An Efficient Password Security of Three-Party Key Exchange Protocol based on ECDLP

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Abstract

In this paper we have proposed an efficient password security of Three-Party Key Exchange Protocol based on Elliptic Curve Discrete Logarithm Problem. Key exchange protocols allow two parties communicating over a public network to establish a common secret key called session key. Due to their significance by in building a secure communication channel, a number of key exchange protocols have been suggested over the years for a variety of settings.Here we have taken two one-way hash functions to built the level of security high.

Key word : Key exchange protocol, Password based, secure communication, off-line dictionary attack,ECDLP.

1 Introduction

Three-party authenticated key exchange protocol is an important cryptographic technique in the secure communication areas, by which two clients, each shares a

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human-memorable password with a trusted server, can agree a secure session key. Over the past years, many three-party authenticated key exchange protocols have been proposed. However, to our best knowledge, not all of them can meet the requirements of security and efficiency simultaneously. Therefore, in this paper, we would like to propose a new simple three-party password based authenticated key exchange protocol. Compared with other existing protocols, our proposed protocol does not require any server's public key, but can resist against various known attacks. Therefore, we believe it is suitable for some practical scenarios.

With the proliferation of the handheld wireless information appliances, the ability to perform security functions with limited computing resources has become increasingly important. In mobile devices such as personal digital assistants (PDAs) and multimedia cell phones, the processing resources, memory and power are all very limited, but he need for secure transmission of information may increase due to the vulnerability to attackers of the publicly accessible wireless transmission channel [1]. New smaller and faster security algorithms provide part of the solution, the elliptic curve cryptography ECC provide a faster alternative for public key cryptography. Much smaller key lengths are required with ECC to provide a desired level of security, which means faster key exchange, user authentication, signature generation and verification, in addition to smaller key storage needs. The terms elliptic curve cipher and elliptic curve cryptography refers to an existing generic cryptosystem which use numbers generated from an elliptic curve. Empirical evidence suggests that cryptosystems that utilize number derived from elliptic curve can be more secure [2]. As with all cryptosystems and especially with public-key cryptosystems, it takes years of public evaluation before a reasonable level of confidence in a new system is established. ECC seem to have reached that level now. In the last couple of years, the first commercial implementations are appearing, as toolkits but also in real-world applications, such as email security, web security, smart cards, etc. The security of ECC has not been proven but it is based on the difficulty of computing elliptic curve discrete logarithm in the elliptic curve group [3].

2 Background

In this section we brief overview of Elliptic Curve over finite field, Elliptic Curve Discrete Logarithm Problem,Key exchange, Elliptic Curve Diffe-Helman(ECDH) and about three-party key exchange protocol.

2.1 The finite field F_p

Let p be a prime number. The finite field F_p is comprised of the set of integers $0, 1, 2, \ldots p-1$ with the following arithmetic operations [5] [6] [7]:

- Addition: If $a, b \in F_p$, then $a + b = r$, where r is the remainder when $a + b$ is divided by p and $0 \leq r \leq p-1$. This is known as addition modulo p.
- Multiplication: If $a, b \in F_p$, then $a.b = s$, where s is the remainder when $a.b$ is divided by p and $0 \leq s \leq p-1$. This is known as multiplication modulo p.
- Inversion: If a is a non-zero element in F_p, the inverse of a modulo p, denoted a^{-1}, is the unique integer $c \in F_p$ for which $a.c = 1$.

