperature distribution along the beam length.

Consider (for instance) the variant of the thermal shock caused by a temperature jump, ΔT, at the ice cover upper surface (the coefficient of heat transfer $\beta \rightarrow \infty$). In this case (*Kovalenko* 1970) the function $T(x, t)$ equals

$$T = \Delta T \ \mathrm{erfc}\left(x / 2\sqrt{at}\right) \tag{13}$$

where a is the coefficient of thermal conductivity.

We obtain from Eqs (12), (13)

$$\sigma_y(x,t) = -1/2EH\alpha\Delta T \ \exp(x^2/4at)/(1-\mu)\sqrt{\pi at} \tag{14}$$

$$K_I = AH^{3/2}(at)^{-1/2} \exp(\ell^2/4at); \quad A = 1/2E\alpha\Delta T/(1-\mu)\sqrt{\pi} \tag{15}$$

Assume that K_I time variation equals its maximal value, i.e. it is related to the maximal energy dissipation. Then

$$t = \ell^2/2a \tag{16}$$

and the value $K_{I\max}$ equals for this regime

$$K_{I\max} \approx AH^{3/2}\sqrt{2}/\sqrt{e}\,\ell \tag{17}$$

where e is the base of natural logarithm.

Using the condition of the crack limit equilibrium (Eq. (9)) $K_{I\max} = K_{Ic}$ we obtain the crack size as a function of a distance between them within the crack system under consideration

$$\ell \approx \left(AH^{3/2}/K_{Ic}\right)\sqrt{2/e} \tag{18}$$

We can use Eq. (18) to obtain other system parameters, e.g., relative crack density. For this aim we assume that the maximal possible crack length correlates with the ice cover thickness $\ell_{\max} \sim h$. Then the distance between the cracks of maximal length equals

$$H_{\max} \approx \left(hK_{Ic}/A\right)^{2/3}\left(e/2\right)^{1/3} \tag{19}$$

and the estimate of the general crack density has the form

$$n_\varepsilon \sim \int_0^h H^{-1}(\ell)d\ell = \frac{1}{3}\left(\frac{2h}{e}\right)^{1/3}\left(\frac{A}{K_{Ic}}\right)^{2/3} \tag{20}$$

The relative crack density (the ratio of amount of cracks of length exceeding a certain value to amount of all cracks) can be represented as follows

$$\frac{n(\ell > \ell_0)}{n_\varepsilon} \approx \frac{I(\ell_0,h)}{I(0,h)} \approx 1-\left(\frac{\ell_0}{h}\right)^{1/3} \quad ; \quad I(c,d)=\int_c^d H^{-1}(\ell)d\ell \tag{21}$$

Note that the thermal parameters of the material are absent in the obtained relations. Moreover, the relation for the relative crack density has an universal character and is determined by the temperature distribution function. One should bear in mind that the given relations provide an upper estimate of the crack density, since using Eq. (19) to characterize the whole ensemble we assumed that the effective element (beam) of larger thickness consists of the smaller size elements. We did not take into account that a part of the elastic energy in the vicinity of larger cracks is spent on creation of small cracks.

Let us estimate the role of thermal cracks in other processes. Being small the main part of cracks formed at a thermal shock are not filled by water and continue to be open during the relaxation process. Hence, the crack surfaces can play the role of an active boundary of heat exchange between atmosphere and ice cover. This effect, in particular, has an influence on the rate of ice accumulation. We represent the integral specific heat flux at the boundary atmosphere-ice cover, q, as follows

$$q \approx q_0\left[1+m(\Delta S/S_0)\right] \tag{22}$$

where q_0 is the heat flux from the ice cover surface free of cracks, S_0 is the initial area of heat exchange, ΔS is the total surface of thermal cracks, and $m \le 1$ is a coefficient taking into account the heat exchange character in thin cracks.

