

Non-linear Stress-strain Relation in Sedimentary Rocks and Its Effect on Seismic Wave Velocity

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(Received: January 2004; Accepted: February 2005)

Abstract

Character of the stress-strain relation $\sigma(\varepsilon)$ of sedimentary rocks depends on the strain level in a range of $\varepsilon \sim 10^{-6} \div 10^{-3}$. Nonlinearity and a hysteresis in the $\sigma(\varepsilon)$ are caused by microplasticity. Static and dynamic elastic modulus and seismic velocity may either increase or decrease with strain. Microplastic strain of saturated rock considerably grows and this results in nonlinear stress-strain relation (equation of state). Therefore, curvature of $\sigma(\varepsilon)$ is explained by microplasticity. In this case the seismic velocity decreases with strain, if the curvature of $\sigma(\varepsilon)$ is negative, while it increases with strain, if the curvature of $\sigma(\varepsilon)$ is positive. In our paper we experimentally show that the longitudinal wave velocity increases with increasing strain amplitude for a dolomite having a positive curvature in the $\sigma(\varepsilon)$. Therefore, stress-strain relation $\sigma(\varepsilon)$ received for large deformations cannot be used for modeling of nonlinear wave propagation at intermediate and small strain levels. Model for nonlinear wave propagation should take into consideration small-strain relations $\sigma(\varepsilon)$.

Key words: the stress-strain relations, microplasticity, hysteresis, the strain-amplitude dependence, nonlinear wave propagation

1. Introduction

Nonlinear effects in rocks are observed at moderate and even small strain levels ($\varepsilon > 10^{-6}$) (Winkler *et al.*, 1979; Mashinsky, 1994; Johnson *et al.*, 1996; Zinzner *et al.*, 1997; Tutuncu *et al.*, 1998a; Xu *et al.*, 1998; Mashinskii and D'yakov, 1999). These effects are caused by nonlinearity and hysteresis of a stress-strain relationship $\sigma(\varepsilon)$. Therefore, studying of the $\sigma(\varepsilon)$ is very important. It defines the equation of state and it is the principal theoretical component in static and dynamic studies.

Dependencies $\sigma(\varepsilon)$ are received from laboratory quasi-static measurements or in situ for large deformations, that is, in the near source region (Boitnott, 1993) and rarely for small ones (McKavanagh and Stacey, 1974). However, small deformations are of a great interest in seismic prospecting and seismology. Direct measurements of the stress-strain relationship $\sigma(\varepsilon)$ have shown that physical nonlinearity of this relationship is caused by microplasticity of rocks (Mashinsky, 1994). Microplasticity essentially changes representation of elastic and nonelastic behavior of rocks and explains dependence of the elastic modulus on strain and the non-closed hysteresis in a simple way. Physical mechanisms of rock's microplasticity may be the same as in metal polycrystals

(movement of dislocations) or any others. Microplasticity does not exclude known mechanisms (stick-slip friction, grain contact adhesion hysteresis, discrete memory) (*Stewart et al.*, 1983; *McCall and Guyer*, 1994; *Johnson et al.*, 1996; *Tutuncu et al.*, 1998b; *Xu et al.*, 1998).

The principal theoretical component in static and dynamic studies is the stress-strain relationship $\sigma(\varepsilon)$. A negative curvature in the $\sigma(\varepsilon)$ corresponds to decrease of the static modulus with stress, while a positive curvature in the $\sigma(\varepsilon)$ corresponds to increase of this modulus with stress (*McCall and Guyer*, 1994). If similar rule acts for the dynamic stress-strain relationship $\sigma(\varepsilon)$, then the dynamic modulus (wave velocity) will also follow this rule, i.e. the modulus will either decrease or increase with stress amplitude depending on character of the curvature.

A possible physical reason of such behaviour of the elastic modulus is as follows. Consider total strain that consist of three main components (*Mashinskii*, 2003):

$$\varepsilon_i = \varepsilon_{i-e} + \varepsilon_{v-e} + \varepsilon_{\mu} \quad (1)$$

where the component ε_{i-e} represents an ideally elastic (“atomic” elastic) Young's modulus $E_{i-e} = \Delta\sigma_i / \Delta\varepsilon_{i-e}$, corresponding to deformation of the monocrystalline grains of rock skeleton. Second and third terms in (1) represent the anelastic component. The strain ε_{v-e} corresponds to viscoelastic behavior of rocks dependent on the magnitude and time of stress. For example, in a Maxwell model, viscoelastic strain (for $\sigma = \text{constant}$) is

$$\varepsilon_{v-e} = \frac{\sigma t_{\sigma}}{\eta_{ef}} = \frac{\varepsilon_i E_i t_{\sigma}}{\eta_{ef}} = \frac{\varepsilon_i t_{\sigma}}{T_{rel}}, \quad (2)$$

where t_{σ} is the time of the stress duration; η_{ef} is an effective viscosity; $T_{rel} = \eta_{ef} / E_i$ is the relaxation time, $E_i = \Delta\sigma_i / \Delta\varepsilon_i$ is instantaneous Young's modulus.

The ε_{μ} is microplastic strain; it is residual deformation. On the one hand, the microplastic strain is the time-independent deformation, however, on the other hand, it is the amplitude-dependent deformation. Instantaneous (local) Young's modulus is given by

$$E_i = \frac{\Delta\sigma_i}{\Delta\varepsilon_{i-e} + \Delta\varepsilon_{v-e}(t_{\sigma}) + \Delta\varepsilon_{\mu}(|\varepsilon|)} \quad (3)$$

where $\Delta\varepsilon_{v-e}(t_{\sigma})$ is the time-dependent viscoelastic component and $\Delta\varepsilon_{\mu}(|\varepsilon|)$ is the strain-dependent microplastic component (with variable sign). The subscript index μ is microplasticity.

