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Distribution System Planning Usign Mixed Integer Programming

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Abstract

This paper presents an improved mathematical model to optimise the size and locations of substations and the network routing problem. The model was formulated to minimise the total cost of the network by determining the optima of the substation locations and power, the load transfers between the demand centers, the feeder routes and the load flow in the network subject to a set of constraints. The computational results of a devised sample problem indicate that the developed optimisation model and its code are adequate for computer aided planning of distribution systems.

1. Introduction

The optimal planning of a distribution system is an important to decrease the cost of installation with regard to construction, materials and equipment of the system. Distribution systems planning is a fairly complex procedure since a large number of variables are involved and also the mathematical modeling is quite a difficult task considering many requirements and restrictions imposed by the configuration of the system.

The main planning approaches of the distribution systems consists of the following methods,

i) The alternative policy method which compares a number of alternative policies and selects the best.

ii) The decomposition approach in which a large optimisation problem is divided into several smaller subproblems and each one is solved separately.

iii) The linear programming and integer programming methods where the constraint conditions are linearised.

iv) The dynamics programming method.

A significant number of studies have been devoted to the optimisation o distribution systems using an computational methods. Knight [1] is one of the pioneering researchers who formulated and solved the optimisation problem of distribution systems using an integer programming technique. A dynamic programming method was utilised by Oldfield and Lang [2] and later by adams and Laughton [3] to make a compromise between the difficulties due to the large number of variables, the complexity of the design process, and the computational advantage to be gained by searching for optimality. Oldfield and Lang have suggested a two-stage planning method in which the processes of design and optimisation are applied consecutively rather than simultaneously. The model used by Adams and Laughton includes cost of linearisation of feeder copper losses and it determines load transfer schemes and substation installation dates by minimising the cost of substation transformer losses. Their dynamic-programming technique that uses the branch and bound algorithm examines all possible combinations of expansion alternatives at each stage of the design. However, this approach does not necessarily generate the optimal expansion plan since minimising the costs for each year does not necessarily minimise the present value of all costs throughout the study period.

Crawford and Holt [4] have employed a linear programming approach utilising the transportation algorithm to optimise substation service areas by minimising the products of demand and the distances from the substation. This technique minimises distribution feeder losses but it does not necessarily arrive at the optimal expansion plan since it does not minimise the present value of costs associated with expansion.

The model developed by Masud [5] consists of a zero-one integer programming approach to optimise substation transformer capacity and a linear programming approach to optimise the load transfers. The procedure involves first minimising substation transformer capacities for each year and then optimising the load transfers. However, it does not minimise the present value of the expansion costs.

Gönen and Foote [6] have developed a mathematical model and utilised mixed integer programming to determine the optimal design of distribution systems. The interesting aspect of their approach is that they linearized the nonlinear cost curve by using piecewise linear equations at the expense of increasing the number of variables.

Carson and Cornfield [7, 8] utilised a heuristic approach in which the discrete cost function is converted into a continuous one. They developed their method to determine the near optimal design of radial networks.

The branch and bound algorithm of the linear programming method have been employed by Hindi and Brammeller [9,10] to find an optimal solution to the design problem. The model is concerned with the optimal locations of transformer substation sites and cable routes.

Ponnavaikko and Rao [20] utilised the Quadratic Mixed Integer Programming method. Their model includes the substation fixed cost, cost of transformation losses and the cost of feeder losses. In this approach, the problem is solved in two steps, first, using the simplex method and second the quadratic mixed integer programming algorithm.

Hsu and Chen [21] developed a knowledge-based expert system for distribution system planning. However, their model has a few drawbacks. The objective function consists only of the cost of feeder losses without including the significant costs such as investment and losses from the substation transformer. Moreover, the optimisation model contains only the feeder capacity limit and the transformer capacity limit without taking other constraints into account.

Jonnavithula and Billinton [23] have formulated the objective function as the sum of outage cost, the cost of feeder resistive loss the cost of investment and maintenance, without the cost of substation transformer loss. However, the distribution feeder routing problem has been considered static expansion planning with a single planning period.