We can estimate the value ΔS for the plane model as follows

$$(\Delta S/S_0) \sim 2\left(n_{(\ell>\ell_{\min})} - n_{(\ell>\ell_{\max})}\right) \tag{23}$$

Taking account of Eqs (19), (21) lead to the following estimate

$$(\Delta S/S_0) \sim 2/3(2/e)^{1/3}(A/K_{Ic})^{2/3}\left(\ell_{\max}^{1/3} - \ell_{\min}^{1/3}\right) \tag{24}$$

Assume that $\ell_{\min} << \ell_{\max}$, then we can evaluate the increment of the heat flux caused by an influence of thermal cracks

$$q - q_0 \approx 2/3 q_0 m(2h/e)^{1/3}(A/K_{Ic})^{2/3} \tag{25}$$

To estimate the rate of ice accumulation we use formula given in (*Pehovich* 1983)

$$v = (dh/dt) = \left(q - q_{w/i}\right)L^{-1} \tag{26}$$

where v is the rate of ice thickness increasing, q is the heat flux from the ice cover to atmosphere, $q_{w/i}$ is the heat flux from water to the lower surface of the ice cover, and L_v is the phase transition heat (melting heat).

Some estimates of the influence of thermal cracks in the ice cover on the relative rate of ice accumulation (such that water temperature is close to the freezing temperature; $q_{w/i} \to 0$) are given in Fig. 3 as we do not take account of the influence of the snow cover

$$(\Delta v / v) \approx kmh^{1/3}\left[E\alpha(\Delta T)/K_{Ic}(1+\mu)\right]^{2/3},\ k \sim 0.26 \tag{27}$$

Occurrence of open cracks (leads) increases the effective area of heat transfer with atmosphere and, hence, increases the rate of freezing. Increasing the air temperature to the melting period leads to occurrence of compressive stresses in the surface layers of ice. The crack closure diminishes the heat transfer.

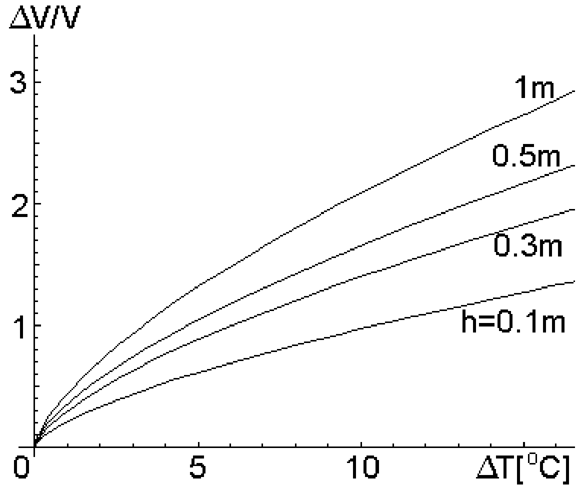

Fig. 3. Interrelation of the increasing of the relative velocity of ice thickness growth and the temperature jump, ΔT, for the ice cover with a system of thermal cracks (K_{Ic} = 0.1 MPa$\cdot m^{1/2}$; $m \sim 1$; E = $5\cdot10^{3}$MPa; $\alpha = 6\cdot10^{-5}$ 1/grad; $\mu \sim 0.3$; $q_{w/i} \to 0$)

4. *Cracking in the whole lake*

Consider the growth of thermal cracks in the ice cover of an isometric reservoir within the assumption that the ice cover contour is not joined to the shore line. The steady thermal stresses caused by the uniform cooling of the homogeneous ice cover create uniform tensile stresses in a surface part of the ice cover. The crack density and trajectory can be related to small variations of stresses. The circulation in conjunction with the heat transfer in the stratified thickness of water leads to occurring of a temperature difference between the central and peripheral zones of the lake at the ice cover surface part. In turn, the temperature difference causes additional tensile hoop stresses in the peripheral zone of the ice cover on the lake. We evaluate an influence of these additional stresses on the thermal cracking of the ice cover.