Studying of the anelasticity of many rock types using stress-strain relationship $\sigma(\varepsilon)$ has shown that the behavior of anelastic component (residual strain) is different: a negative curvature of $\sigma(\varepsilon)$ takes place often, while a positive curvature of it is seldom observed (*Mashinskii*, 1989; 1994; 2001). A negative curvature of $\sigma(\varepsilon)$ means that the

elastic modulus decreases with stress and a positive curvature means that it increases with stress. It follows from the equation (3).

Usually the components $\Delta\varepsilon_{i-e}, \Delta\varepsilon_{v-e}(t_\sigma), \Delta\varepsilon_\mu(|\varepsilon|)$ increase with increasing stress for the most rock types. Therefore, the elastic modulus E_i in (3) decreases. However in individual rocks the anelastic component (at least microplastic component) can decrease with stress. It takes place, for example, in dolomite and argillite (*Mashinsky, 1994*). The decrement of anelastic component $\Delta\varepsilon_{anelastic} = \Delta\varepsilon_{v-e}(t_\sigma) + \Delta\varepsilon_\mu(|\varepsilon|)$ means that the modulus E_i in (3) increases. The decrement of $\Delta\varepsilon_{v-e}$ is possible if the relaxation time T_{rel} depends on the stress. Then the value $\Delta\varepsilon_{v-e}(T_{rel})$ ($t_\sigma = const$) decreases and the modulus E_i increases with stress under condition that the increment of T_{rel} occurs as well. This is just a hypothesis and an additional study is required to test it. On the contrary, the microplastic strain can decrease with increasing stress and, consequently, the modulus can increase owing to microplastic component. The non-standard behaviour of the modulus under unloading is possible as well owing to microplasticity. The classic viscoelastic mechanism does not suppose such behavior.

There is an explanation of a difference between measured static and dynamic Young's moduli. It is caused by different anelastic contributions to the stress-strain relationship that behaves as a function of strain amplitude and frequency (energy and strain rate). The increment of frequency, that is, the speed deformation leads to the decrement of the time-dependent viscoelastic component ($\Delta\varepsilon_{v-e}(t_\sigma)$). Then according to the equation (3), the modulus E_i increases with increasing frequency. As the microplastic component ($\Delta\varepsilon_\mu(|\varepsilon|)$) is time-independent, it does not contribute to deformation, if strain (stress) does not change. However, the microplastic component contributes to deformation, if the strain changes. The microplastic component can either increase or decrease with strain. The decrement of the microplastic contribution with increasing strain is possible, owing to the "resorptional" (diffused) effect. It leads also to the increment of the modulus with increasing strain (*Mashinsky, 1994*).

Microplastic contribution in the total strain varies with strain level and strongly influences $\sigma(\varepsilon)$. The stress and strain change in wide range at propagation of seismic waves. Therefore it is important to analyse the relationship $\sigma(\varepsilon)$ in various deformation ranges and compare saturated and dry rocks. As seismic wave velocities and attenuation depend on the strain amplitude for a wide variety of rocks, non-linear and non-unique stress-strain relations can affect these parameters. Some results received in this direction are presented in this paper.

2. *Technique of research*

The experimental curves $\sigma(\varepsilon)$ were analyzed in a range of $\varepsilon \sim 4 \cdot 10^{-6} - 10^{-3}$. The purpose of experiments was tracing of changes of $\sigma(\varepsilon)$ and of the elastic moduli with change of the strain level. The three-dot bend technique and discrete loading - unloading was used (*Tushinsky and Plochov, 1985; Mashinsky, 1994*), Figure 1. The sample 1 is a rectangular parallelepiped (length $l = 70\text{mm}$, width $b = 5-10\text{mm}$, thickness $h = 2-$

5mm) that lays on two fulcrums 2. Force F as loads of various weight is applied in the middle of a sample. The deflection of a sample f is measured with a help of an optical microscope 4. An easy long probe 3 is fixed at an end face of a sample for increase of sensitivity of measurements. Length of optical lever L is 10 times more than a half of length of a sample l . There is a small head with a vision line as a triangle on the end of the probe. The optical eyepiece-micrometer is established opposite the vision line. Readings on a microscope scale are made by eye. Small deflection ($\alpha < 3^\circ$) and the relation $l/h \geq 7$ provide the deformation of one-axis stretching. The maximal deflection f_{max} does not exceed 0.3mm and the deviation of a vision line is $H_{max} = 10f_{max} = 3\text{mm}$. The constant loading was put for imitation of lithostatic pressure of 1.5MPa. Speed of loading in quasi- static regime was maintained as a constant and corresponded to a seismological frequency range ($\sim 0.01\text{-}2\text{Hz}$).

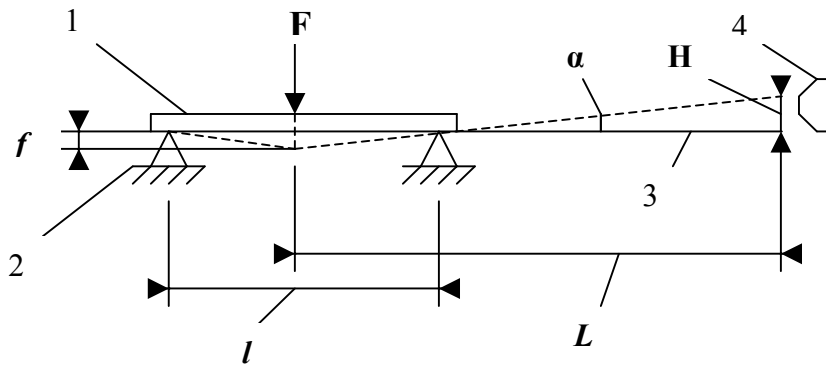


Fig. 1. Experimental scheme. 1 - a sample; 2 - basic prisms; 3 - the lengthening stake with a head at an end; 4 - an eyepiece-micrometer (microscope); F - upload force; f - a deflection of a sample; α - a deflection angle of stake; H - a deviation of vision mark; l - length of a sample; L - the optical lever.