In this study a general mathematical model of the optimal design problem of distribution systems is formulated and programmed using mixed integer programming. The model was solved using a computer code developed in Fortran 77. The program was been tested in real systems described in [11,12]. The developed model can assist the designer to select the optima of,

i) Substation locations,

ii) Substation transformer sizes,

iii) Load transfers between substations and between demand centers,

iv) Feeder routes subject to a set of specified constraints.

2. Mathematical Model

A distribution system can be modeled effectively as a mixed integer programming problem with the substations as sources and the loads on the feeders as demands. In this study, the objective function was designed, as the minimisation of the present value of the capital, i.e., the fixed cost of distribution system installation and the present value of the variable costs associated with the power losses, subject to restrictions which relate substation transformer capacities and feeder ratings to system load projections [11,12]. The objective function was formulated as,

$$\min Z = \sum_{i=1}^{N_S} a_i \delta_i + \sum_{i=1}^{N_S} \sum_{j=1}^{N_D} b_{ij} P_{ij} + \sum_{i=1}^{N_T} \sum_{j=1}^{N_D} c_{ij} L_{ij} \beta_{ij} + \sum_{i=1}^{N_T} \sum_{j=1}^{N_D} d_{ij} L_{ij} P_{ij} \tag{1}$$

where N_S, N_T and N_D denote the integer numbers of potential substation,total nodes and demand nodes, respectively. The other variables are defined in the nomenclature.

The optimisation problem was subject to the following constraint equations which formulate the limitations imposed by the network conditions and design variables. The notation of the variables in equations below are given in the nomenclature .

i) The load demanded by the consumer at each node should be supplied in all conditions. This is expressed mathematically as,

$$\sum_{i=1_{i \neq j}}^{N_s} (P_{ij} - P_{ji}) \geq P_{tj} \qquad j = 1, 2, 3, \ldots, N_D \tag{2}$$

ii) The power transmitted through each line should not be above its thermal power capacity, i.e.,

$$P_{ij} \leq P_{ij}^{\max} \beta_{ij} \qquad i = 1, 2, 3, \ldots N_T, \quad j = 1, 2, 3, \ldots, N_D \tag{3}$$

iii) There should not be any exit line from an unselected transformer and the number of line exits from a selected transformer should not be greater than N_F,

$$\sum_{j=1}^{N_d} \beta_{ij} \leq N_F \delta_i \qquad (i = 1, 2, \ldots, N_S) \tag{4}$$

iv) The maximum number of substations to be installed in a given area is assumed to be $N_{\max}$,

$$\sum_{i=1}^{N_s} \delta_i \leq N_{\max} \tag{5}$$

v) The power flow in the lines is unidirectional,

$$\beta_{ij} + \beta_{ji} \leq 1 \quad i = 1, 2, \ldots, N_D \tag{6}$$

vi) The total power loss in the distribution system should be less than the assumed maximum power loss of the network,

$$(A \cos\varphi + BP_{ij})100 \leq Z_{\max} P_{total} \cos\varphi \ i = 1, 2, 3, \ldots, N_D, \ j = 1, 2, \ldots, N_D \tag{7}$$

vii) The constraint of the mathematical programming is specified as,

$$P_{ij} \geq 0 \quad i = 1,2,3,\ldots,N_D,\ j = 1,2,\ldots,N_D \tag{8}$$

ix) If a distribution substation is to be built at site i then,

$$\delta_i = 1,\ \text{otherwise}\ \delta_i = 0$$

If the line between the nodes i and j is selected then,

$$\beta_{ij} = 1,\ \text{otherwise}\ \beta_{ij} = 0 \tag{9}$$

The optimisation problem consists of minimising the objective function given by equation (1) subject to inequality constraint equations (2) to (9).

