第 28 卷 第 6 期
2004年6月

高能物理与核物理
HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS

Vol.28, No.6
Jun., 2004

On Debye Mass in Hot Dense QED*

FANG Jie[1)] HOU De-Fu

(Institute of Particle Physics, Huazhong Normal University, Wuhan 430079, China)

Abstract Starting from the Schwinger-Dyson equation for the photon propagator in QED, we derive a useful relation between the Debye mass and the thermodynamic pressure. Using this relation, we calculate the two-loop Debye mass from relevant pressures at finite temperature and chemical potential. The result indicates that the two-loop correction decreases the Debye screening mass. We also discuss the magnetic mass in QED plasma.

Key words Debye mass, self-energy, thermodynamic pressure

1 Introduction

The Debye mass is a very important physical parameter which describes the collective motion of a hot and/or dense plasma. Electrically charged particles in this plasma react to electromagnetic fields and cause screening of static electric fields at large distance. The inverse screening length is called Debye mass. There have been many works about it[1—5]. But they all result from direct calculation of the self-energy and the gauge fields. The approach could be very complicated, especially when dealing with two or higher loop self-energy diagrams. Here we adopt an alternative method. First we will derive a useful relation between the zero-zero component of the full photon self-energy and the thermodynamic pressure. Pressure is given by vacuum diagrams, which are easier to be handled than the corresponding self-energy diagrams. We calculate the leading order contribution and its loop correction of the Debye screening mass in QED from the leading order and two-loop pressures at finite temperature and chemical potential. Furthermore, we will also present the numerical results and compare them with the analytic results.

2 Two-loop correction to the Debye mass

According to Schwinger-Dyson equation for the photon propagator in QED, one can write the full photon self-energy with full electron propagators and a full vertex (depicted in Fig.1) in imaginary time formalism as:

$$\Pi_{\mu\nu}(Q) = e^2\int \frac{d^4 P}{(2\pi)^4}\mathrm{Tr}[\gamma_\mu S(P)\Gamma_\nu(P,P+Q)S(P+Q)], \tag{1}$$

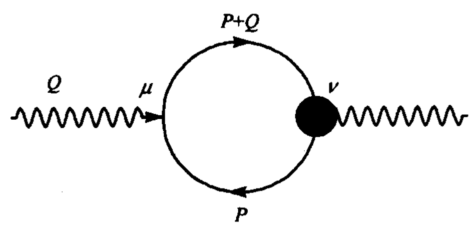


Fig.1. The full photon propagator.
Wavy line: photons. Solid lines: electron. Filled circle: all kinds of corrections.

where S is the full electron propagator and Γ_ν the full electron-photon vertex. Using the general QED Ward identity in the derivative form:

$$\Gamma_\nu(P,P) = \frac{\partial S^{-1}(P)}{\partial P_\nu}, \tag{2}$$

we can simplify Eq.(1) as:

$$\Pi_{\mu\nu}(0,q\to 0) = -e^2\int \frac{d^4 P}{(2\pi)^4}\mathrm{Tr}\left(\gamma_\mu \frac{\partial S(P)}{\partial P_\mu}\right). \tag{3}$$

After analytical continuation to real time, S is a function of $P_0-\mu$, where μ is the electron chemical potential,

Received 29 September 2003, Revised 9 December 2003
* Supported by NSFC (10135030, 10005002)
1) E-mail: fjie@iopp.ccnu.edu.cn

617—620

and one may replace the P_0 derivative with a μ derivative. From this observation, we can get:

$$\Pi_{00}(0, q \to 0) = -e^2 \int \frac{d^4 P}{(2\pi)^4} \mathrm{Tr}\left(\gamma_0 \frac{\partial S(P_0 - \mu)}{\partial P_\nu} \right). \quad (4)$$

Tabing Fourier transformation, one finds:

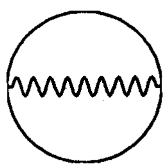
$$\Pi_{00}(0, q \to 0) = -e^2 \left(\frac{\partial n}{\partial \mu} \right)_T, \quad (5)$$

where n is the particle density defined by $n = \mathrm{Tr}(\hat{\Psi}^\dagger(0,0)\hat{\Psi}(0,0))$. By employing thermodynamical formulation, we can rewrite Eq. (5) as:

$$\Pi_{00}(0, q \to 0) = -e^2 \left(\frac{\partial^2 P(\mu, T)}{\partial \mu^2} \right) \equiv -m_{el}^2, \quad (6)$$

where P is the pressure, and m_{el}^2 is the Debye mass. This is our main result.