For the sake of simplicity we write the quasisteady distribution of temperature along the disk shaped ice cover in the following parabolic form

$$T(r) = T_0 - \Delta T(r/R)^n \tag{28}$$

where T_0 is the temperature in the central part of the disk, R is its radius and ΔT is the temperature difference between the central and peripheral zones of the ice cover.

A system of radial cracks is initiated at the ice cover surface at multiple cracking. These cracks subdivide the ice cover on a system of wedge shaped beams. The hoop stresses in such beams equal (similarly to Eq. (13)) to

$$\sigma_\theta \approx \frac{E\Delta T H_0 \alpha n}{2(1+\mu)R}\left(\frac{r}{R}\right)^n + \sigma_0(x) \tag{29}$$

where H_0 is the beam width at the disk periphery, $\sigma_0(x)$ are the steady uniform thermal stresses. Conditions of the limit equilibrium of a crack in a thin wedge shaped beam have the following form (similarly to Eq. (12)):

$$K_{Ic} \approx \sigma\sqrt{H}\ ; \quad H = H_0(r/R) \tag{30}$$

If the steady stresses are small we obtain from Eqs (30) and (31) the relation between the parameter H_0 and the length of the equilibrium radial crack ℓ (Fig. 4):

$$H_0 \approx \left(2K_{Ic}R(1+\mu)/nE\Delta T\left(1-(\ell/R)\right)^{n+0.5}\right)^{2/3} \tag{31}$$

That is the cooling of the circular lake periphery leads to formation of cracks of length comparable with the basin radius (Fig. 4). These cracks separate the basin area on small amount of weakly joined large fragments. This feature can be used at the estimate of ice cover evolution under the action of other sources of loading (wind loading, winter navigation, etc.).

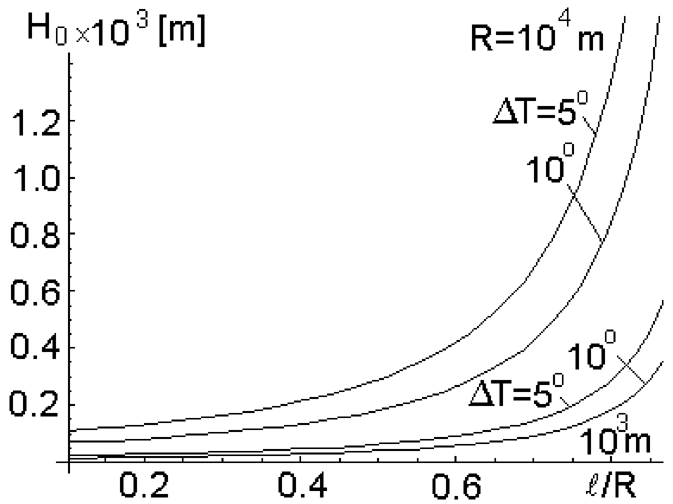

Fig. 4. Interrelation between the crack mouths at the outer contour of an ice cover disk, H_0, and the relative length of the radial cracks, ℓ/R, for the disk-shaped ice cover of radius R (K_{Ic} = 0.1 MPa$\cdot m^{1/2}$; n = 2; E = $5\cdot10^3$MPa; $\alpha = 6\cdot10^{-5}$ 1/grad; $\mu \sim 0.3$).

Similarly to Eq. (22) we can find the interrelation between the relative crack density and their length. In particular, the relative amount of cracks of length larger than a fixed value can be evaluated as follows

$$\frac{n(\ell > \ell_0)}{n_\varepsilon} \approx \frac{J(\zeta_0,1)}{J(0,1)} \approx \left(1 - \frac{\ell}{R}\right)^{8/3} \; ; \; J(a,b) = \int_a^b H_0^{-1}(\zeta) d\zeta \; ; \; \zeta = \frac{\ell}{R}; \; n = 2 \tag{32}$$

Similar estimates for conditions of the inversion of the temperature field in spring show that multiple fracture of the ice cover by radial cracks nucleating in the central zone of the lake (reservoir) is possible.

