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A Pseudo Spot Price Algorithm Applied to the Pumped-Storage Hydraulic Unit Scheduling Problem

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Abstract

A lossy electric power system that contains thermal units and a pumped-storage hydraulic unit is considered in this paper. The total cost of thermal units in an operation cycle is minimized under some possible electric and hydraulic constraints of the units. The operation cycle is divided into time intervals where the system loads are assumed to remain constant.

The proposed solution technique has two main parts. In the first part, the active thermal generations and active generation or pumping power of the pumped-storage unit in all time intervals are determined by using units' active generation incremental costs and pseudo spot price of bought active powers. Calculation of active flows of transmission lines, which are used in the determination of the active generations and pumping power of the pumped-storage unit, are obtained by means of Newton-Raphson AC load flow calculation. In the second part, which we call the outer loop, the pseudo active generating/pumping incremental cost value of the pumped-storage hydraulic unit is determined according to its net water usage.

The proposed dispatch technique considers the minimum and maximum reservoir storage limits of the pumped-storage unit, the upper and lower active generation limits of thermal units, and the upper and lower active pumping and generation power limits of the pumped-storage unit in a lossy power system.

The proposed technique was demonstrated on an example power system and the obtained results are presented.

1. Introduction

The main function of pumped-storage hydraulic units in electric power systems is to store cheap surplus electric energy that is available during off-peak load levels as hydraulic potential energy. This is done by pumping water from the lower reservoir of a pumped-storage unit to its upper reservoir. The stored hydraulic potential energy is then used to generate electric energy during peak load levels. Pumped-storage units are generally operated over daily or weekly periods. The operation of a pumped-storage unit over a period can reduce the total thermal generation cost in a power system.

In reference [1], a short-term hydrothermal generation coordination problem including pumped-storage and battery storage energy systems is solved by using multi-pass dynamic programming. Another pumped-storage unit scheduling problem where the pumped storage unit is jointly owned by several utilities is solved by using dynamic programming in [2]. None of the above papers consider transmission losses in their solutions. Some previous papers that do not consider transmission losses in pumped-storage unit scheduling problem can also be found in [3]. Yet another pumped-storage hydraulic unit scheduling problem in a lossy electric power system is solved by using a genetic algorithm in [4].

In the proposed solution technique, the active generations of the generating units are adjusted by changing purchased active powers by each bus in the system. The bought active power by a bus is changed according to the difference between the pseudo spot price of the bought active power and the incremental active generation cost of the bus. The spot price of the bought active power is determined by increasing the seller bus's incremental active generation cost according to the transmission loss percentage that occurs during transmission of the bought active power. In the active generating/pumping power adjustment procedure, the pumped-storage unit can change its operation mode from generation to pumping or vice versa. The pumping power of the pumped-storage unit is adjusted like its generating power. The total net water amount used by the pumped-storage hydraulic unit is controlled by adjusting the unit's pseudo active generating/pumping incremental cost.

The pseudo spot price of electricity is used in [5] to solve the dispatch problem for the specific loading of an electric power system that contains only thermal units.

2. Problem Formulation

The optimization problem which will be considered in this paper can be given mathematically as follows:

$$\text{Minimize } F_T = \sum_{j=1}^{j_{\max}} \sum_{n \in N_s} F_n(P_{n,j}) t_j \quad (R)^1 \tag{1}$$

subject to hydraulic and electric constraints

$$\begin{aligned} P_{load,j} + P_{loss,j} + |P_{PH,j}| - \sum_{n \in N_s} P_{n,j} &= 0, \quad j \in \mathbf{J}_{pump} \\ P_{load,j} + P_{loss,j} - P_{GH,j} - \sum_{n \in N_s} P_{n,j} &= 0, \quad j \in \mathbf{J}_{gen} \end{aligned} \tag{2}$$

$$P_n^{\min} \le P_{n,j} \le P_n^{\max}, \quad n \in N_s, \quad j = 1, \ldots, j_{\max} \tag{3}$$

