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Production Planning for Multi-site Batch Plants with the MILP Method

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Production planning for multi-site batch plants to maximize total profits should establish simultaneous allocations to the plants considering available production times and transportation of products from the plants to the distribution centers. Since some intermediate products of batch plants are unstable, combining products often reduces the cycle times. Therefore, production sequences are required to estimate production times, but production sequences depend upon allocation, which cannot be determined without estimation of production times. A straightforward mathematical formulation for such production planning results in a mixed-integer non-linear programming (MINLP) problem, which may require excessive computational time even for small problems. Optimal production planning requires that allocations for each batch plant are produced based on product combinations (product mixes), of which only the productivities are improved. This study introduced product mixes as the units of allocation. The method of allocation is to determine the optimal numbers of individual product mixes for each batch plant. Cycle times of product mixes are constant, so the required production time at each plant may be approximated as the sum of the cycle times of product mixes. Based on product mixes with predetermined cycle times, manufacturing costs, and transportation costs, the mixed-integer linear programming (MILP) problem is solvable and can simultaneously establish the allocation and the transportation of products with maximum total profits. The effectiveness of this proposal is demonstrated through an example problem.

Keywords
Production planning, Batch plant, Product mix, MILP

Introduction

Various process industries, such as chemicals, foods, cosmetics, pharmaceuticals and so on, use batch plants to produce small lots of various high value-added products. More than one product may be produced in individual batch plants at different locations and transported to distribution centers. To maintain business competitiveness, coordination of all activities from production to distribution stages is important for batch process management.

Batch process management has been investigated extensively during the last decade, and can be divided into four layers, *i.e.* strategic and business, production planning, scheduling, and batch process control layers. In the strategic and business layer, long-term policies of capacity and configuration design are decided. These layers have been widely investigated[1),5),7),10)]. In the scheduling layer, available resources and plant status are coordinated under the constraints of due date, and many scheduling methods have been proposed[3),6),14)]. In the batch process control layer, unit processes and process sequences are monitored through the Distributed Control System (DCS)[13)]. However, the production planning layer for deciding the products and quantities during the mid-term has been studied mainly for a single batch plant[11),12),15)].

The present study will focus on production planning for process industries operating multi-site batch plants to produce more than one product according to the demands from distribution centers. In such a situation, allocation to the batch plants and transportation of products to the centers should be determined simultaneously under the constraints of maximum profit or minimum cost, because each plant has different product recipes and production costs according to the equipment specification and different transportation costs according to the location. In assembly industries, this type of production planning for the multi-site plants can be formulated as a mixed-integer linear programming (MILP) problem, because the production times required for individual tasks can be estimated as the sums of the processing times of the products allocated to the plants. In batch plants, the shapes of Gantt Charts of products are often unchangeable because some intermediate

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products are unstable, so the production times required for individual process units cannot be directly estimated as the sums of the processing times of the allocated products. A straightforward mathematical formulation of this type of production planning on the multi-site batch plants becomes a mixed-integer non-linear programming (MINLP) problem, which is naturally difficult to solve even if small.

For the single batch plant, production planning problems have been mainly investigated by equipment assignment of parallel production lines to maximize productivity based on developed heuristic methods[11)~13)]. In other words, for the single batch plant the production times required for individual processing units can be properly estimated for the given demand. If such heuristic methods are adopted for multi-site batch plants, production planning problems with multi-dependent loops must be solved; loops for allocating production and loops for evaluating the production time of the allocation in each plant. Unfortunately, it is impossible to optimize demand allocation through such loops, because demand allocation cannot be decided without evaluation of production time constraints, but the production times can only be estimated after the allocation is established. A MILP formulation has been proposed assuming no waiting time constraints and unlimited storage capacities for intermediate products[2)]. These assumptions are unrealistic for batch plants, so this approach cannot be applied to production planning for multi-site batch plants.

