A Method of Evaluating Discrepant Data

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Abstract In present paper several methods of evaluating discrepant data are introduced and compared each other briefly. The advantage and disadvantage of these methods(WM: weighted mean, LRSW: Limitation of relative statistical weights, MBAYS: Modified Bayesian Technique, NR: Normalized Residuals, and RA: Rajeval technique) are outlined. On the basis of these analysis and comparison, a new method i.e. Double-Mean method (DM) of evaluating discrepant data was proposed. The Double-Mean method takes into account the experimental uncertainties of different authors and all the available experimental information is fully considered. Thus the evaluated values of DM are less dependent on discrepant data and the uncertainties of the evaluated values are more reliable, i.e., the deviation between the evaluated results and the “true” values smaller than the other evaluation methods. The application of using the measured half-life and γ emission probability for ^{7}Be is given as an example. The results of half-life and γ emission probability deduced from the present work are $T_{1/2}$=(53.282±0.012)d and P_{γ}=(10.45±0.04)%, respectively.

Key words evaluation method, discrepant data, Double-Mean method

1 Introduction

The nuclear data evaluators often meet discrepant set of data. They must decide whether they really are, which ones are the more doubtful, and determine the “best” method of deriving a “best” value and its standard deviation from the discrepant set of data. In order to solve this problem, many data evaluation procedures have been proposed by several authors in recent years. These methods proposed assume that the incorrect uncertainties are responsible for the data discrepancy and usually modify them by a common factor by keeping unchanged the measured values.

In present paper several methods of evaluating discrepant data are introduced and compared briefly. The advantage and disadvantage of these methods are outlined. On the basis of these analysis and comparison, a new method i.e. Double-Mean method of evaluating discrepant data is proposed. The application of using the measured half-life and γ emission probability for ^{7}Be is given as an example.

2 Data evaluation methods

In the following discussion, x_i and σ_i refer to individual data and their associated uncertainties respectively and N refer to the number of measurements. The nuclear data evaluation methods using statistical techniques to analyze the discrepant data sets are summarized as follows.

2.1 Unweighted mean(UWM)

The unweighted mean for N measurements is given by

$$x_{\rm u}=\frac{\sum x_i}{N}\ , \tag{1}$$

with associated uncertainty

$$\sigma_{\rm u}=\sqrt{\frac{\sum(x_i-x_{\rm u})^2}{N(N-1)}}\ . \tag{2}$$

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2.2 Weighted mean(WM)

The weight associated with measurement i is given by

$$w_i = 1/\sigma_i^2 \tag{3}$$

and the total weight is given by

$$W = \sum_{i=1}^{N} w_i \ . \tag{4}$$

The weighted mean is then given by

$$x_{\mathrm{w}} = \frac{\sum w_i x_i}{W} \tag{5}$$

with the larger of the internal error

$$\sigma_{\mathrm{w}} = \sqrt{1/W} \tag{6}$$

and external error

$$\sigma_{\mathrm{e}} = \sqrt{\frac{\sum \frac{(x_i - x_{\mathrm{w}})^2}{\sigma_i^2}}{N-1}} \sigma_{\mathrm{w}} \ . \tag{7}$$

2.3 Limitation of relative statistical weights (LRSW)[1, 2]

A relative statistical weight is defined as w_i/W. To avoid any single datum having too much influence on determining the weighted mean, LRSW prescribes that no single datum should have a relative statistical weight greater than 0.50 when determining the weighted mean of a data set. The uncertainty of any datum should be increased until its relative statistical weight is reduced to 0.50. Then LRSW procedure compares the unweighted mean with the new weighted mean. If their uncertainties overlap, i.e.

$$\left| x_{\mathrm{u}} - x_{\mathrm{w}} \right| \leqslant \sigma_{\mathrm{u}} + \sigma_{\mathrm{w}} \ , \tag{8}$$

the weighted mean should be adopted. If their uncertainties did not overlap, the data were inconsistent and it would be safer to use the unweighted mean. In either case the uncertainty is increased, if necessary, to cover the most precise value in the data set.

2.4 Normalized residuals(NR)

The normalized residuals method was introduced by James et al.[3], in which the uncertainties of only the discrepant data are adjusted. Such discrepant data are identified on the basis of the normalized residual R_i which is defined as:

$$R_i = \sqrt{\frac{w w_i}{w - w_i}} (x_i - w_{\mathrm{w}}) \ . \tag{9}$$

A limiting value of the normalized residual R_0 for a set of n values is defined as:

$$R_0 = \sqrt{1.8 \ln N + 2.6} \quad \text{for} \quad 2 \leqslant N \leqslant 100 \tag{10}$$

If any value in the data set has $|R_i| > R_0$, the weight of the value with the largest R_i is reduced until the normalized residual is reduced to R_0. This procedure is repeated until no normalized residual is greater than R_0. The weighted mean is then recalculated with the adjusted weights.