$$\begin{aligned} &P_{GH}^{\min} \le P_{GH,j} \le P_{GH}^{\max}, \quad j \in \mathbf{J}_{gen} \quad \text{or} \\ &q_{GH}^{\min} \le q_{GH}(P_{GH,j}) \le q_{GH}^{\max}, \quad j \in \mathbf{J}_{gen} \end{aligned} \tag{4}$$

$$\begin{aligned} &P_{PH}^{\min} \le |P_{PH,j}| \le P_{PH}^{\max,} \quad j \in \mathbf{J}_{pump} \quad \text{or} \\ &q_{PH}^{\min} \le q_{PH}(|P_{PH,j}|) \le q_{PH}^{\max}, \quad j \in \mathbf{J}_{pump} \end{aligned} \tag{5}$$

[1] R represents a fictitious monetary unit

$$V^{\min} \leq V_j \leq V^{\max}, \quad j = 1, \ldots, j_{\max} \tag{6}$$

$$V_0 = V_{j_{\max}} = V^{start}. \tag{7}$$

Since the beginning and ending reservoir water volume values of the pumped-storage unit are the same, the total net water amount used by the pumped-storage unit must be equal to zero. That is to say, the total water amount which is used for generation, must be equal to the total pumped water amount.

$$q_{spent\ TOT} - q_{pump\ TOT} = q_{net\ spent} = 0 \tag{8}$$

where

$$q_{spent\ TOT} = \sum_{j \in \mathbf{J}_{gen}} q_{GH}(P_{GH,j}) t_j \tag{9}$$

$$q_{pump\ TOT} = \sum_{j \in \mathbf{J}_{pump}} q_{PH}(|P_{PH,j}|) t_j \tag{10}$$

In the above equations,
F_T = total cost of thermal units in the operation cycle, (R),
$j, j_{\max}$ = time interval index and number of time intervals respectively,
$P_{n,j}, P_{GH,j}$ = the n^{th} thermal and the pumped-storage hydraulic units' generation powers in the j^{th} time interval respectively (MW),
$P_{PH,j}$ = the pumped-storage unit's pumping power, which is a negative quantity, in the j^{th} time interval, (MW),
$F_n(P_{n,j})$ = cost rate of the n^{th} thermal unit in the j^{th} time interval, (R/h),
t_j = length of time interval $j, (h)$,
$P_{load,j}, P_{loss,j}$ = total system load (excluding the pumping power of the pumped-storage hydraulic unit if it is operated in pumping mode) and loss in the j^{th} time interval respectively, (MW),
$P_n^{\min}, P_n^{\max}$ = lower and upper generation limits of the n^{th} thermal unit, (MW),
$P_{GH}^{\min}, P_{GH}^{\max}$ = lower and upper generation limits of the pumped-storage hydraulic unit, (MW),
$P_{PH}^{\min}, P_{PH}^{\max}$ = lower and upper pumping power limits of the pumped-storage hydraulic unit, (MW),
$q_{GH}(P_{GH,j})$ = discharge rate of the pumped-storage hydraulic unit, (*acre-ft/h*)[2]
$q_{PH}(|P_{PH,j}|)$ = pumping rate of the pumped-storage hydraulic unit, (*acre-ft/h*),
$q_{GH}^{\min}, q_{GH}^{\max}$ = lower and upper discharge rate limits of the pumped-storage hydraulic unit, (*acre-ft/h*),
$q_{PH}^{\min}, q_{PH}^{\max}$ = lower and upper pumping rate limits of the pumped-storage hydraulic unit, (*acre-ft/h*),
V_j = stored water volume in the upper reservoir of the pumped-storage hydraulic unit *at the end of* the j^{th} time interval, (*acre-ft*),
$V^{\min}, V^{\max}$ = minimum and maximum reservoir storage limits of the pumped-storage hydraulic unit, (*acre-ft*),
$q_{net\ spent}$ = net spent water amount by the pumped-storage hydraulic unit during the operation cycle, (*acre-ft*),

[2] 1 acre-fet = 1233.5 m ^{3}

$q_{spent\ TOT}, q_{pump\ TOT}$ = total water amount which is spent for generation and total pumped water amount respectively, (*acre-ft*),
V^{start}, V^{end} = starting and ending stored water volumes in the upper reservoir of the pumped-storage hydraulic unit, (*acre-ft*),
N_s = set which contains all thermal units in a given power system.
$\mathbf{J}_{gen}, \mathbf{J}_{pump}$ = sets which contain all time intervals where the pumped-storage unit is operated in generation and pumping modes respectively. They are mutually exclusive sets.