Product combinations frequently have shorter cycle times than the sums of the cycle times of the individual products because of unstable intermediate products. Such combinations are called product mixes in this paper. The term product mix has often been used to describe production quantities of multiple products[4),8),9)], but our definition is different. This cycle time reduction leads to productivity improvement. Therefore, optimal production planning for multi-site batch plants producing more than one product will allocate products to the plants based on these product mixes, and the required production times can be approximately estimated as the sums of the cycle times of these product mixes. Cycle times of product mixes with fixed production sequences are constant, so planning can focus on the product mixes as units of demand allocation. Lists of these product mixes can be prepared with cycle times, manufacturing costs for individual plants, and transportation costs of individual products from plants to centers, so the planning involves allocation of the numbers of individual product mixes to each plant, and the production time of the allocations will be the summation of the cycle times of these product mixes. In other words, by introducing product mixes as units of demand allocation, the production planning problem for the multi-site batch plants can be formulated into a solvable MILP problem.

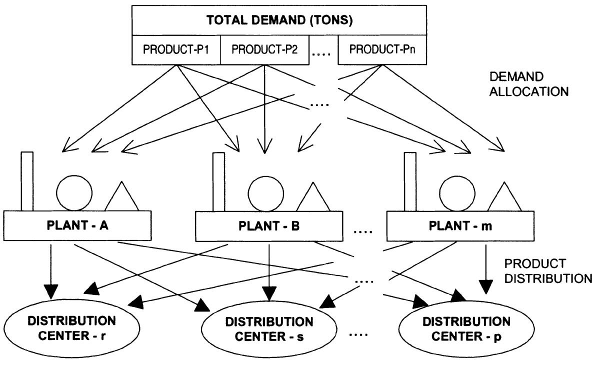

Fig. 1 Production Planning for Multi-site Plants Producing Multiple Products Required at Several Distribution Centers

The present study describes the process of problem analysis, explains the product mix approach and MILP formulation, and demonstrates the effectiveness of this proposal with an example problem.

1. Problem Analysis

The outline of production planning on multi-site batch plants, which provide more than one product for several distribution centers, is shown in **Fig. 1**. Production processes should be performed on a batch basis, using fixed volumes based on the characteristics of the facilities, such as the volumes of the reactors, and then the allocation planning will establish the numbers of batches of individual products for the various plants. Such production planning will optimize the numbers of batches of products allocated to plants and the numbers of products distributed from plants to the centers under four constraints: available production times, product requirements, relationships between production and distribution, and relationships between distribution and product requirements.

Most batch plants consist of unit processes that include chemical reaction, separation, mixing, etc., during which some unstable intermediates are produced. These unstable intermediate products must have zero waiting time or any fixed waiting time specified in the product recipes, so the shapes of the Gantt Charts of products are often unchangeable. There is a critical unit process in each product which determines the cycle time, and products often have different critical unit processes. Therefore, combination of products can reduce the cycle times, leading to productivity improvement. For example, consider the production of 100 tons of product P1 and 115 tons of product P2 in a batch plant. The Gantt Charts of these products are shown in **Fig. 2** and the batch sizes of are both 1 ton/batch. Cycle times of P1 and P2 are both 10 h, but

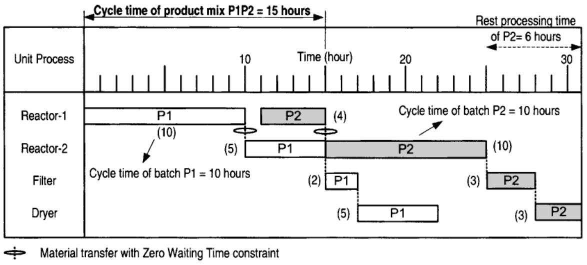

Fig. 2 Gantt Chart of Product Mix P1P2

the critical unit processes are Reactor-1 for P1 and Reactor-2 for P2. The cycle time of product combination P1P2 becomes 15 h, and it is less than the summation of the cycle times of P1 and P2 of 20 h. In this example, the changeover times are neglected to simplify the explanation, but the cycle times can be determined with consideration of changeover times, if these values are known. To estimate the cycle time for a combination of more than two products, the optimal production sequence among all possible sequences must be selected because the cycle time depends on the production sequence.

If 100 lots of batch P1 and 115 lots of batch P2 are produced, the time required is about 2150 h (1000 (= 100 × 10 for batch P1) + 1150 (= 115 × 10 for batch P2)). On the other hand, for 100 lots of product mix P1P2 and 15 lots of P2, about 1650 h is required (1500 (= 100 × 15 for product mix P1P2) + 150 (= 15 × 10 for batch P2)). There is no explicit relationship between required production times and numbers of allocated products, so estimation of production times requires enumeration of all possible production sequences, and calculation of production times. As a result, production planning for multi-site batch plants may be formulated as MINLP problems.