Next we will use this relation to calculate the Debye screening mass of QED. The two-loop pressure of QED can be calculated from the two-loop vacuum diagram (shown as following),



which is easier to be handled than the two-loop self-energy. The QED pressure in two-loop approximation can be expressed as:

$$P = -\frac{1}{6} e^2 T^2 \int \frac{d^3 p}{(2\pi)^3} \frac{N_F(p)}{E_p} - \frac{1}{2} e^2 \int \frac{d^3 p}{(2\pi)^3} \int \frac{d^q p}{(2\pi)^3} \frac{1}{E_p E_q} \times \left\{ \left(1 + \frac{2m^2}{(E_p - E_q)^2 - (\boldsymbol{p} - \boldsymbol{q})^2} \right) \times [N_F^-(p) N_F^-(q) + N_F^+(p) N_F^+(q)] + \left(1 + \frac{2m^2}{(E_p + E_q)^2 - (\boldsymbol{p} - \boldsymbol{q})^2} \right) \times [N_F^-(p) N_F^+(q) + N_F^+(p) N_F^-(q)] \right\}, \quad (7)$$

where $N_F = N_F^+ + N_F^-$, the fermion occupation numbers are:

$$N_F^\pm(p) = \frac{1}{e^{\beta(E_p \pm \mu) + 1}}$$

and $E_p = \sqrt{\boldsymbol{p}^2 + m^2}$. Terms which are independent of distribution in Eq. (7) represent the energy shift of the vacuum and are not of interest to us. Inserting Eq. (7) into Eq. (6), we obtain the two-loop correction of Debye screening mass.

For non-zero mass, it is diffcult to obtain an analytical expression of Debye mass m_{el}^2. Here we present some numerical results in a group of curves. We make $\boldsymbol{q}$ at z-axis direction, and $\boldsymbol{p}$ have an angle θ with respect to the z-axis. Then we can perform the integrals in Eq. (7) with following parameters: $e = \frac{1}{137}$, $m_e = 0.511\mathrm{MeV}$. We show temperature dependence of the leading order contribution $m_{el}^{2(0)}$ and its two-loop correction $m_{el}^{2(2)}$ with different chemical potentials in Fig. 2 (a) and (b), respectively. We also present our numerical results about the chemical potential dependence of the leading order contribution $m_{el}^{2(0)}$ and its two-loop correction $m_{el}^{2(2)}$ with different temperatures in Fig. 3 (a) and (b), respectively. We see that the two-loop correction gives small negative contribution to the Debye screening mass. To this end, we take the approximation $m_e = 0$ in the state of QED plasma. We can obtain the analytical expressions, which can be compared with the numerical results.

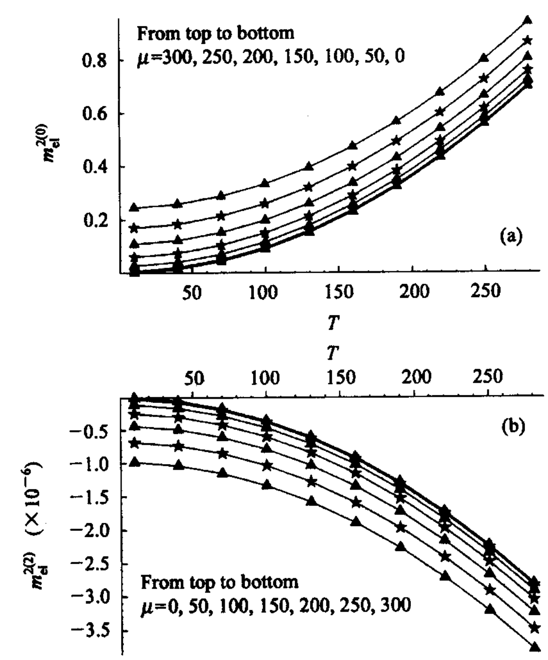


Fig.2. The temperature dependence of the Debye mass at different chemical potential. (a) The leading-order contribution; (b) The two-loop correction; (Triangles and stars denote data points.)