The stress and strain in a sample is calculated using formulas (Mashinsky, 1994):

$$\sigma = 3lF / 2bh^2. \quad (4)$$

$$\varepsilon = 0.6hH / l^2. \quad (5)$$

The absolute error of measurement of strains is $\pm 0.5 \cdot 10^{-6}$. It does not exceed 2 % in moderate strain range, and reaches 12 % in small strain range. The diagrams $\sigma(\varepsilon)$ were plotted and values of the Young's modulus were determined using calculated stresses and strains. The determination of the absolute value of the Young's modulus was not put here because our interest was relative change of an average Young's modulus. It was determined from the curve $\sigma(\varepsilon)$ as: $E_i = \partial\sigma_i / \partial\varepsilon_i$. Reproducibility of curves was checked up by a number of duplicating measurements of $\sigma(\varepsilon)$ from identical samples. The rocks were studied from different locations. The duplicating samples were cut out from the same piece of each rock. Deviations of the duplicating curves from some average curve did not exceed several percents. It means that the slopes of the $\sigma(\varepsilon)$

duplicating curves do not differ from one another several percents, i.e. the modulus changes are insignificant.

The important feature of the load-unload technique is the opportunity to separate the components of deformation from each other, i.e. elastic component (convertible) from microplastic one (irreversible). The sample is loaded by steps (discretely: $\sigma_1 > \sigma_2 > \sigma_3 > \dots > \sigma_{\max}$). The total strain ($\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots$) is measured at each step after loading, then the loading is removed and residual (microplastic) strain ($\varepsilon_{\mu 1}, \varepsilon_{\mu 2}, \varepsilon_{\mu 3}, \dots$) is measured. The elastic component ε_{Hi} is defined as a difference between total ε_i and residual $\varepsilon_{\mu i}$ strains ($\varepsilon_{Hi} = \varepsilon_i - \varepsilon_{\mu i}$ etc). Similar operation is made at unloading ($\sigma_{\max} \rightarrow \sigma_1$). Thus, the residual strain is known at each step of loading – unloading.

The object of our study were samples of sandstone, argillite, marl from deposits of Western Siberia, Table 1. Measurements were carried out under normal conditions on “dry” and water-saturated rocks. The estimation of influence of saturation was made qualitatively. Saturation was hygroscopic by vapors of chloride of ammonium. It provides formation of an intergrain liquid (*Dachnov, 1985*). The maximal saturation corresponds to the humidity of 79%, while partial saturation corresponded to humidity of ~ 20%. Saturation was defined by weighing of a sample. It should be remembered, however, that dry samples are not absolutely dry (*Zinszner et al., 1997*). Before measurements, samples have lain for two months during summer time in a box at room temperature.

Table 1. Rock samples used.

Rock type	Number of samples	Depth (m)	Density (kg/m ³)	Average Young's modulus (GPa)	Porosity (%)
Sandstone	6	1200	2.53	4	19
Argillite	4	2800	2.65	13	11.5
Marl	5	2700	2.74	12.3	2.4

The initial diagram $\sigma(\varepsilon)$ was obtained for the $\varepsilon_{\max I} \sim 10^{-3}$. Such a relationship $\sigma(\varepsilon)$ is often referred to as an equation of state, see, for example *Boitnott (1993)*. Then the diagrams were received for lower levels of ε_{\max} and compared to the initial diagram. Diagrams of the second and third strain levels had the maximal strain approximately 10 and 100 times lower than the initial one, i.e. $\varepsilon_{\max III} < \varepsilon_{\max II} < \varepsilon_{\max I}$.

3. Results of experiments

Generally the diagram $\sigma(\varepsilon)$ is close to a linear one for $\varepsilon_{\max I} \sim 10^{-3}$. It shows a behavior of rocks for large strains. However, this apparent linearity does not mean linearity for lower strain levels. Nonlinear deviations may not be seen in the small strain region in large-scale plots. I illustrate it by three examples of marl, argillite, sandstone (Figure 2, 3 and 4, respectively). Diagrams $\sigma(\varepsilon)$ for $\varepsilon_{\max I} \sim 10^{-3}$ are shown on Figure 2a, 3a, 4a. The $\sigma(\varepsilon)$ for $\varepsilon_{\max II}$ are shown on Figure 2b, 3b, 4b and Figure 2c, 3c, 4c is for

$\varepsilon_{\max\text{III}}$ ($\varepsilon_{\max\text{I}} > \varepsilon_{\max\text{II}} > \varepsilon_{\max\text{III}}$). The loading is shown by continuous line, and unloading by dotted one. The deviation from average of the duplicating curves are insignificant. The residual strain after the final unloading ($\Sigma\varepsilon_{\mu}$) is a total microplastic strain for σ_{\max} . The energy of microplastic strain ΔW_{μ} can be calculated as the square of a hysteresis loop, and the elastic energy W_H is equal to the square of the triangle below the hysteresis loop. A total energy is $W = W_H + \Delta W_{\mu}$. The value $\delta = \Delta W_{\mu}/W \sim \varepsilon_{\mu i(\max)}/\varepsilon_{i(\max)}$ defines a microplastic contribution.