2.5 Rajeval(RA)

Rajput and MacMahon[4] proposed this method in 1992. This method shares the same basic principle as the normalized residuals method in that the uncertainties of only the more discrepant data are adjusted. The method comprises three stages:

(1) Outliers in the data set are detected by calculating the quantity y_i

$$y_i = \frac{x_i - x_{\mathrm{u}i}}{\sqrt{\sigma_i^2 + \sigma_{\mathrm{u}i}^2}} \ , \tag{11}$$

where $x_{\mathrm{u}i}$ is the unweighted mean of all the data set excluding x_i, and $\sigma_{\mathrm{u}i}$ is the standard deviation associated with $x_{\mathrm{u}i}$. The critical value of $|y_i|$ is 1.96 at 5% significance level for a two-tailed test. Measurements with $|y_i| > 3 \times 1.96$ are considered to be outliers and may be excluded from further stages in the evaluation.

(2) Inconsistent measurements that remain in the data set after the population test are revealed by calculating a standardized deviate Z_i:

$$Z_i = \frac{x_i - x_{\mathrm{w}}}{\sqrt{\sigma_i^2 - \sigma_w^2}} \tag{12}$$

for each Z_i the probability integral

$$P(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-t^2}{2}\right) \mathrm{d}t \tag{13}$$

is determined. The absolute difference between $P(z)$ and 0.5 is a measure of the central deviation (CD). A critical value of the central deviation (cv) can be determined by the following expression:

$$cv = [0.5^{N/(N-1)}] \quad \text{for} \quad N > 1 \ . \tag{14}$$

(3) If the central deviation of any value is greater than the critical value, that value is regarded as inconsistent. The uncertainties of the inconsistent values are adjusted to σ_i':

$$\sigma_i' = \sqrt{\sigma_i^2 + \sigma_{\rm w}^2} \ . \tag{15}$$

An iteration procedure is adopted in which $\sigma_{\rm w}$ is recalculated each time and added in quadrature to the uncertainties of those values with $CD > cv$. The iteration process is terminated when all $CD < cv$.

2.6 Modified bayesian technique(MBAYS)[5, 6]

A Bayesian data evaluation technique has been proposed by Gray et al[5]. Nothing is assumed to be known about the extent to which the experimentalists estimated their uncertainties incorrectly, and therefore an uninformative prior density is used as an error probability density function. The recommended value is the weighted mean with a standard deviation given by [6] as following,

$$\sigma_{\rm b} = \sqrt{\frac{\sum \frac{(x_i - x_{\rm w})^2}{\sigma_i^2}}{N-2}}\ \sigma_{\rm w} \ . \tag{16}$$

2.7 Double mean (DM)

UWM is influenced by outliers in the data and takes no account of the fact that different authors made measurements of different precision, so some of the measured information is lost and therefore to be avoided if possible.

The WM can be heavily influenced by discrepant data with small quoted uncertainties, and would only be acceptable where the reduced chi-squared is small, i.e. close to unity. If the value of chi-squared is very high, indicating inconsistencies in the data.

The LRSW still chooses the weighted mean but inflates its associated uncertainty to cover the most precise value. In this case, therefore, both the LRSW value and its associated uncertainty are heavily influenced by the most precise value of the data set.

The NR and RA have been developed to address the problems of the other techniques and to maximise the use of all the experimental available information. They use different statistical techniques to reach the same objective: that is to identify discrepant data and to increase the uncertainties of only such data to reduce their influence on the final weighted mean. In general the Rajeval Technique makes larger adjustments to the uncertainties of discrepant data than the Normalized Residuals Technique does, and has a lower final uncertainty.

The MBAYS method use the weighted mean as the recommended value and alter only the magnitude of the recommended uncertainty. Compared the NR to RA method, MBAYS can derive the more reliable uncertainty[6] because both NR and RA method sometimes underestimate uncertainties and sometimes overestimate them.