2.1. Determination of Active Generation and Pumping Powers in a Time Interval

Since application of the iterative calculation procedure described in this section is the same for all time intervals, only the one in time interval j will be given here.

An AC load flow is performed with the active thermal generations and active generation or pumping power of the pumped-storage unit which were determined in the last iteration. At the beginning of the iterative procedure, AC load flow is performed with the chosen initial active thermal generations and active generation or pumping power of the pumped-storage unit. The active power flows and the losses of each line in the system are obtained from this AC load flow calculation. Afterwards, the total (pseudo) cost rate for time interval j is calculated as

$$\begin{aligned} TCR_j^{odd} &= \lambda_{GH,j}^{old} P_{GH,j}^{old} + \sum_{n \in N_s} F_n(P_{n,j}^{old}), \quad j \in \mathbf{J}_{gen} \quad (R/h) \\ TCR_j^{odd} &= \lambda_{PH,j}^{old} P_{PH,j}^{old} + \sum_{n \in N_s} F_n(P_{n,j}^{old}), \quad j \in \mathbf{J}_{pump} \quad (R/h) \end{aligned} \tag{11}$$

where $\lambda_{GH,j}^{old}, \lambda_{PH,j}^{old}$ represent pseudo active generation and pumping incremental cost (R/MWh) for the pumped-storage hydraulic unit calculated in the previous iteration. Note that $P_{PH,j}^{old}$ in (11) is a *negative* quantity. As long as the generation or pumping power of the pumped-storage unit does not violate its generation or pumping limits in a time interval, its generation or pumping incremental cost value in that interval remains equal to the value that is determined at the beginning of the current outer loop. The determination of the pseudo active generation and pumping incremental cost values of the pumped-storage hydraulic unit when it violates its generating or pumping limits is explained in Section 2.1.1. The active generation incremental cost of each thermal unit is calculated according to

$$\lambda_{n,j} = \frac{dF_n(P_{n,j})}{dP_{n,j}} \quad (R/MWh), \quad n \in N_s. \tag{12}$$

The amount of power that is sold or bought by a bus is taken as active power flow entering or leaving the considered bus. They are fictitious. They are used only to find optimal active thermal generations and generation or pumping powers of the pumped-storage unit. The active powers transmitted between any two busses in time interval j are represented as $TP_{ik,j}$. The first two indices indicate that active power flow is from bus i to bus k. The second index also indicates that $TP_{ik,j}$ is the active power flow at bus k's border. The last index shows the considered time interval. With this notation, $-TP_{ki,j}$ represents an active power flow in time interval j going from bus i to bus k *at the border of bus* i The (pseudo) spot active power price of bus i at bus k's border in time period j is represented by $SP_{ik,j}$. It is calculated as

$$SP_{ik,j} = \lambda_{i,j}\left(1 + \frac{-TP_{ki,j} - TP_{ik,j}}{TP_{ik,j}}\right) \quad (R/h). \tag{13}$$

In the calculation of $SP_{ik,j}$, the active generation incremental cost of bus i is *increased* according to the *active loss percentage* that occurs during the transmission of active power from bus i to bus k. Thus the inclusion of active transmission losses into the solution procedure is made possible.

In the proposed dispatch algorithm, it is supposed that there is a mechanism at every bus of the system which decides how much active power is to be bought from other buses. In the determination of new active bought powers, the active generation incremental cost of the bus which buys active power $(\lambda_{k,j})$ and the spot price of active bought power $(SP_{ik,j})$ are used according to

$$TP_{ik,j}^{new} = \left(1 + \frac{\lambda_{k,j} - SP_{ik,j}}{\lambda_{k,j}}\right) TP_{ik,j}^{old}. \tag{14}$$

If the active generation incremental cost of unit k is higher than spot price of the bought active power from bus $i(\lambda_{k,j} > SP_{ik,j})$, the new bought active power from bus i becomes higher than its previous value. In the reverse situation, the new bought active power from bus i becomes lower than its previous value.