Now, we calculate the leading contribution of Debye mass and compare it with its two-loops correction. Insert-

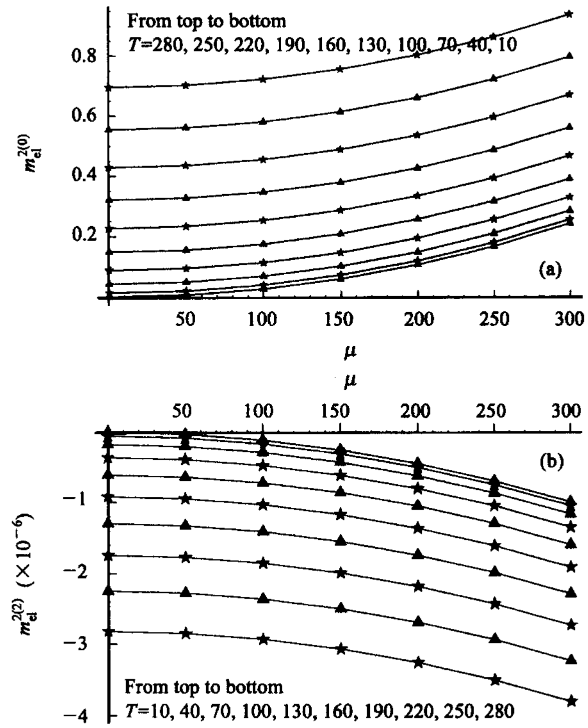


Fig.3. The chemical potential dependence of the Debye mass at different temperature. (a) the leading contribution and (b) the two-loop (Triangles and stars denote data points.).

ing the leading-order contribution of QED pressure[1,6,7]:

$$P^{(0)} = \frac{1}{6\pi^2}\left(\frac{7\pi^4 T^4}{60} + \frac{\mu^2\pi^2 T^2}{2} + \frac{\mu^4}{4}\right),$$

into Eq. (6), one easily obtains

$$m_{\rm el}^{2(0)} = -\Pi_{00}(0, q\to 0) = \left(\frac{e^2 T^2}{6} + \frac{e^2\mu^2}{2\pi^2}\right). \quad (8)$$

In the approximation $m_e = 0$, we can obtain the analytic expression of the two-loop pressure of QED plasma from Eq.(7):

$$P^{(2)} = -\frac{e^2}{288}\left(5T^4 + \frac{18}{\pi^2}\mu^2 T^2 + \frac{9}{\pi^4}\mu^4\right),$$

and hence the two-loop correction of the Debye screening mass:

$$m_{\rm el}^{2(2)} = -\Pi_{00}(0, q\to 0) = -\frac{e^4 T^2}{8\pi^2} - \frac{3e^4\mu^2}{8\pi^4}. \quad (9)$$

We present the comparison between the numerical and analytical results in Fig.4. We can see that their difference is very small, this is because the electron mass is very small with respect to the realistic temperature and chemical potential scales in QED plasma.

Now, let's look at the special diagonal components of the photon self-energy which are relevant to magnetic screening. Starting with Eq.(4), we write

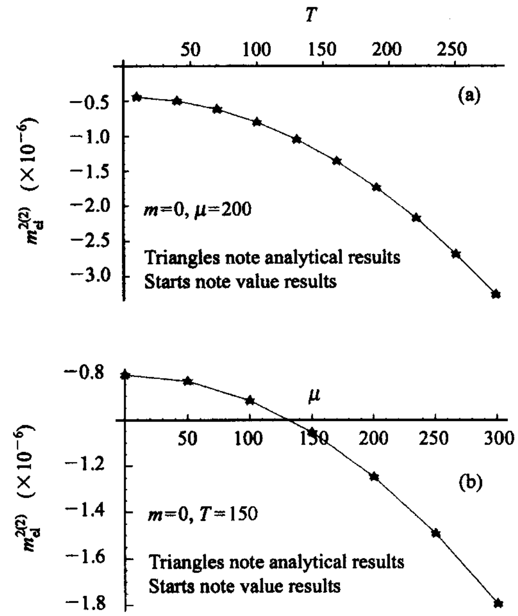


Fig.4. The comparison between analytical and numerical results at (a) μ = 200MeV and (b) T = 150MeV.