Previously, the “exhausting” of residual strain effect is shown with repeated measurements on the same sample (Mashinsky, 1994). The residual strain is not present if a stress level in repeated loading does not exceed the initial one. It appears only at higher stress levels. The unloading modulus E_{un} is always higher than loading modulus E_{lo} , and this can be explained by microplastic influence. The total strain ε_i is higher during loading due to a microplastic component $\varepsilon_{\mu i}$: $E_{loi} = \partial\sigma_i/\partial(\varepsilon_i = \varepsilon_{Hi} + \varepsilon_{\mu i})$. The total strain ε_i at the same stress $\partial\sigma_i$ is less during unloading as it consists only of elastic component $\varepsilon_i = \varepsilon_{Hi}$. Therefore, the modulus $E_{uni} = \partial\sigma_i/\partial(\varepsilon_i = \varepsilon_{Hi}) > E_{loi}$. In other words, the unloading goes without microplasticity by removal of elastic stress with a higher modulus.

From Figures 2, 3, 4 it is seen that average Young’s moduli of marl, argillite and sandstone are not constant on strain level $\varepsilon_{\max\text{I}}$, $\varepsilon_{\max\text{II}}$, $\varepsilon_{\max\text{III}}$. Namely, transition from $\varepsilon_{\max\text{I}}$ to $\varepsilon_{\max\text{II}}$ and then $\varepsilon_{\max\text{III}}$ results in change of average Young’s modulus. The decrement of the strain leads to the decrement of the modulus from 12.7 to 10.5 GPa in marl and from 16.3 to 10.6 GPa in argillite, that is, changes the correspondent moduli by 17% and 35%, respectively. In sandstone the decrement of the strain leads to the increment of the modulus from 2.2 to 5.0 GPa, i.e. it results in significant (2-3 times) increment of Young’s modulus. The value of microplastic contribution δ in marl, argillite and sandstone increases with decreasing strain from 0.07 to 0.67; from 0.05 to 0.4 and from 0,56 to 0,75, respectively. The microplastic contribution for marl and argillite is more pronounced for small strains than for large strains. However, for sandstone the microplastic contribution is relatively large for the whole strain range.

Thus, change of $\sigma(\varepsilon)$ is observed at change of an energy level. The elastic modulus ($E_{sti} = \Delta\sigma_i/\Delta\varepsilon_i$) may either increase or decrease, as Figure 5 shows. Young’s modulus of marl and argillite increases with stress, while Young’s modulus of sandstone decreases with stress. The most significant change of the modulus is observed in sandstone. Its nonlinear change is caused by microplasticity. The increment of the modulus is accompanied by decrement of δ (e.g. marl and argillite). On the contrary, the decrement of modulus is caused by preservation of the large contribution of microplasticity or even by grow of δ with stress, as Figure 5 shows. In general, the microplasticity influences the behavior of the total strain and, consequently, the behavior of modulus.

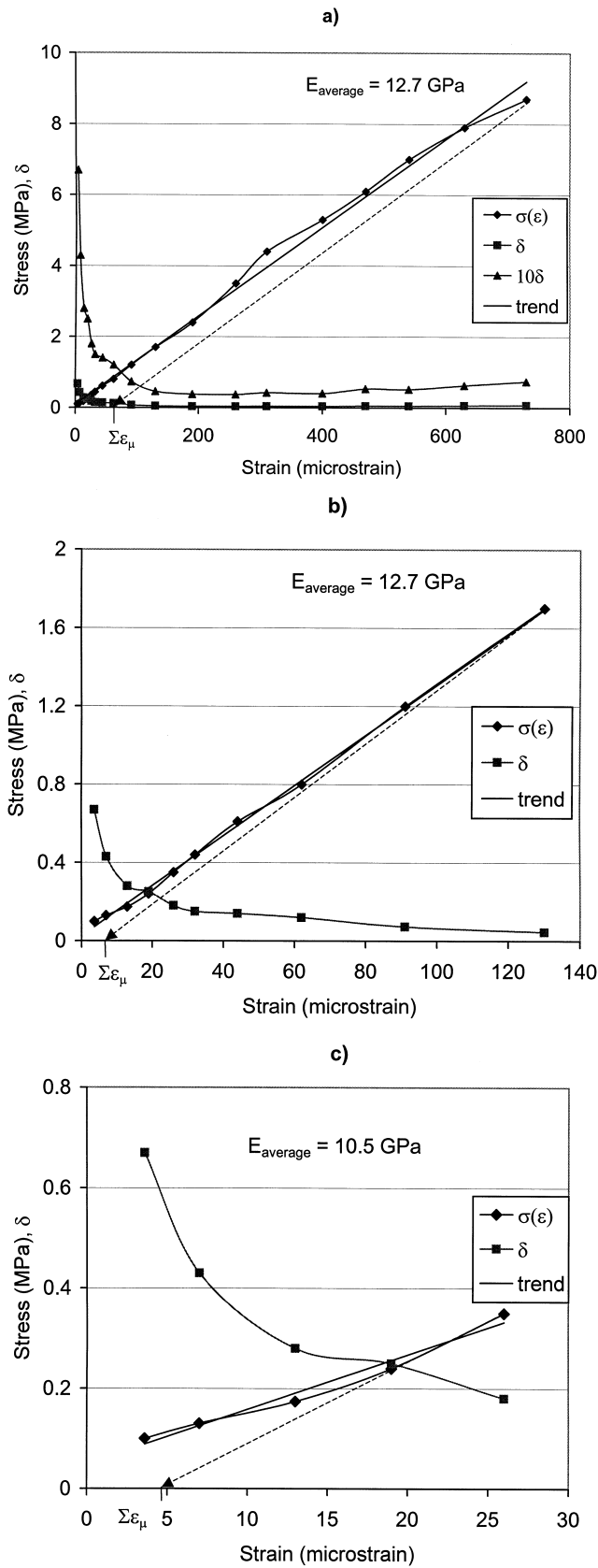


Fig. 2. Diagrams $\sigma(\epsilon)$ for marl from the depth of 2700m; a - ϵ_{maxI} , b - ϵ_{maxII} , c - ϵ_{maxIII} . A continuous line denotes loading, a dotted line denotes unloading.