To reduce the number of iterations (load flow), for operation points which are far from the solution point, new bought active powers are calculated according to (14). About the solution point, if the new active bought powers are calculated according to (14), since the changes in active bought powers cannot be small enough, an increase instead of a decrease in the total active generation cost rate could be obtained. To overcome this problem, active bought powers should be changed in a small amount about the solution point. Therefore, new active bought powers in this case are calculated according to

$$TP_{ik,j}^{new} = \left[1 + \alpha_j\left(\frac{\lambda_{k,j} - SP_{ik,j}}{\lambda_{k,j}}\right)\right] TP_{ik,j}^{old} \tag{15}$$

α_j in (15) is taken equal to 1.0 initially. If an increase in the total active generation cost rate occurs, the value of α_j is decreased in a specific amount, and the calculation procedure described in this section is performed again. This procedure is repeated until either a decrease in the total active generation cost rate occurs or the value of α_j becomes less then its predefined $\alpha_j^{\min}$ value.

For the bus to which the pumped-storage unit operating in pumping mode is connected ($k = PH$), new bought active powers are calculated according to the relative difference between pumping incremental cost and spot price of bought active powers according to (14) or (15). When the pumped-storage unit is operating in pumping mode, there is no active generation in the bus. Therefore, the bus's incremental cost, which is used in determination of the bought active powers from bus k, is calculated according to (16). Consequently, two different incremental cost values are used for the bus where the pumped-storage unit operating in pumping mode is connected; one for the calculation of bought active powers from the other buses and the other one for the calculation of the powers purchased *from* that bus.

If there is a bus to which no generating unit is connected, bought active powers by this bus become optimal when the spot prices of all bought active powers are equal at this bus's border. To be able to calculate new bought active powers for such buses, incremental active generation cost of those buses should be calculated first. The incremental active generation cost of such buses is calculated as weighted average of spot prices of bought active powers with respect to bought active powers.

$$\lambda_{k,j} = \frac{\sum\limits_{i \in \left\{ \substack{\text{All buses from which} \\ \text{bus } k \text{ buys active power}} \right\}} TP_{ik,j} SP_{ik,j}}{\sum\limits_{i \in \left\{ \substack{\text{All buses from which} \\ \text{bus } k \text{ buys active power}} \right\}} TP_{ik,j}}, \quad k \notin N_s, k \neq GH \tag{16}$$

After calculation of the incremental active generation cost for bus k, all new active bought powers are determined according to (14) or (15). If bus k at which there is no active generation is connected to bus g at which there is no active generation either $(i = g)$, the incremental cost of bus g should have already been calculated according to (16), before calculating $\lambda_{k,j}$. Therefore the active generation incremental cost values of the buses at which there is no active generation have to be calculated *in a specific order.*

After finding all bought active powers for all buses of the system, active powers sent from other ends of the transmission lines (sold active powers) can approximately be calculated by adding the *old* active transmission losses ($P^{old}_{loss\ ik,j}$) (obtained from the last load flow calculation) to the respective bought active powers.

$$-TP^{new}_{ki,j} = TP^{new}_{ik,j} + P^{old}_{loss\ ik,j} \tag{17}$$

After calculation of bought and sold active powers at each bus in the system, for buses whose active power balance are distorted, new active generations or pumping power are calculated to reestablish their active power balance according to

$$P^{new}_{k,j} = P_{load\ k,j} - \sum_{a \in \left\{ \substack{\text{All buses con-} \\ \text{nected to bus } k} \right\}} TP^{new}_{ak,j} \tag{18}$$