The present study shows how to transform this production planning problem into a form solvable by generally available tools with reasonable computational effort, similar to MILP problems in assembly plants.

2. Product Mix Approach

To overcome the limitations of the current approach for solving production planning at multi-site batch plants, we propose the use of product mixes (product combinations with fixed production sequences) as the new units of product allocation. Product combinations are generated from products. For example, if a batch plant can produce products P1, P2, and P3, the possible product combinations are P1, P2, P3, P1P2, P1P3, P2P3, and P1P2P3. Theoretically there are $\sum_{i=1}^{n}(n!/(n-i)!n!)$ product combinations from n products. For a product combination of more than two products, all possible production sequences must be examined. For the case of product combination P1P2P3, the possible separation sequences are P1P2P3 and P1P3P2. Not all combinations are suitable for product mixes, and only product combinations with shorter cycle times than the sum of the cycle times of individual products are selected. Lists of product mixes with cycle times can be prepared for each plant beforehand, and product mixes proposed as units of demand allocation. The allocation to each plant will correspond to the numbers of individual product mixes allocated to the plant, and the production times can be estimated as the sums of the cycle times of product mixes. For the same example shown in **Fig. 2**, comparing cycle times of P1 and P2, and product mix P1P2, 100 lots of product mix P1P2 and 15 lots of P2 are selected, and the required production time is about 1650 h. Additional production time is necessary for estimating the required production time from the sum of cycle times. In this example, this additional time is 16 h: 6 h for adjusting between product mix P1P2 and P2, and 10 h for rest processing time of the last P2, but we neglect such additional times in this study because the production horizon is relatively longer than the additional time, and in actual cases some margins may be considered in the constraints of available production times. The same result is obtained by considering all possible production sequences, estimating all required production times, and choosing the minimum production time.

The objective function of production planning for multi-site batch plants with multi-site distribution centers is total profits or total costs, and the constraints are available production times in the batch plants, relationships between products and demands, relationships between products and production in the batch plants, and relationships between production and demands in the distribution centers. The decision variables of this production planning are the numbers of individual product mixes allocated to the batch plants and the numbers of individual products distributed from the plants to the centers. Our proposal is to establish linear functions for the objective function and constraints. Manufacturing costs of product mixes, penalty costs and transportation costs can be prepared beforehand, so the objective function becomes a linear function of the numbers of allocated product mixes and distributed products. As mentioned above, required production times of individual batch plants can be estimated as the sum of the cycle times of product mixes allocated to the plants. The other three constraints can be also formulated into linear functions based on the numbers of allocated product mixes and distributed products. Integer variables are used for the numbers of individual product mixes allocated to the batch plants. Continuous or integer variables can be used for the numbers of distributed products depending on the distribution modes and product characteristics, so this study uses continuous variables. Therefore, production planning for multi-site batch plants can adopt a MILP approach. Considering the size of the MILP problem in process industries, the number of products produced in a batch plant is not so large as in an assembly plant. We consider that the optimal solution may be found in a reasonable time. This approach of product mixes is also applicable to production planning for a single batch plant, as a special case of planning for multi-site batch plants.

3. MILP Formulation

Production planning for multi-site batch plants can be formulated into a MILP problem. In this example, the total profit without penalty costs is adopted as the objective function to simplify the explanation of the formulation. Our approach can consider the penalty costs, if necessary. As mentioned above, four constraints can be formulated into linear functions of the number of allocated product mixes and units of distributed products. To represent the MILP model, the following variables and parameters are defined.

Decision variables:

x_{kj} = number of product mixes k at plant j (integer) ≥ 0,
y_{ijm} = quantity of product i delivered from plant j to distribution center m (ton) ≥ 0,

Parameters:

I = number of products,
J = number of plants,
M = number of distribution centers,
K_j = number of product mixes at plant j,
T_j = available production time of plant j (h),
CT_{kj} = cycle time of product mix k at plant j (h),
A_j = allowance time for plant j (h),
B_{ikj} = batch size of product i in product mix k of plant j (ton/batch),
D_{im} = demand of product i at center m (ton),
DC_{ijm} = transportation cost of product i from plant j to center m (US\$/ton),
MC_{kj} = manufacturing cost of product mix k of plant j (US\$/batch),
P_{kj} = price of product mix k of plant j (US\$/cycle).