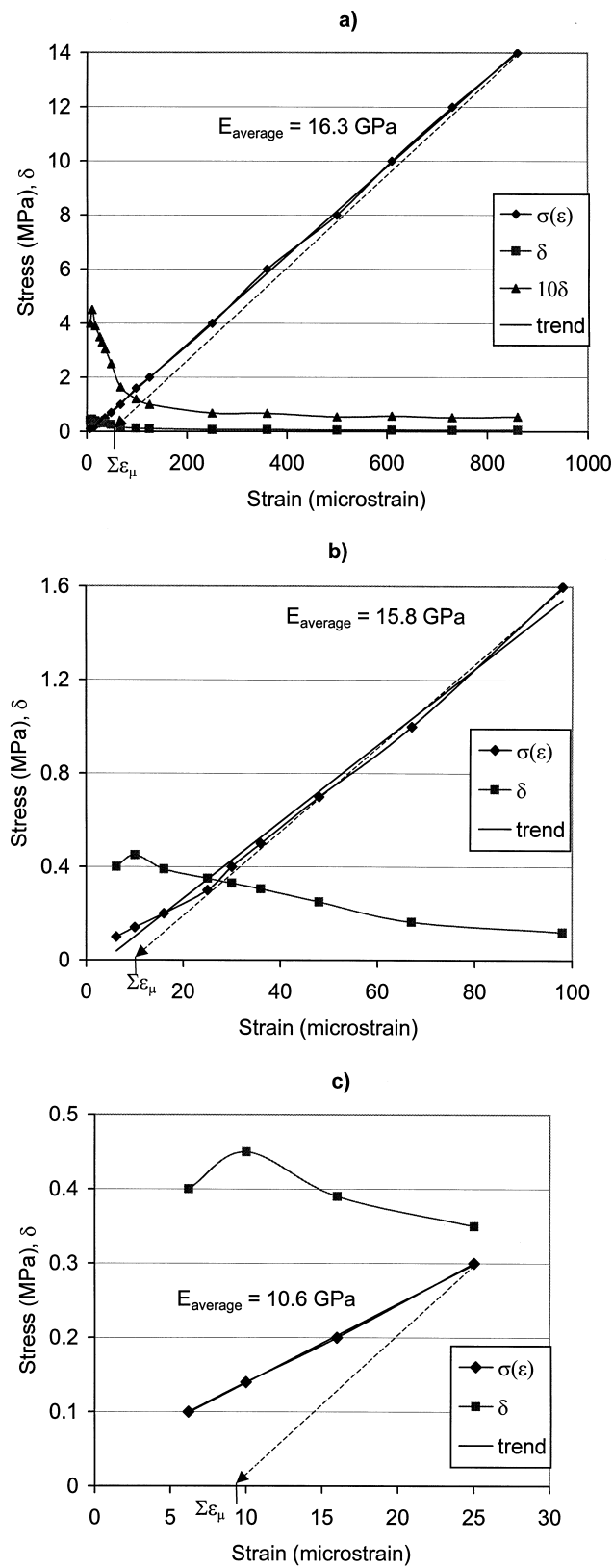


Fig. 3. Diagrams $\sigma(\epsilon)$ of bitumized argillite, depth 2800m; a - ϵ_{maxI} , b - ϵ_{maxII} , c - ϵ_{maxIII} . A continuous line denotes loading, a dotted line denotes unloading.