The equations (8) and (9) are called the constraints of canonical programming. Normally, the power flow in a network should not be negative, i.e., there can not be a flow from the demand node to the supply node. Equation (8) guarantees this condition. Equation (9) is the constraint imposed by the mixed integer programming method itself. This equation ensures that the binary integer variables are either zero or one. The inequalities are converted to equalities by the addition of slack variables which is taken into account within the computer code.

The power loss computed by the design optimisation program should be within an acceptable limit. The previous optimisation implementations did not include this constraint which is considered to be a significant factor in the design of distribution systems. The constraint equation (7) satisfies this condition and it constitutes a contribution to the mathematical modeling of the optimal design of distribution systems.

In general, the power loss in a feeder is given by,

$$P_z = 3R_h I^2 \tag{10}$$

where R_h and I denote the resistance and current of the feeder, respectively.

The ratio of power loss is given by,

$$z = \frac{P_z}{P_{total}} 100 \tag{11}$$

where P_z denotes the power loss in the feeder.

The power loss of a line as a function of the current of equation (10) is shown in Figure 1. Since linear programming is used in the mathematical model, it is necessary to assume a linear relationship between unit power loss and power. To this end, a tangent to the minimum cross section line with maximum slope is taken into account. This tangent line represents the maximum power loss that may occur in the lines. The tangent equation is given by,

$$P_{Z_{tangent}} = 0.448S - 13428571.4 \tag{12}$$

where S denotes the power across the line.

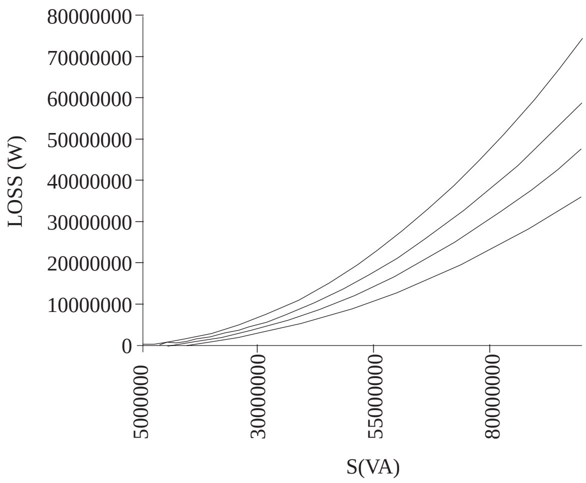

Figure 1. Power Loss of a feeder at 34.5 kV

3. Formulation of Costs

The objective function was defined as the sum of present value of total annual feeder costs and total annual substation costs. The formulations of the feeder and substation costs are described.

3.1. Feeder Cost

In general the distribution feeder costs consist of, (a) the cost of investment, (b) the cost of lost energy due to I^2R_h losses in the feeder and (c) the cost of lost demand (i.e., the cost of lost capacity) due to I^2R_h losses. The cost of investment is the biggest cost component which includes material and labor costs involving feeders. Hence, the total cost of a given distribution feeder can be formulated as [11-14],

$$TAFC = AIC + AEC + ADC \tag{13}$$

where,

TAFC : total annual feeder cost per unit length
AIC : annual investment cost per unit length
AEC : annual energy cost per unit length
ADC : annual demand cost per unit length.

The annual investment cost (i.e., fixed cost of a given feeder) is the installation cost of the feeder multiplied by the fixed cost rate of the feeder. This fixed cost rate or the so-called carrying charge rate of the feeder includes mainly the cost of capital, taxes, insurance, operation and maintenance, depreciation, and possible others. This can be expressed as,

$$AIC = IC_F i \tag{14}$$

where the variables are defined in the nomenclature [11-14].

The annual energy cost due to I^2R_h losses in the feeders is calculated using the following calculation,

$$AEC = 3I^2R_h f_E F_{LL} F_{LS} 876010^{-3} \tag{15}$$

where f_E, F_{LL} and F_{LS} denote energy cost per kWh, load location factor and the loss factor, respectively [11,12].

The load location factor is a per unit value which is considered to be that point on a feeder having distributed loading where the total feeder load can be assumed to be concentrated for the purpose of I^2R_h loss calculations.