The MILP formula for production planning at multi-site batch plants for deciding the optimal numbers of individual product mixes allocated to the batch plants and quantities of products distributed from the plants to the centers is as follows:

$$\text{maximize}\left\{\sum_{j=1}^{J}\sum_{k=1}^{Kj} x_{kj}(P_{kj}-MC_{kj}) - \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{m=1}^{M} y_{ijm}DC_{ijm}\right\} \tag{1}$$

Table 1 Demand and Product Prices

Distribution center	Demand product [ton]		
	P1	P2	P3
DC 1	250	120	260
DC 2	200	125	360
DC 3	300	200	200
Total demand	750	445	820
Price [US\$/ton]	200	280	200

Subject to:

$$\sum_{k=1}^{Kj} x_{kj}CT_{kj} + A_j \leq T_j,\ j=1,2,\cdots,J \tag{2}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{Kj} B_{ikj}x_{kj} \leq \sum_{m=1}^{M} D_{im},\ i=1,2,\cdots,I \tag{3}$$

$$\sum_{m=1}^{M} y_{ijm} = \sum_{k=1}^{Kj} B_{ikj}x_{kj},\quad i=1,2,\cdots,I \text{ and } j=1,2,\cdots,J \tag{4}$$

$$\sum_{j=1}^{J} y_{ijm} \leq D_{im},\ i=1,2,\cdots,I \text{ and } m=1,2,\cdots,M \tag{5}$$

The objective function is stated in Eq. (1), in which total sales and manufacturing costs are functions of numbers of product mixes, and transportation costs are calculated from the quantities of distributed products. Inequalities in Eq. (2) are constraints for evaluating available production times considering the allowance times, in which the required production time for each plant is approximated as the sum of cycle times of allocated product mixes. Constraints between the products and demands from the centers are shown in Eq. (3). These products for each plant are calculated from the numbers of product mixes and their batch sizes. In Eq. (4), the constraints of balancing between production and distribution are presented. In Eq. (5), the constraints ensure that distribution is less than the demand at the centers.

4. Example Problem

An example problem is presented to demonstrate the effectiveness of this proposal. In this example, three batch plants are planned to produce three products to serve demands from three distribution centers. The demand information is shown in **Table 1**. The available production times of these plants are given in **Table 2**, in which each plant consists of one unit process for each operation stage. The batch sizes and manufacturing costs of products are given in **Tables 3** and **4**, respectively. Product recipes are shown in **Fig. 3**, and transportation costs from the plants to the centers in **Table 5**.

According to the product recipes and the available unit processes, we should prepare lists of product mixes and their properties such as production sequences, cycle time, prices, and manufacturing costs for each plant before establishing allocation. Cycle times are calculated according to the production sequences of product mixes and the Gantt Charts of product recipes, and prices and manufacturing costs of product mixes are prepared from the values for the component products. The lists of product mixes for each plant are shown in **Tables 6**, **7**, and **8**. In this case, product mixes for plants A and B consist of all products because the critical unit processes of the three products are different, so seven product mixes are available for plants A and B. However, for plant C, products P2

Table 2 Available Production Time and Allowance Time

Resources	Plant A	Plant B	Plant C
Production time [h]	3000	2800	3000
Allowance time [h]	40	40	40

Table 3 Batch Sizes [ton/batch]

Product	Plant A	Plant B	Plant C
P1	2.0	2.5	2.0
P2	1.8	2.0	1.5
P3	2.0	1.8	2.2

Table 4 Manufacturing Cost of Products [US$/batch]

Product	Plant A	Plant B	Plant C
P1	192	170	162
P2	174	192	162
P3	168	166	190

Table 5 Transportation Costs from Plants to Centers [US$/ton]