2.2 Elliptic Curve over F_p

Let $p \geq 3$ be a prime number. Let $a, b \in F_p$ be such that $4a^3 + 27b^2 \neq 0$ in F_p. An elliptic curve E over F_p defined by the parameters a and b is the set of all solutions $(x, y), x, y \in F_p$, to the equation $y^2 = x^3 + ax + b$, together with an extra point O, the point at infinity. The set of points $E(F_p)$ forms a abelian group with the following addition rules [9]:

1. Identity : $P + \mathcal{O} = \mathcal{O} + \mathcal{P} = \mathcal{P}$, for all $P \in E(F_p)$
2. Negative : if $P(x, y) \in E(F_p)$ then $(x, y)+(x, -y) = \mathcal{O}$, The point $(x, -y)$ is dented as -P called negative of P.
3. Point addition: Let $P((x_1, y_1), Q(x_2, y_2) \in E(F_p)$,then $P + Q = R \in E(F_p)$ and coordinate $(x_3, y_3)$of R is given by $x_3 = \lambda^2 - x_1 - x_2$ and $y_3 = \lambda(x_1 - x_3) - y_1$ where $\lambda = \frac{y_2 - y_1}{x_2 - x_1}$
4. Point doubling : Let $P(x_1, y_1) \in E(K)$ where $P \neq -P$ then $2P = (x_3, y_3)$ where $x_3 = (\frac{3x_1^2+a}{2y_1})^2 - 2x_1$ and $y_3 = (\frac{3x_1^2+a}{2y_1})(x_1 - x_3)$- y_1

2.3 Elliptic Curve Discrete Logarithm Problem (ECDLP)

Given an elliptic curve E defined over a finite field F_p,a point $P \in E(F_p)$ of order n, and a point $Q \in < P >$,find the integer $l \in [0, n-1]$such that $Q = lP$. The integer l is called discrete logarithm of Q to base P,denoted $l = log_p Q$ [9].

2.4 Key exchange

Key exchange protocols allow two parties to agree on a secret shared secret key that they can use to do further encryption for a long message. One of these protocols is the Diffie-Hellman, which is the most used one. The Elliptic curve Diffie-Helman is considered as an extension to the standard Diffie- Hellman.

2.5 Elliptic Curve Diffie-Helman

Elliptic curve Diffie-Helman protocol (ECDH) is one of the key exchange protocols used to establishes a shared key between two parties. ECDH protocol is based on the additive elliptic curve group. ECDH begin by selecting the underlying field $F(P)$ or $GF(2^k)$, the curve E with parameters a,b and the base point P. The order of the base point P is equal to n. The standards often suggest that we select an elliptic curve with prime order and therefore any element of the group would be selected and their order will be the prime number n [5]. At the end of the protocol,the communicating parties end up with the same value K which is a point on the curve.

2.6 Three-Party Key Exchange Protocol

Recently, Chang proposed a practical three-party key exchange (C-3PEKE) protocol with round efficiency [4]. It allows two parties A and B to share an easy-to-remember password with a trusted server S. S acts as a coordinator between two communication parties to complete mutual authentication. Once authentication is achieved, two parties can share a session key to encrypt and decrypt their communication. A practical 3PEKE protocol should comply with the following requirements [4]:

1. The session key should be agreed by the communication parties instead of being assigned by the server directly.
2. Except the password, no extra secret information should be needed - the public key for example.
3. The server has to authenticate both communication parties.
4. Computation and round efficiencies should be provided at the same time.

3 Proposed Protocol

Let A and B be two clients who wish to establish a session key, and S be a trusted authentication server with which client A has registered a password pw_A . Then

the 3PAKE protocol runs among the three parties, S, A and B, with the following parameters established:

- Let the elliptic curve E defined over a finite field F_p two field elements $a, b \in F_p$, which defined the equation of the elliptic curve E over F_p i.e $y^2 = x^3 + ax + b$ in the case $p \geq 3$, where $4a^3 + 27b^2 \neq 0$.
- Let M and N be any two group elements in $E(F_p)$.
- Two one-way hash functions $\mathcal{G}$ and $\mathcal{H}$, where the output are the elements of F_p
- Iteration Count is the number to be randomly choosed and both the hash function will be executed that nos of time. Let the number be $c \in [1, n-1]$. So we have to compute the hash $\mathcal{G}$ and $\mathcal{H}$ c times.

The proposed protocol follows the follows the following steps.