The loss factor is the ratio of the average power loss over a years period to the peak loss occurring in that period. This can also be defined as the ratio of the actual total kWh losses to what the kWh losses would have been if the peak losses had continued throughout the 8760 hours in the year. The loss factor is the annual power loss divided by the annual peak which approximated by the following equation,

$$F_{LS} = 0.3F_{LD} + 0.7F_{LD}^2 \tag{16}$$

where FLD denotes load factor [6].

The annual demand cost maintains an adequate system capacity in order to supply the $\mathrm{I}^2\mathrm{R}_h$ losses in the feeder conductors which is expressed as,

$$ADC = 3I^2R_hF_{LL}F_r\{(c_Gi_G) + (c_Ti_T) + (c_Si_S)\}10^{-3} \tag{17}$$

where F_r, c_G, c_T, c_S denote the reserve factor, cost of the generation system, cost of the transmission system, the substation, respectively [11,12]. The reserve factor is the ratio of total generation capability to the sum of total load and losses to be supplied.

3.2. Total Annual Equivalent Feeder Cost

For the analyses of conductor use the present value of leveled annual fixed charges on the total line capital investment plus annual expenses for line losses is considered [13,14].

$$TAFC_{PW} = \sum_{n=1}^{NYE}\left(1 + \frac{i}{100}\right)^{-n}(AIC + AEC + ADC) \tag{18}$$

A total annual equivalent feeder cost of distribution system with respect to the power flow across a feeder is shown in Figure 2. The curve is drawn using equation (18). The envelope curve shown in Figure 2 is the one that minimizes the cost. However, since mixed integer programming is a linear programming method it is necessary to use a linear equation representative of the nonlinear cost curve. This is achieved by using a tangent line passing through the minimum of the envelope curve. Since the fixed charge approach was assumed in the mathematical model, the cost per unit line length is approximated by the following line equation ,

$$TAFC_{PW_{tangent}} = 3602.4S + 1200735.2 \tag{19}$$

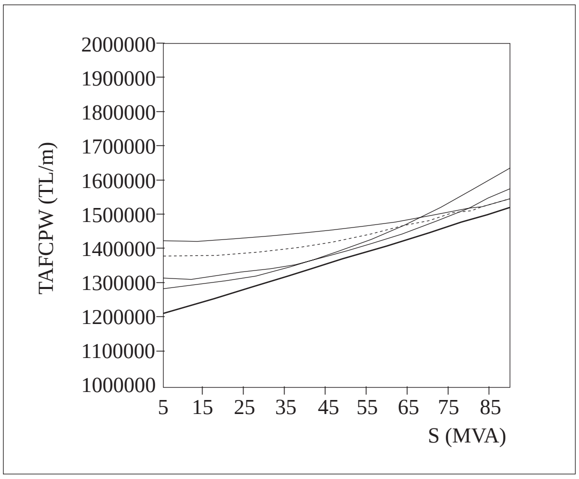

Figure 2. Annual Total feeder cost

3.3. Substation Cost

The data required for each substation are its capacity, its location and its fixed and variable costs. The substation fixed cost includes the cost of transformers and other equipment at the substation and the cost of construction. The substation variable cost includes the cost of power losses in substation transformers, annual operating and maintenance costs [11,12].

The total cost of a substations is formulated as,

$$TASC = AIC_{tr} + AVC_{tr} \tag{20}$$

where AIC_{tr} and AVC_{tr} denote annual investment cost of the substation and annual variable cost of the substation.

The annual investment cost of the substation is assumed to be a given percentage of the total cost of the material expenses required to build up the substation. This is expressed as,

$$AIC_{tr} = IC_{tr} i \tag{21}$$

where IC_{tr} and i denote the investment cost of substation and fixed charge rate, respectively.