On the basis of the above analysis and comparison, a new method of evaluating discrepant data is proposed in the present work, i.e. the DM. The DM procedure compares the MBAYS mean with the NR mean. If their uncertainties overlap, i.e.

$$\left| x_{\rm N} - x_{\rm b} \right| \leqslant \sigma_{\rm N} + \sigma_{\rm b} \ , \tag{17}$$

the mean of the MBAYS, NR and RA with the larger of the three uncertainties should be adopted. If their uncertainties do not overlap, the mean of the NR and RA with the larger of the two uncertainties should be adopted.

3 Application

3.1 Half-life

All the measured values and their uncertainties of the half-life of ^{7}Be are collected and listed in Table 1 with the chronological order of their publication. In Table 1 it also listed the results of applying several evaluation method as each new data point is added to the set.

Table 1 shows that there are significant differences from the ways of the evaluation techniques in the case less number of discrepant data. The NR and RA recover much more quickly than the other techniques.

From Table 1 it is easy to get:

$$\chi_{\rm b} = 53.292,\ \sigma_{\rm b} = 0.012,\ \chi_{\rm N} = 53.282,\ \sigma_{\rm N} = 0.006;$$

$$\chi_{\rm N} - \chi_{\rm b} = 0.01;\ \sigma_{\rm b} + \sigma_{\rm N} = 0.018,$$

$$|\chi_{\rm N} - \chi_{\rm b}| < \sigma_{\rm b} + \sigma_{\rm N} \ .$$

According to the DM method, A value of (53.282±0.012)d can be adopted as the current best estimate of the half-life of ^{7}Be.

3.2 γ emission probability

^{7}Be is ε decay to ground state and the first excited state (477.6210keV) of ^{7}Li. Only 477.6035keV γ-ray can be emitted. The measured values and their uncertainties are collected and listed in Table 2 with the chronological order of their publication.

From Table 2 it is easy to get:

$$\chi_{\mathrm{b}}=10.449, \quad \sigma_{\mathrm{b}}=0.041, \quad \chi_{\mathrm{N}}=10.449,$$

$$\sigma_{\mathrm{N}}=0.044; \quad \chi_{\mathrm{N}}-\chi_{\mathrm{b}}=0;$$

$$\sigma_{\mathrm{b}}+\sigma_{\mathrm{N}}=0.085, \quad |\chi_{\mathrm{N}}-\chi_{\mathrm{b}}|<\sigma_{\mathrm{b}}+\sigma_{\mathrm{N}} .$$

According to the DM method, A value of (10.45±0.04)% can be adopted as the current best estimate of the γ emission probability P_{γ}(477.6keV) for ^{7}Be.

Table 1. Comparision of the measurement and evaluation by several evaluation methods of the half-life of ^{7}Be(all data in days).

authors	Exp.	WM	LRSW	NR	RA	MBAYS	DM
Segre[7]	52.93(22)	52.93(22)	52.93(22)	52.93(22)	52.93(22)	52.93(22)	52.93(22)
Kraushaar[8]	53.61(17)	53.356(329)	53.270(340)	53.311(200)	53.311(200)	53.356(329)	53.326(339)
Bouchez[9]	53.0(4)	53.320(233)	53.278(332)	53.209(153)	53.126(281)	53.320(330)	53.218(330)
Wright[10]	53.5(2)	53.372(167)	53.372(238)	53.372(108)	53.471(119)	53.372(205)	53.405(205)
Eugland[11]	53.1(3)	53.341(143)	53.341(269)	53.341(101)	53.354(121)	53.341(165)	53.345(165)
Vaninbroukx[12]	53.20(55)	53.336(126)	53.336(274)	53.336(100)	53.363(112)	53.336(141)	53.345(141)
Merritt[13]	53.284(6)	53.284(7)	53.310(82)	53.284(6)	53.284(6)	53.284(8)	53.284(8)
Johlige[14]	53.52(10)	53.285(8)	53.356(72)	53.285(6)	53.326(47)	53.285(9)	53.299(47)
Cressy[15]	53.0(3)	53.285(8)	53.345(62)	53.285(6)	53.284(6)	53.285(9)	53.285(9)
Lagoutiue[16]	53.17(17)	53.285(8)	53.328(57)	53.285(6)	53.284(6)	53.285(8)	53.285(8)
Rutledge[17]	53.284(4)	53.284(4)	53.284(5)	53.284(3)	53.284(3)	53.284(4)	53.284(4)
Jaeger[18]	53.12(7)	53.284(5)	53.284(6)	53.284(5)	53.284(3)	53.284(5)	53.284(5)
Huh[19]	53.42(1)	53.297(12)	53.302(18)	53.285(8)	53.335(32)	53.297(13)	53.306(32)
Norman[20]	53.107(22)	53.294(14)	53.232(52)	53.284(8)	53.242(36)	53.294(15)	53.273(36)
Norman[20]	53.174(37)	53.293(14)	53.228(56)	53.283(7)	53.204(24)	53.293(14)	53.260(24)
Norman[20]	53.195(52)	53.292(13)	53.226(58)	53.282(7)	53.204(21)	53.292(14)	53.259(21)
Norman[20]	53.311(42)	53.292(13)	53.231(53)	53.282(7)	53.238(18)	53.292(13)	53.271(18)
Liu[21]	53.270(19)	53.292(12)	53.233(51)	53.282(6)	53.267(12)	53.292(13)	53.280(13)
Liu[21]	53.275(25)	53.292(12)	53.235(49)	53.282(6)	53.271(10)	53.292(12)	53.282(12)