Remind, that the variations of the steady temperature fields are small and, hence, the distances between the cracks are larger as compared with the case of the thermal shock. Further, the small value of thermal stresses do not provide conditions for nucleation of cracks on the basis of the local ice strength criterion. Crack initiation can be provided by existence of local defects in the ice cover. Hence, the possible range of crack lengths in the system of multiple cracking becomes narrower. Note, that one can classify the state of ice cover in the basins relative to its propensity to one of two following fracture types: 1) fracture initiation within the surface layer of the ice cover after attaining the thermal stress level determined by the ice tensile strength σ_f; or 2) thermal shock initiation of growth of pre-existed cracks (faults). Different approaches to such classification can be suggested (see, e.g., *Liu and Fleck*, 1998).

We will compare the limit states using the estimate of the relative critical temperature jump. Let us introduce the dimensionless parameter N, which characterizes the tendency to these fracture types, as a ratio of the temperature jump needed for attaining the limit stress, σ_f, at the ice cover surface to the jump which causes initiation of growth of cracks of depth ℓ with their spacing $2H$. By incorporating Eqs. (1) and (18) we obtain

$$N = m_* \left(\sigma_f H^{3/2} / K_{Ic} \ell\right), \quad m_* = \left(1/\sqrt{2\pi e}\right) \sim 0.25 \tag{33}$$

The first type of fracture dominates if $N < 1$ while the second at $N > 1$.

Note, that Eq. (33) is fulfilled within the framework of the aforementioned assumptions about the thermal shock at the surface of the ice cover. The ice cover of different basins can be characterized by the presence of surface cracks or cracklike defects or their systems. The ice properties and loading conditions adjust the specific scenario of the fracture process at the same existing surface crack (defect) system. For instance, the critical value of the parameter $N \approx 1$ is attained in the case described by Eq. (33) for $\ell \sim 0.05$ at typical values of the fracture toughness $K_{Ic} \sim 0.1 MPa \cdot m^{1/2}$ and tensile strength $\sigma_f \sim 1$ MPa (*Bogorodski and Gavrilo* 1980, *Liu and Fleck* 1998). Hence, new cracks will be formed in the ice cover if the length of the existing defect (crack) is less than $2H = 0.14$m.

5. Conclusion

The presented results show that a series of thermomechanical processes exists which can lead to fracture of ice cover in bounded basin. These processes can be subdivided on global (at the scale of the basin) and local (at the scale of ice thickness). The global processes adjust the geometry of violations of ice cover in whole and creation in ice cover horizontal loads acting, in particular, on different structures frozen in ice cover as well as on dams. The local processes lead to formation of a system of multiple cracks and influence on local heat exchange and strength of ice under the action of local loads, in particular, related to contact loading. The observed structures of thermal cracks in ice cover can serve as an indicator of thermomechanical processes in ice basin.

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Appendix: Bearing capacity of ice cover

Open thermal cracks essentially weaken the bearing capacity of ice cover. In particular, an interrelation occurs between the critical specific load and area where this load is distributed.

As an example we consider the conditions of ice cover breach under the action of the uniform load of intensity P distributed on an isometric area element (Fig. A.1). Assume that breach is caused by the shear loads acting along the area boundary and that the characteristic size of the area correlates with the distance between the thermal cracks of certain length. These cracks form the loaded area element.

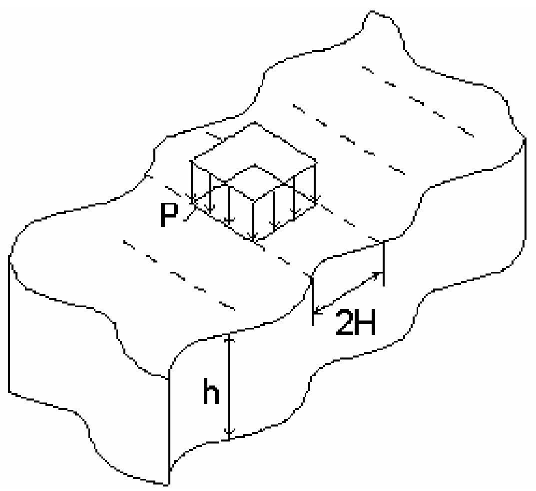

Fig. A1. Scheme of loading of an ice cover plate with a system of parallel crack at contact fracture.