$$\Pi_{ii}(0, q\to 0) = -e^2\int\frac{d^4P}{(2\pi)^4}\mathrm{Tr}\left(\gamma_i\frac{\partial S(P)}{\partial P_i}\right) = -e^2 T\sum_n\int\frac{d^3P}{(2\pi)^3}\mathrm{Tr}(\boldsymbol{\gamma}\cdot\nabla S(p)) = -e^2 T\frac{1}{(2\pi)^3}\sum_n \mathrm{Tr}\left(\oint d\boldsymbol{\sigma}\cdot\boldsymbol{\gamma}S(P)\right) = 0. \quad (10)$$

Here we use Gauss theorem and the fact that $S(p)$ vanishes at infinity. This proves that $\Pi_{ii}(0, q\to 0)$ vanishes to all orders in perturbation theory: the magnetic mass is necessarily zero for the QED plasma, which is in contrast to that in the QCD plasma case. There is a magnetic gluon screening mass for hot QCD. The calculation for such mass in hot QCD is highly nontrivial, where the non-perturbative resummation is required[4,8,9].

3 Conclusion

In summary, starting from the Schwinger-Dyson equation for the photon propagator in QED, we derive a useful relation between full Debye mass and the full thermodynamic pressure. Therefore one can calculate the Debye mass by only evaluating vacuum diagrams, which give the pressure, instead of calculating more complicated self-energy diagrams at finite temperature and chemical potential. As examples, we calculate the leading order contribution and the two-loop correction of the Debye mass from the corresponding pressures at finite temperature and chemical potential. We present the numerical results and compare them with the analytic ones at $m_e = 0$ limit. We show that the two-loop correction decreases the Debye screening mass. We also prove that, in contrast to the hot QCD case, the magnetic mass in QED plasma is zero. We note that the relations between Eq. (4) and Eq. (10) is general valid for all orders. Therefore, one can obtain higher order Debye screening mass from higher order vacuum diagrams.

The authors are grateful to Prof. Li Jia-Rong for valuable discussion.

References

1 Kapusta J I. Finite Temperature Field Theory. Cambridge: Cambridge University Press, 1989. 124

2 LI Jia-Rong. Introduction to Quark Matter Theory. Changsha: Hunan Eduation Press, 1989. 78

3 Braaten E, Nieto A. Phys. Rev. Lett., 1994, **73**:2402

4 HOU De-Fu, LI Jia-Rong. Z. Phys., 1996, **C71**:503

5 Schneider R A. Phys. Rev., 2002, **D66**:036003

6 Bellac M Le. Thermal Field Theory. Cambridge: Cambridge University Press, 1996. 68

7 HOU De-Fu, Heinz U, LI Jia-Rong. Chin. Phys. Lett., 1999, **16**: 709—711

8 Pisarski R D. Phys. Rev. Lett., 1989, **63**:1129; Braaten E, Pisarski R D. Nucl. Phys., 1990, **B337**:569

9 Arnold P, Yaffe L G. Phys. Rev., 1995, **D52**:7208

热密QED中的德拜质量*

方洁[1) 侯德富

(华中师范大学粒子物理研究所 武汉 430079)

摘要 从QED中光子传播子的Schweinger-Dyson方程出发,得到一个有用的德拜质量和热力学压强之间的关系. 利用这个关系以及有限温度与有限化学势下的相关压强计算了德拜质量的双圈修正. 其结果显示双圈修正减少了德拜屏蔽质量. 最后还讨论了QED等离子体中的磁质量.

关键词 德拜质量 自能 热力学压强

2003-09-29收稿,2003-12-09收修改稿

*国家自然科学基金(10135030, 10005002)资助

1) E-mail:fjie@iopp.ccnu.edu.cn