Note: the digits in brackers are uncertainty of the results.

Table 2. Comparision of the measurement and evaluation by several evaluation methods of γ emission probability P_{γ}(477.6keV) for ^{7}Be.

authors	Exp.	WM	LRSW	NR	RA	MBAYS	DM
Tayor[22]	0.1032(16)	0.1032(16)	0.1032(16)	0.1032(16)	0.1032(16)	0.1032(16)	0.1032(16)
Poenitz[23]	0.1042(18)	0.10364(120)	0.10370(127)	0.10364(120)	0.10364(120)	0.10364(120)	0.10364(120)
Goodier[24]	0.1035(8)	0.10354(66)	0.10357(85)	0.10354(66)	0.10354(66)	0.10354(28)	0.10354(66)
Balamuth[25]	0.1010(45)	0.10349(66)	0.10348(82)	0.10349(66)	0.10349(66)	0.10349(32)	0.10349(66)
Davids[26]	0.1061(23)	0.10369(63)	0.10375(73)	0.10369(63)	0.10369(63)	0.10369(47)	0.10369(63)
Donoghue[27]	0.1060(50)	0.10372(63)	0.10379(72)	0.10372(63)	0.10372(63)	0.10372(43)	0.10372(63)
Knapp[28]	0.1090(50)	0.10380(62)	0.10389(70)	0.10380(62)	0.10380(62)	0.10380(48)	0.10380(62)
Mathews[29]	0.1070(20)	0.10409(59)	0.10416(66)	0.10409(59)	0.10409(59)	0.10409(56)	0.10409(59)
Norman[30]	0.980(50)	0.10400(59)	0.10405(62)	0.10400(59)	0.10400(59)	0.10400(58)	0.10400(59)
Evans[31]	0.1040(70)	0.10400(59)	0.10405(61)	0.10400(59)	0.10400(59)	0.10400(54)	0.10400(59)
Fisher[32]	0.1061(17)	0.10423(56)	0.10423(73)	0.10423(56)	0.10423(56)	0.10423(53)	0.10423(56)
Skelten[33]	0.1049(7)	0.10449(44)	0.10449(44)	0.10449(44)	0.10449(44)	0.10449(41)	0.10449(44)

Note: the digits in brackers are uncertainty of the results.

4 Comparison with other evaluations

The present recommended half-life and γ emission probability were compared with other evaluations and listed in Table 3.

Table 3. Comparison of the evaluated half-life and γ emission probability (477.6keV γ-ray) for ^{7}Be radionuclide.

$T_{1/2}$/d	P_γ(%)	references
53.282±0.012	10.45±0.04	present work
53.22±0.06	10.44±0.04	[34]
53.29±0.07	10.52±0.06	[35]
53.23±0.06	10.60±0.20	[36]

5 Conclusion

Several methods of evaluating discrepant data are introduced and compared each other briefly. The advantage and disadvantage of these methods are outlined. On the basis of these analysis and comparison, a new method of evaluating discrepant data is proposed in present work, i.e. the Double-Mean method(DM).

The DM procedure compares the MBAYS mean with the NR mean. If their uncertainties overlap, the mean of the MBAYS, NR and RA with the larger of the three uncertainties should be adopted. If their uncertainties did not overlap, the mean of the NR and RA with the larger of the two uncertainties should be adopted.

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评价有分歧实验数据的新方法

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摘要 简要介绍了有分歧实验数据的几种数据处理方法, 分析比较了这些方法的各自优缺点, 在此基础上提出了一新处理方法——二次平均法, 并以 ^{7}Be 的半衰期和 γ 发射几率为例, 说明了其具体应用.

关键词 评价方法 有分歧实验数据 二次平均法

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