From	Product	To distribution center		
		DC 1	DC 2	DC 3
Plant A	P1	10	16	18
	P2	15	20	24
	P3	12	18	20
Plant B	P1	16	10	16
	P2	20	15	20
	P3	18	12	18
Plant C	P1	18	16	10
	P2	24	20	15
	P3	20	18	12

Table 6 List of Product Mixes and Their Properties for Plant A

Product mixes	Cycle time [h]	Prices [US$]	Manufacturing costs [US$]
P1	12	400	192
P2	14	504	174
P3	11	400	168
P1P2	20	904	366
P1P3	18	800	360
P2P3	22	904	342
P1P2P3	28	1304	534

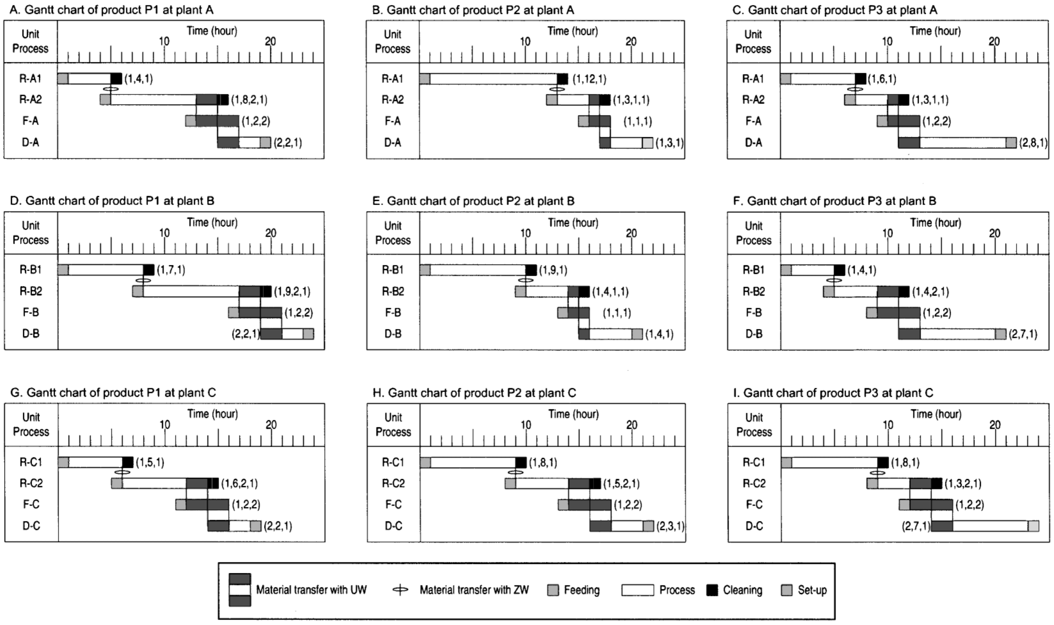

Fig. 3 Product Recipes

and P3 have the same critical unit of Reactor RC-1. Product combinations including products P2 and P3 are not suitable for plant C, so only five product mixes are available for plant C.

After these preparations, a MILP program is constructed to solve the production planning problem. According to the product mixes and other input information, the MILP program involves 19 integer variables, 27 continuous variables, and 24 constraints. The number of integer variables corresponds to the number of product mixes, and are used to represent the number of product mixes for allocating the production to the plants. The number of continuous variables representing the distribution of products from the plants to the centers is related to the number of plants, centers, and products.

The MILP program was solved through optimization software XPRESS-MP on a Sun Ultrasparc computer. Optimal solution of the MILP was found in a few minutes (CPU time). Results of numbers of selected product mixes are presented in **Table 9**. Based on these results, optimal production planning with production quantities and transportation of products is shown in **Table 10**, and required production times in **Table 11**.