- Step -I : A chooses a random number t from the interval $[1, n-1]$, computes the point $P = t.Q$ and $P' = P + M.pw_A$ and sends $A\|P'$ to B.
- Step -II : B selects a random number s from the interval $[1, n-1]$, computes $R = s.Q$ and $R' = R + N.pw_B$ and sends $A\|P'\|B\|R'$ to S.
- Step-III : Upon receiving , S first recovers P and R by computing $P = P' - M.pw_A$ and $R = \tilde{R} - N.pw_B$. Next S select random number u from $[1, n-1]$ and computes $\tilde{P} = u.P$ and $\tilde{R} = u.R$, then compute the following

$$\begin{aligned} pw'_A(1) &= pw_A.\mathcal{G}(A\|S\|P) \\ pw'_A(2) &= \mathcal{G}(pw'_A(1)) \\ &\vdots \\ pw'_A(c) &= \mathcal{G}(pw'_A(c-1)) \end{aligned}$$

Finally we get $pw'_A = \mathcal{G}(pw'_A(c))$
Similarly $pw'_B(1) = pw_B.\mathcal{G}(B\|S\|P))$

$$\begin{aligned} pw'_B(2) &= \mathcal{G}(pw'_B(1)) \\ &\vdots \\ pw'_B(c) &= \mathcal{G}(pw'_B(c-1)) \end{aligned}$$

Finally we get $pw'_B = \mathcal{G}(pw'_B(c))$

$$\begin{aligned} \tilde{P}' &= pw'_B.\tilde{P} \\ \tilde{R}' &= pw'_A.\tilde{R} \end{aligned}$$

and sends $\tilde{P}'\|\tilde{R}'$ to B.

- Step -IV : After having received $\tilde{P'}\|\tilde{R'}$, B computes
$$\begin{aligned}
\tilde{pw'_B}(1) &= pw_B.\mathcal{G}(B\|S\|R)\\
\tilde{pw'_B}(2) &= \mathcal{G}(\tilde{pw'_B}(1))\\
&\vdots\\
\tilde{pw'_B}(c) &= \mathcal{G}(\tilde{pw'_B}(c-1))\\
pw'_B &= \mathcal{G}(\tilde{pw'_B}(c))\\
K &= s.pw'^{-1}_B.(\tilde{P'})\\
L_1(1) &= \mathcal{G}(A\|B\|K)\\
L_1(2) &= \mathcal{G}(L_1(1))\\
&\vdots\\
L_1(c) &= \mathcal{G}(L_1(c-1))\\
L_1 &= L_1(c) \text{ and sends } \tilde{R'}\|L_1 \text{ to } A.
\end{aligned}$$

- Step-V : With $\tilde{R'}\|L_1$ from B, A computes
$pw'_A = pw_A.\mathcal{G}(A\|S\|P)$, $K = t.pw'^{-1}_A.(\tilde{R'})$ and verifies that L_1 is equal to $L_1(c)$ by computing $L_1(1) = \mathcal{G}(A\|B\|K), L_1(2) = \mathcal{G}(L_1(1)) \ldots L_1(c) = \mathcal{G}(L_1(c-1))$. If the verification fails, then A aborts the protocol. Otherwise, A computes the session key SK as
$$\begin{aligned}
SK(1) &= \mathcal{H}(A\|B\|K)\\
SK(2) &= \mathcal{H}(SK(1))\\
&\vdots\\
SK(c) &= \mathcal{H}(SK(c-1))\\
SK &= SK(c)
\end{aligned}$$
and sends $L_2 = \mathcal{G}(A\|B\|K)$.

- Step-VI : B verifies the correctness of L_2 is equal to $L_2(c)$ by checking the equation $L_2(1) = \mathcal{G}(B\|A\|K), L_2(2) = \mathcal{G}(L_2(1)) \ldots L_2(c) = \mathcal{G}(L_2(c-1))$. If it holds, then B computes the session key $SK = \mathcal{H}(A\|B\|K)$. Otherwise, B abort the protocol.