It is clear from the preceding analysis that the maximal size of the area is related to a distance between the cracks of length commensurable with the ice thickness. Then the bearing capacity of the ice cover is created by its buoyancy.

On the other hand, the breach mechanism for small area elements and crack depth small as compared with the ice thickness is rather related to the bending of the ice cover on the hydraulic foundation than to the ice shear. Hence, our estimates correspond to the area elements and cracks of an intermediate size.

We represent the critical breach conditions as follows

$$P \cdot F \approx \left(h - \ell \frac{L_1}{L} \right) L \cdot \sigma^* + Fgh(\rho_1 - \rho_2) \quad , \quad \ell \leq h \tag{A.1}$$

where F is the loaded ice cover area, L is its contour length, L_1 is the length of the contour part with a crack, σ^* is the shear strength, ρ_1 and ρ_2 are the densities of water and ice, respectively.

We obtain from Eq. (A.1) for an isometric area element ($F \sim (2H)^2$; $L \sim 8H$)

$$P \approx \frac{2\sigma^*}{H}(h-\ell)+gh(\rho_1-\rho_2); \quad \ell \le h \tag{A.2}$$

where H is the half-width of the loaded area (the half-spacing between the cracks). Using Eq. (18) we obtain the following interrelation between the area size and its specific bearing capacity in case of thermal cracks presence,

$$P \approx 2\sigma^*\left(\frac{h}{H}-\frac{L_1}{L}\frac{A\sqrt{H}}{K_{Ic}}\sqrt{\frac{2}{e}}\right)+gh(\rho_1-\rho_2) \quad , \quad \ell \le h \tag{A.3}$$

Note, that the value of the critical load P readily decreases with increasing the area size (Fig. A.2).

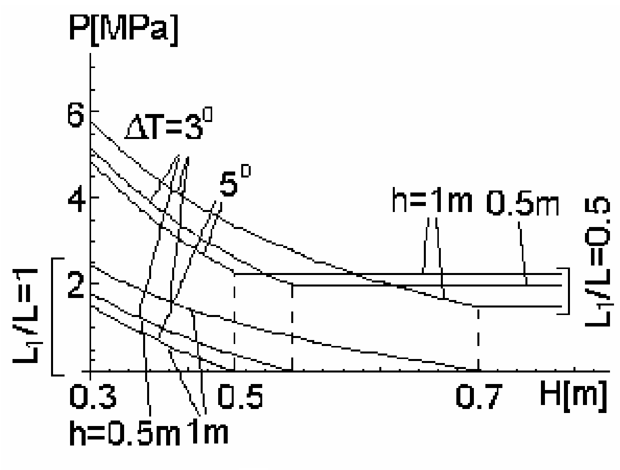

Fig. A2. Example of an estimate of ice cover bearing capacity at a variation of the loaded zone size.

The parameter (L_1/L) characterizes a part of the contour length of the loaded area containing the thermal cracks. The remaining part of the contour is assumed to be free of damage and its fracture at breach is evaluated using the average cleavage stresses in the section of the ice plate. For instance, (L_1/L) ~ 0.5 if the thermal cracks form a system of parallel cracks (due to deformation constraint at the lake periphery), since the cracks can outline the loaded area along two opposite sides, while two other sides (Fig. A.1) remain conditionally unloaded. In this case the bearing capacity of ice on the area of width $2H$ is influenced by thermal cracks during the period preceding the moment when the cracks become the through ones. Then the residual bearing capacity is only supported on the area sides being free of thermal cracks.