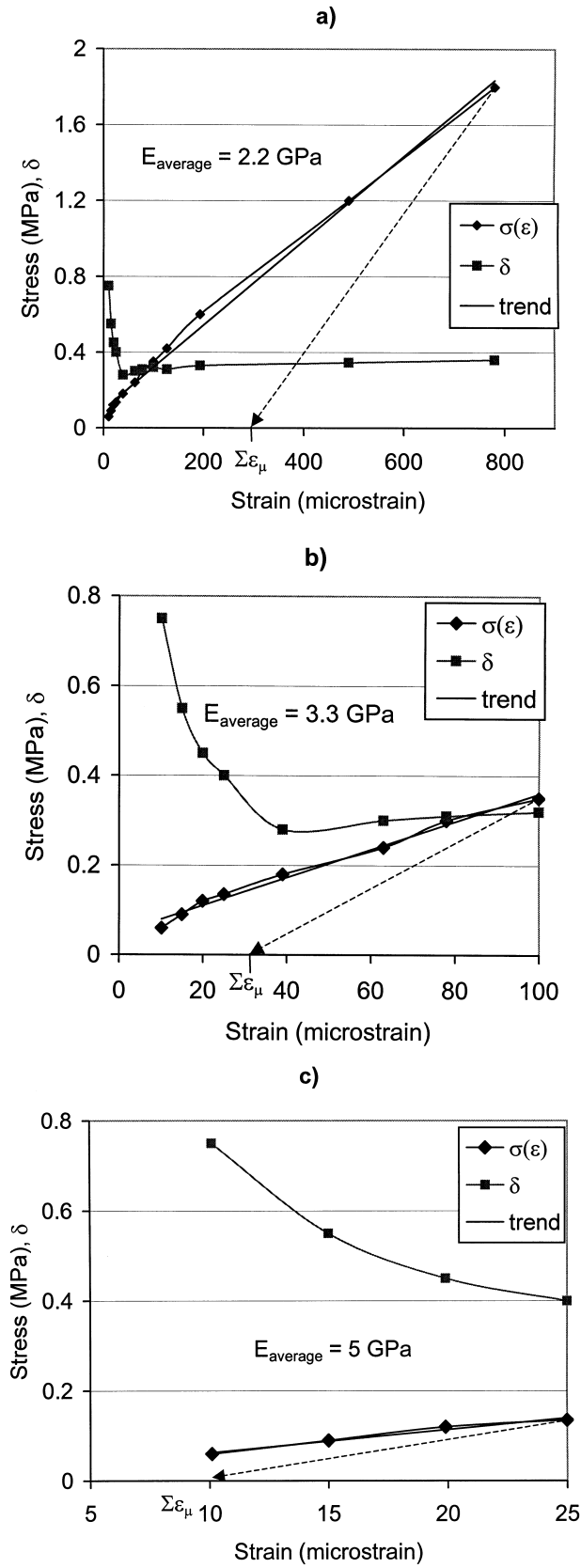


Fig. 4. Diagrams $\sigma(\epsilon)$ for sandstone from the depth of 1200m; a - ϵ_{maxI} , b - ϵ_{maxII} , c - ϵ_{maxIII} . A continuous line denotes loading, a dotted line denotes unloading.

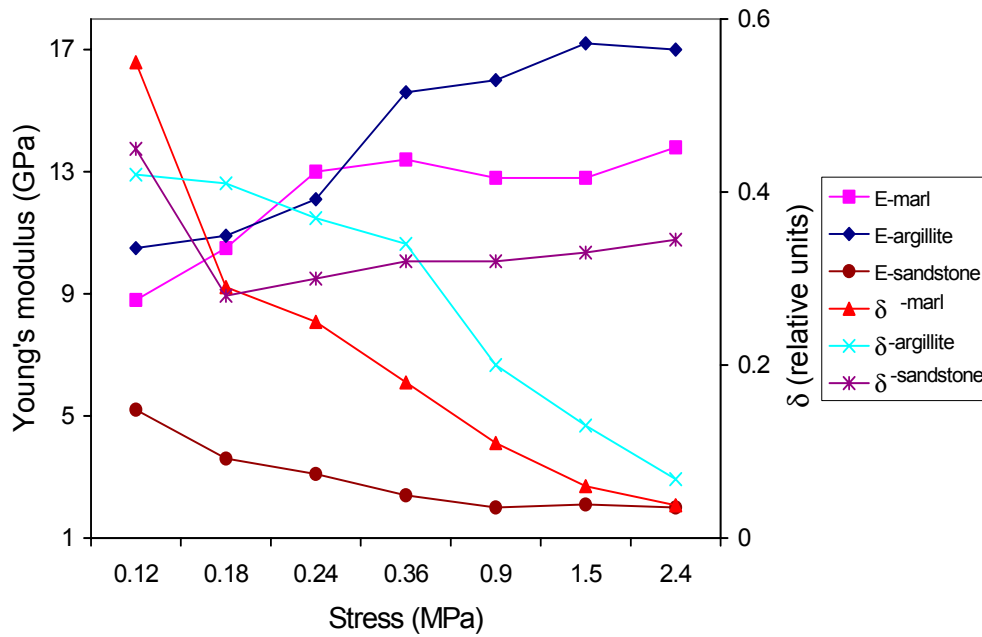


Fig. 5. Dependence of the quasi-static Young's modulus (E) and δ -parameter on stress for marl, argillite and sandstone.

The microplastic strain in rocks grows with a water saturation. Figure 6 presents hysteresis loops for dry and saturated sandstone and alevrolite (*Mashinskii and Zapivalov, 2003*). These data was obtained by the same method as in this paper. The figure qualitatively compares two diagrams, in which the maximum stresses differ in 8 times. In the first case (Figure 6a) the $\sigma(\epsilon)$ is obtained for maximum stress ($\sigma_{\max I}$) achieving 0.56 MPa ($\sim \epsilon_{\max I}$). In the second case (Figure 6b) the maximum stress ($\sigma_{\max II}$) did not exceed 0.07 MPa ($\sim \epsilon_{\max II}$). The shape of the $\sigma(\epsilon)$ is different for different stress levels. The dry sandstone has a narrow and sharp hysteresis loop, for the higher stress level which considerably enlarges with saturation. Dry alevrolite has an ellipse-shape loop that becomes a beak-shaped with saturation. Residual strains increase with saturation. The changes of the hysteresis loops occur also at smaller level and the residual strains are big enough (Figure 6b). The narrow and sharp loop remains in dry sandstone, but the overflow disappears. Saturated sandstone as well as dry alevrolite shows a sharp hysteresis loop. The strongest nonlinear changes are observed in saturated alevrolite at lower stress level. Therefore, for the same rock the character of the hysteresis loop is defined by the stress level and the saturation degree.