3.1 Verification of Correctness of 3PAKE

The correctness of 3PAKE can be verified as
$K = s.pw'^{-1}_B.\tilde{P'} = s.pw'^{-1}_B.pw'_B\tilde{P} = s.\tilde{P} = s.u.P$
$= sut.Q$
and $K = t.pw'^{-1}_A.\tilde{R'} = t.pw'^{-1}_A.pw'_A\tilde{R}$
$t.\tilde{R} = tu.R = tus.Q$

4 Security discussions

Theorem 1 *3PAKE does not leak any information that allows to verify the correctness of password guesses.*

Proof : Since $\mathcal{G}$ is a one-way hash function is executed c times and s,u and t are all random numbers, so the protocol does not leak any information that allow the adversary to verify the correctness of password guesses.

Theorem 2 *3PAKE is secure against off-line password guessing attacks.*

Proof : If the hacker intends to tract out the password, first he has to find out the iteration count c which is a random number and process that number of times. Further he has to solve Elliptic Curve Discrete Logarithm problem(ECDLP) which is computationally infeasible takes fully exponential time . So we can say it is secured against off-line password guessing attacks.

5 Conclusion

In this research a new protocol for exchanging key between two parties with a trusted Server has been defined. This new protocol has two major advantages over all previous key exchange protocol, first this protocol does not leak any information that allow the adversary to verify the correctness of password guesses. The second one is that this protocol does not leak any information that allows to verify the correctness of password guesses. The proposed protocol is also easy to implement.The security of our system is based on Elliptic Curve Discrete Logarithm Problem (ECDLP). The primary reason for the attractiveness of ECC over systems such as RSA and DSA is that the best algorithm known for solving the underlying mathematical problem (namely, the ECDLP) takes fully exponential time. In contrast, sub-exponential time algorithms are known for underlying mathematical problems on which RSA and DSA are based, namely the integer factorization (IFP) and the discrete logarithm (DLP) problems. This means that the algorithms for solving the ECDLP become infeasible much more rapidly as the problem size increases than those algorithms for the IFP and DLP. For this reason, ECC offers security equivalent to RSA and DSA while using far smaller key sizes.The attractiveness of ECC will increase relative to other public-key cryptosystems as computing power improvements force a general increase in the key size. The benefits of this higher-strength per-bit include higher speeds, lower power consumption, bandwidth savings, storage efficiencies, and smaller certificates.

References

[1] Murat Fiskiran A and B Ruby Lee *Workload characterization of elliptic curve cryptography and other network security algorithms for constrained environments. Proc. IEEE Intl. Workshop on Workload Characterization, pp: 127-137, 2002.*

[2] De Win E. and B Preneel *Elliptic curve public-key cryptosystems - an introduction. State of the Art in Applied Cryptography, LNCS 1528, pp: 131-141,1998.*

[3] Aydos M., E Savas and C .K .KoV *1999. Implementing network security protocols based on elliptic curve cryptography. Proc. fourth Symp. Computer Networks, pp: 130-139, 1999*

[4] Y.F. Chang *A Practical Three-party Key Exchange Protocol with Round Efficiency. International Journal of Innovative Computing, Information and Control,Vol.4, No.4, April 2008, 953960.*

[5] N. Koblitz. *A course in Number Theory and Cryptography ,2nd edition* Springer-Verlag-1994

[6] K. H Rosen *"Elementary Number Theory in Science and Communication", 2nd ed., Springer-Verlag, Berlin, 1986.*

[7] A. Menezes, P. C Van Oorschot and S. A Vanstone *Handbook of applied cryptography. CRC Press, 1997.*

[8] D. Hankerson, A .Menezes and S.Vanstone. *Guide to Elliptic Curve Cryptography, Springer Verlag, 2004.*

[9] *"Certicom ECC Challenge and The Elliptic Curve Cryptosystem" available :http://www.certicom.com/index.php.*