To calculate the loss cost of a substation it is necessary to take into account the core and copper losses. These are formulated, respectively, as,

$$AVC_{tr_{Fe}} = (ga_K + 8760 f_E) P_0 \tag{22}$$

and

$$AVC_{tr_{cu}} = (ga_K + 8760\theta f_E) P_{cun} (S/S_n)^2 \tag{23}$$

where the variables are denoted in the notation list. The annual loss cost of the substation is the sum of the cost of core and copper losses i.e.

$$AVC_{tr} = AVC_{tr_{Fe}} + AVC_{tr_{cu}} \tag{24}$$

In these calculations it is assumed that the substation is 154 kV/34.5 kV. The cost of a substation as a function of the load is shown in Figure 3. Using the same approach as that used to derive equation (19), the linear relation between the substation cost and the load is expressed as,

$$TASC_{tangent} = 188235.29S + 6666666667.00 \tag{25}$$

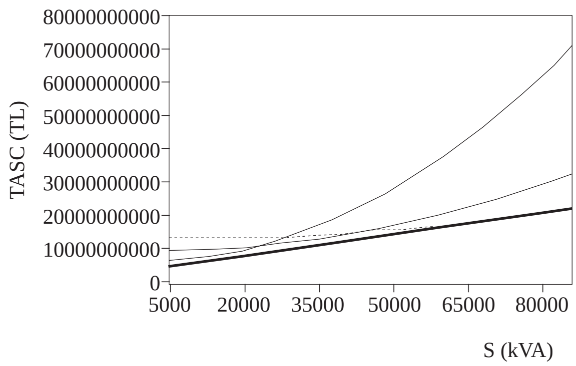

Figure 3. Annual Total transformer cost

4. An Applicational Example

To test the solvability of the model a sample problem was devised and solved. Consider the example plan shown in Figure 4 taken from [20]. However, some data necessary for this study, did not exist in [20]. The data for this study was collected from various Turkish sources. The problem was to select the optimal 34.5 kV feeder routings and the optimal locations for 154/34.5 kV substations in order to feed the demands at the 34.5/10 kV substations in the area. The general data and the load data are given in Tables 1 and 2. The feasible potential sites for locations of the grid substations and feasible routes for the 34.5 kV feeders are shown in Figure 4. It was assumed that there are two possible locations for the transformers.

Table 1. Data of example network

Cost of energy	2700 (TL/k Wh)	Interest rate	% 14
Load factor	0.38	Production cost	178 10^8 (TL/MW)
Power factor	0.9	Distribution substation cost	20 10^8 (TL/kVA)
Power loss ratio	% 16	Production fixed cost rate	% 21
Reserve factor	1.15	Transmission fixed cost rate	% 18
Nominal voltage	34.5 kV	Substation fixed cost rate	% 18

Table 2. Load data

Node No	Demand (MVA)	Node No	Demand (MVA)
3	5	7	3
4	3	8	5
5	4	9	6
6	3		

The studied example was a problem with eight demand locations, two feasible potential sites for constructing substations and sixteen feasible branch elements. This resulted in a forty variable problem with fifty two constraints according to the model described in the previous sections. The optimisation computer was developed according to the cutting plane algorithm which is one of the algorithms of the mixed integer programming method [11, 12]. The implementation of the optimisation program to the assumed design example generated the substation 1 with 50 MVA as the optimal solution. The resulting optimal distribution system is shown in Figure 4. The cost of the network was determined to be 1,61 10^{10} TL. The power flowing through each line segment are shown in Table 3. The overall results confirm that the losses in the network were within the assumed limits while the cost of design was minimised. The traditional manual calculations of the voltage drops using the resulting design obtained in Figure 4 were found to remain within acceptable limits. Thus, the mathematical model and the computer code were considered to be satisfactory for computer aided design of a medium distribution system.