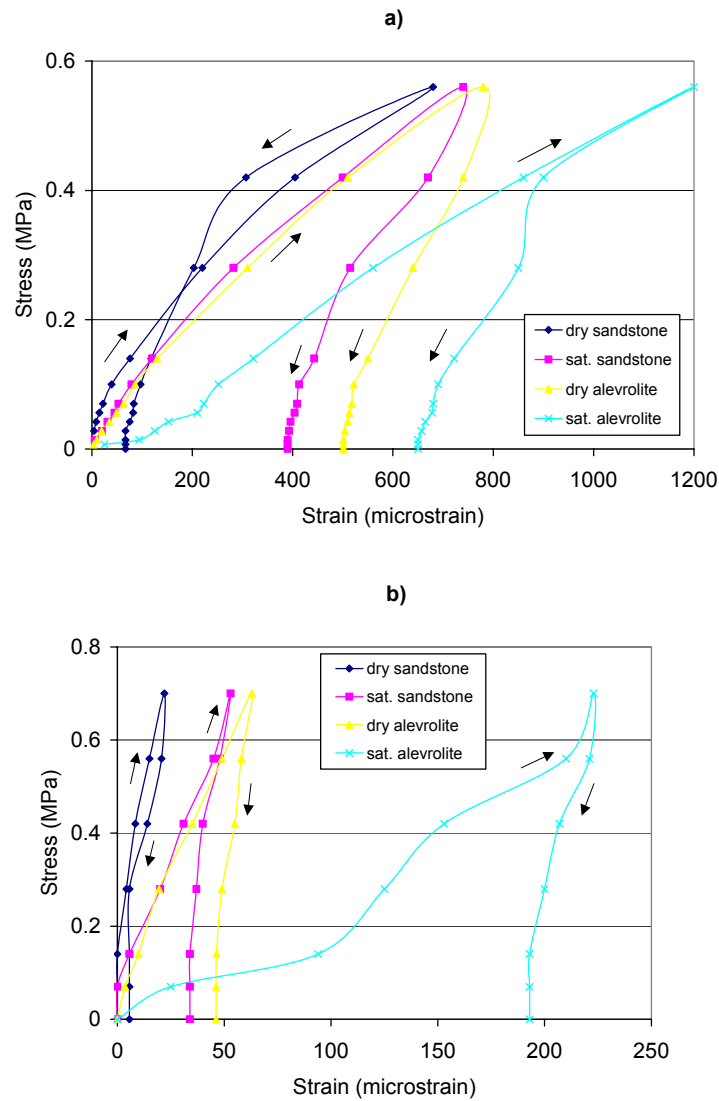


Fig. 6. Hysteresis loops $\sigma(\epsilon)$ of sandstone and alevrolite in dry (dr) and moist-saturated (sat) condition. (a) Stress maximum $\sigma_{\max I} = 0.56$ MPa; (b) Stress maximum $\sigma_{\max II} = 0.07$ MPa. Arrows: loading – unloading.

It is noticeable that the character of the curvature of $\sigma(\epsilon)$ influences the sign of the modulus - stress relation, that is, increment or reduction of the modulus with stress (*McCall and Guyer, 1994*). Therefore, the dependence of seismic wave velocity on curvature of $\sigma(\epsilon)$ can be established. It was established, for example, for a dry dolomite which shows a positive curvature $\sigma(\epsilon)$ (*Mashinskii, 2002*). The compressional wave velocity of Madra dolomites was measured under axial stress of 1-60 MPa. Madra dolomite shows a positive curvature in the $\sigma(\epsilon)$ and grow of quasi-static and dynamic velocities with increasing stress (Figure 7a). Wave velocity increases with amplitude in qualitative conformity with the behaviour of the modulus (Figure 7b). The velocity increment achieved the value of 1.2% for the stress of 5MPa and the strain amplitude range used. It is an unusual result which points out the fundamental importance of the

stress-strain relations in nonlinear model. Usually the velocity decreases with increasing strain amplitude.