According to the optimal numbers of selected product mixes for each plant, we can evaluate the accuracy of the production time approximation obtained as the sum of the cycle times of product mixes, because the additional times for different product mixes and the rest processing time of the last batch are not considered. To estimate additional times, we must arrange the selected product mixes for each plant into a production sequence, and the order within the product mixes should be decided even if the cycle time of product mixes is constant. The maximum additional time of plant A is 18 h, consisting of 4 h for adjusting product mixes P2P3P1 and P3, 0 h for adjusting product mixes P3 and P3P2, and 14 h for the rest processing time of the last product mix P3P2. For plant B, the additional time is 20 h, consisting of 0 h for adjusting product mixes P2 and P2P3P1, 5 h for adjusting product mixes P2P3P1 and P3P2, and 15 h for the rest processing time of the last product mix P3P2. For plant C, because

Table 7 List of Product Mixes and Their Properties for Plant B

Product mixes	Cycle time [h]	Prices [US$]	Manufacturing costs [US$]
P1	13	500	170
P2	11	560	192
P3	10	360	166
P1P2	22	1060	362
P1P3	21	860	336
P2P3	18	920	358
P1P2P3	29	1420	528

Table 8 List of Product Mixes and Their Properties for Plant C

Product mixes	Cycle time [h]	Prices [US$]	Manufacturing costs [US$]
P1	10	400	162
P2	10	420	162
P3	10	440	190
P1P2	19	820	324
P1P3	17	840	352

Table 9 Selected Product Mixes

Plant	Selected product mixes	Number
Plant A	P3	6
	P2P3	22
	P1P2P3	86
Plant B	P2	9
	P2P3	64
	P1P2P3	52
Plant C	P1P3	174

Table 11 Approximation of Required Production Time

	Production time [h]
Plant A	2998
Plant B	2799
Plant C	2998

Table 10 Optimal Production Quantity and Transportation of Products

Plant	Product	Production quantity [ton]	Transportation of produced products [ton]		
			DC 1	DC 2	DC 3
Plant A	P1	172.0	150.0	22.0	0.0
	P2	194.4	120.0	0.0	74.4
	P3	228.0	228.0	0.0	0.0
Plant B	P1	130.0	0.0	130.0	0.0
	P2	250.0	0.0	125.0	125.0
	P3	208.8	0.0	208.8	0.0
Plant C	P1	348.0	0.0	48.0	300.0
	P2	0.0	0.0	0.0	0.0
	P3	382.8	31.6	151.2	200.0

only one product mix is selected, the maximum additional time is 14 h for the rest processing time of the last product mix P1P3. This investigation shows the maximum additional times are less than 1% of required production times, so the additional times have no significant impact in our proposal.

Considering the problem size as an implication of product combinations, the number of product mixes in the example problem is almost double the number of products. In practice, applying our proposal will greatly decrease computational time compared to using a straightforward MINLP approach. This example problem shows that our product mix approach is effective for optimal production planning for multi-site batch plants.

Conclusion

Operating multi-site plants to produce more than one product is becoming mainstream in batch process management. To improve performance, a reasonable production planning method for establishing simultaneous production allocation to multi-site batch plants and transportation of products is necessary. However, existing methods cannot be applied directly because it is difficult to properly estimate required production times from the allocated products for batch plants. We have proposed an efficient production planning method in which product mixes are introduced as the units of allocation. According to our proposal, production planning for multi-site batch plants can be formulated into a MILP problem and the example problem demonstrated the effectiveness of this method.

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要　　旨

MILPを用いた複数サイトのバッチプラント生産計画

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複数サイトのバッチプラント生産計画問題は，生産時間の制約のもとで各プラントへの需要の配分とプラントからセンターへの製品の配送を同時に決定しなければならない。バッチプラントでは中間製品が不安定なことが多々あり，製品の組合せ法により生産サイクルタイムの短縮につながることがある。このため，必要生産時間の推定には製品の生産シーケンスが不可欠となるが，このシーケンスは各プラントに配分された需要に依存し，またこの配分は生産時間の推定なくしては不可能である。つまり，この生産計画問題を直せつに定式化すると混合整数非線形計画（MINLP）問題となる。最適生産計画では，組み合わせることによりサイクルタイムを短縮できる製品の組合せ（product mix）に基づいて生産が行われることから，これらを需要配分の単位とすることを本論文では提案している。product mixに対してサイクルタイム，生産コストおよび配送コストをあらかじめ用意しておけば，需要の配分問題とは各バッチプラントで生産する各々のproduct mixの数を決定することになり，必要生産時間の推定は配分されたproduct mixのサイクルタイムの合計として近似することが可能となる。この生産計画問題を混合整数線形（MILP）問題に定式化できることを，また例題を用いてその有効性を示している。

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