Table 3. Output of the computer program

From Node	To Node	Integer Variable	Power flow (MVA)	From Node	To Node	Integer Variable	Power flow (MVA)
1	9	1	6	1	2	0	0
1	7	1	8	3	2	0	0
7	8	1	5	2	4	0	0
1	5	1	7	2	6	0	0
1	3	1	8	2	5	0	0
3	4	1	3	2	7	0	0
5	6	1	3	4	6	0	0

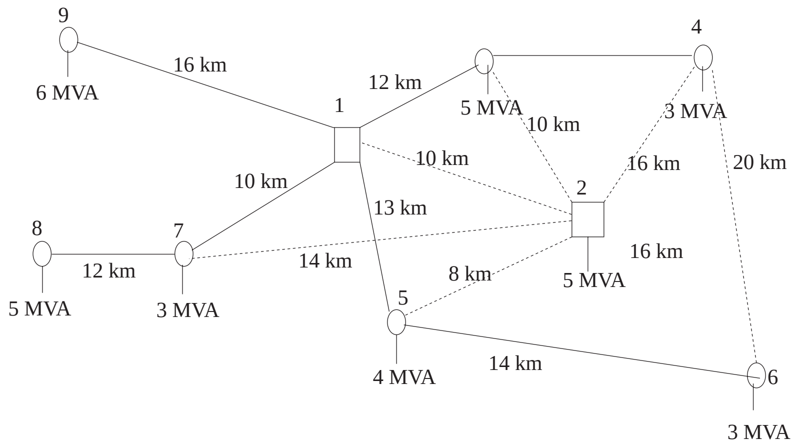

Figure 4. Sample System

5. Conclusions

A general mathematical model for the optimisation of distribution systems was improved. The model was coded using mixed integer programming with has two different algorithms: the branch and bound and the cutting plane algorithms. To the author's knowledge the cutting plane algorithm was applied for the first time in this study for a faster computation [15]. The optimisation program minimized the total cost of the distribution system as the objective function by determining the optima of the number, locations and powers of substation, the routes of the feeders and the power losses within the network subject to a set of constraints.

In previous studies of design optimisation of distribution systems, the operating cost due to losses of the transformers was not included in the mathematical models. In this respect the approach used to derive the cost equation (25) and its inclusion in the optimisation model constituted a contribution to the optimal design of distribution systems. Furthermore, this formulation makes it possible to obtain the optimal power of the transformer as an output of the computer code instead of using estimates from engineering practice. The other contribution was that the constraint of power loss within the network was incorporated in the mathematical model. This was taken into account with the constraint equation (7).

The overall results confirmed that the losses in the network remained within acceptable limits while the cost of design was minimised. Hence, the mathematical model and the computer code were found to be satisfactory for computer aided design of a distribution system according to the results obtained from the applicational example.

Nomenclature

Z	: Objective function,
a_i	: Present value of fixed costs of substations i,
b_{ij}	: Present value of variable costs of substation i,
c_{ij}	: Present value of fixed costs of feeder between nodes i and
j, d_{ij}	: Present value of variable costs of feeder between nodes i and
j, P_{ij}	: Power flow in the branch between nodes i and j,
δ_i	: Binary integer variable which denotes the decision to select or not to select site i,
β_{ij}	: Binary integer variable which denotes the decision to select a branch between nodes i and j,
P_{tj}	: Load demand at node j,
N_F	: Total number of feeders that can emanate from substation,
P_{max}	: Upper bound of branch flow from node i to j,
R_h	: Resistance of conductor,
$Cos\varphi$	: Power factor,
IC_F	: Investment cost o the feeders,
i	: Annual fixed charge rate,
f_E	: Cost of energy
i_G	: Annual fixed charge rate applicable to generation system,
i_T	: Annual fixed charge rate applicable to transmission system,
i_S	: Annual fixed charge rate applicable to substation,
IC_F	: Investment cost of the feeders,
S_n	: Nominal power of transformer,
L_{ij}	: Distance between i and j nodes.
NYE	: Number of years to be studied
n	: nth. year

P_0 : Transformer core losses
θ : Transformer loading factor
P_{total} : Total power of the distribution system
$z_{\max}$: Ratio of the power loss
A : Fixed part of the power loss tangent equation
B : Variable part of the power loss tangent equation

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