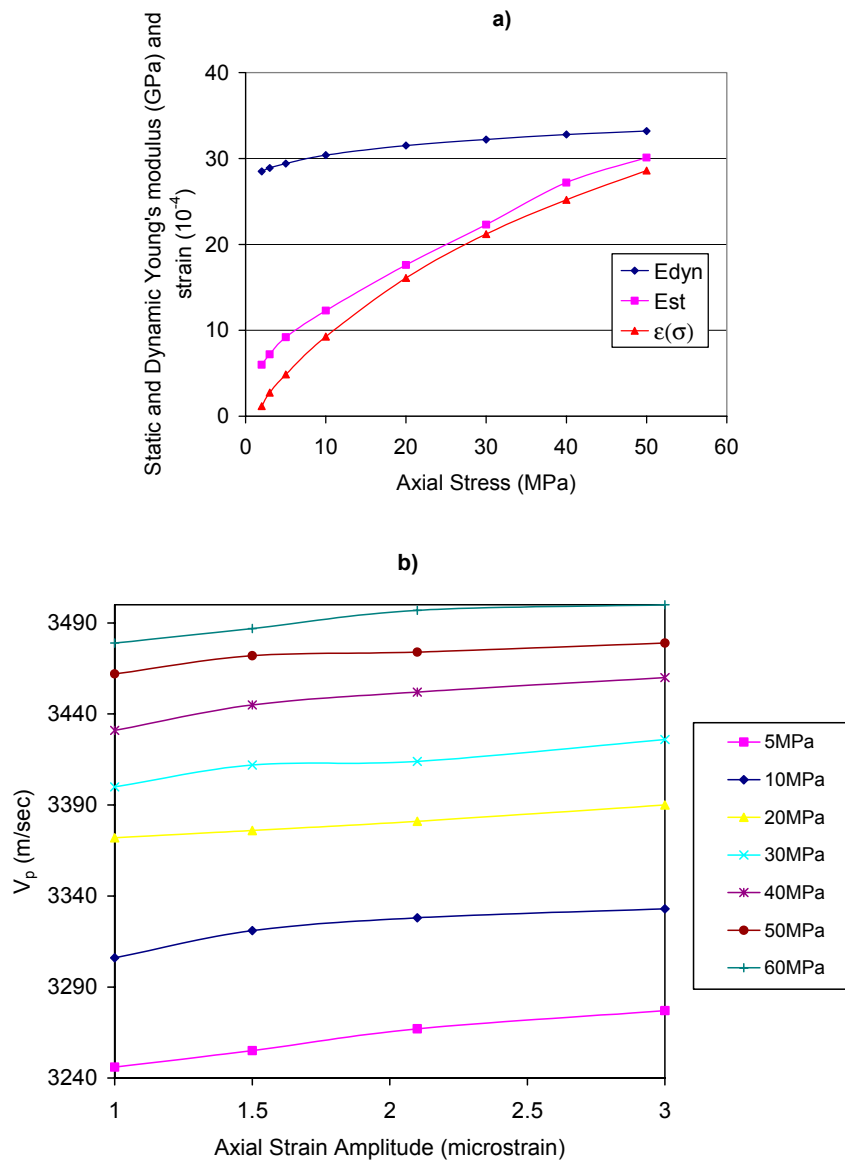


Fig. 7. (a) Dependence of the static (E_{st}) and dynamic (E_{dyn}) Young's modulus on axial stress, and the strain-stress relation $\epsilon(\sigma)$; (b) Dependence of compressional velocity (V_p) on axial strain amplitude on the different stress in Madra dolomite of 2.4% porosity.

4. Discussion of results

The character of the stress-strain relation depends on a energy level. The microplasticity contributes to the nonlinear change of the $\sigma(\epsilon)$. The curvature of the $\sigma(\epsilon)$ depends on the contribution of viscoelastic and microplastic components, which, in turn, depend on the strain range. Saturation of rock increases the destructive role of microplasticity. The consolidated rocks may have a so-called "nought" limit of elasticity when there is a residual strain, but an elastic strain is absent. This effect was earlier no-

ticed only in the soft rocks and loose mediums. Microplastic shifts occur on borders of grains, cracks etc. The thin water pellicles concentrate in these areas and saturation creates favorable conditions for microplasticity.

The dependences of the dynamic modulus (velocity) on the strain amplitude were received in laboratory and field experiments (*Mashinskii and D'yakov, 1999; Mashinskii et al., 1999*) where it was shown that the stress-strain curves may have both convex and concave shape, while static and dynamic modulus increase with strain for a concave curve (argillite) and decrease for convex curve (sandstone). In some previous theoretical and experimental works the investigators have taken no notice of that (*Winkler et al., 1979; Stewart et al., 1983; Tutuncu et al., 1998a,b*). They specify only reduction of a modulus (velocity) on strain amplitude. However, the theoretical work by *McCall and Guyer (1994)* is in agreement with our data.

I suppose that the main reason why our data show a new distinctive result is the difference in lithology and microstructure of the rock used. The most previous experiments were performed on sandstones (Berea, Navajo, Meule, Fontainebleau, Massilon) and rarely on other rocks (limestones, shales, granite). As for dolomite, I have not found any experimental data.

The dependence of seismic wave velocity on the strain amplitude is connected with the attenuation process of elastic waves. Consideration of the attenuation mechanisms (*Spencer, 1981; Stewart et al., 1983; Johnson et al., 1996; Tutuncu et al., 1998b; Xu et al., 1998*) shows that these mechanisms do not take into consideration a microplastic effect. Known mechanisms operate in a presence of a microstructure, whereas microplasticity is possible in without-structure solid media (for example, in monocrystals of natural quartz) (*Mashinskii et al., 2001*). The microplasticity causes the dependence of seismic velocity on the amplitude of strain so far as a the microplastic deformation is the amplitude dependent component.

Our experiments have confirmed the known fact that the loading and unloading moduli are not equal (*Johnson et al., 1996; Xu et al., 1998*). I explain this by microplastic anelasticity. The present study shows that dynamic relationships $\sigma(\varepsilon)_d$ are different for various amplitudes of a seismic wave $(\sigma_{\max i}, \varepsilon_{\max i})_d$ at each lithostatic pressure. Therefore in real conditions the peculiar dependence $\sigma(\varepsilon)_d$ would take place in the corresponding static and dynamic range.

5. The conclusions

Shortly, conclusions are reduced to the following.

1. The stress-strain relation is dependent on a strain range and is non-linear. There are the non-closed hysteresis loops and the residual strains caused by microplasticity.
2. The water saturation essentially changes $\sigma(\varepsilon)$.
3. Young's modulus may increase and decrease with strain. The dependence of seismic velocity on the amplitude of strain can be caused by microplastic anelasticity.

4. A difference of moduli of loading and unloading is explained by microplasticity.
5. The non-linearity of $\sigma(\varepsilon)$ and its dependence on strain level should be taken into consideration in the elaborated nonlinear theory.

Therefore, the stress-strain relations received at different strain levels have the qualitative and quantitative difference. It is necessary to reconsider the traditional approach, in which the small-amplitude seismic wave propagation is described by the stress-strain relation experimentally obtained for the near source region (e.g. large strain). Such a stress-strain relation cannot be applied to the models with small strain levels. I suppose that any phenomenological model may not predict a behavior of the stress-strain dependence for the given strain level and physical condition, therefore, an empirical study of the stress-strain relation in a wide amplitude and frequency ranges is necessary.

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