where $P_{load\ k,j}$ is the active load value (excluding pumping power for the case $k = PH$) connected to bus k in time period j. The bought and sold active powers by bus k are taken as *positive* and *negative* quantities in the above summation. If bus k is a bus where a thermal unit is connected $(k \in N_s)$, the $P^{new}_{k,j}$ generation value cannot be less than its minimum generation limit. *But if bus k is a bus where the pumped-storage unit is connected* ($k = GH$ or PH), the $P^{new}_{k,j}$ *value can transfer from generation power to pumping power (i.e. from positive valued power (generation power) to negative valued power (pumping power or load)) or vice versa.* If the considered bus is a bus to which no unit is connected, the active power imbalance of this bus ($P^{error}_{k,j}$) is made zero by correcting the bought active powers from other buses. Equation (19) gives the active power imbalance for such a bus:

$$P^{error}_{k,j} = P_{load\ k,j} - \sum_{a \in \left\{ \substack{\text{All buses con-} \\ \text{nected to bus } k} \right\}} TP^{new}_{ak,j}, \quad k \notin N_s, \quad k \neq GH, PH \tag{19}$$

$P^{error}_{k,j}$ is made zero by correcting the active powers bought from other buses. If $P^{error}_{k,j} > 0$, bus k needs to buy *extra* active power by the amount of $P^{error}_{k,j}$. Therefore, the bought active powers from the other buses are increased *inversely proportional* to the spot price of the corresponding bought active powers.

$$T_{dk,j}^{new,\ corrected} = T_{dk,j}^{new} + \frac{(1/SP_{dk,j})P_{k,j}^{error}}{\sum\limits_{c \in \left\{ \substack{\text{All buses from which} \\ \text{bus } k \text{ buys active power}} \right\}} (1/SP_{ck,j})}, \quad \text{if} \quad P_{k,j}^{error} > 0 \tag{20}$$

Subscript d in (20) represents a bus from which bus k buys active power. Increasing the active bought powers by bus k according to (20) gives the highest and the lowest increases in the active bought powers that have the lowest and the highest spot prices respectively. Also, the total amount of increase in the bought active powers becomes always equal to the active power imbalance of bus k. If $P_{k,j}^{error} < 0$, bus k needs to *decrease* its bought active power by the amount of its power imbalance. Therefore, the bought active powers from the other buses are decreased in *proportion* to the spot price of the corresponding bought active powers:

$$T_{dk,j}^{new,\ corrected} = T_{dk,j}^{new} + \frac{SP_{dk,j}P_{k,j}^{error}}{\sum\limits_{c \in \left\{ \substack{\text{All buses from which} \\ \text{bus } k \text{ buys active power}} \right\}} SP_{ck,j}}, \quad \text{if} \quad P_{k,j}^{error} < 0 \tag{21}$$

Decreasing the active bought powers by bus k according to (21) gives the highest and the lowest decreases in the active bought powers that have the highest and the lowest spot prices respectively. Again, the total amount of decrease in the bought active powers becomes always equal to the active power imbalance of bus k.

Later, the effect of corrections on active bought powers at buses where no unit is connected is reflected on the other buses, and the new corrected thermal generations ($P_{n,j}^{new,\ corrected}, n \in N_s$) and generating or pumping power of the pumped-storage unit ($P_{GH,j}^{new,\ corrected}$ or $P_{PH,j}^{new,\ corrected}$) are determined by using (18). With the new corrected thermal generations and generating or pumping power of the pumped-storage unit, an AC load flow is performed and new total cost rate, TCR_j^{new}, is calculated according to

$$\begin{aligned} TCR_j^{new} &= \lambda_{GH,j}^{new} P_{GH,j}^{new,\ corrected} + \sum_{n \in N_s} F_n(P_{n,j}^{new,\ corrected}), \quad j \in \mathbf{J}_{gen} \quad (R/h) \\ TCR_j^{new} &= \lambda_{PH,j}^{new} P_{PH,j}^{new,\ corrected} + \sum_{n \in N_s} F_n(P_{n,j}^{new,\ corrected}), \quad j \in \mathbf{J}_{gen} \quad (R/h) \end{aligned} \tag{22}$$

where $\lambda_{GH,j}^{new}$ and $\lambda_{GH,j}^{new}$ represent new pseudo generation and pumping incremental cost values. Their determination is described in Section 2.1.1. After that, the stopping criteria, which is given as

$$\begin{aligned} \Delta TCR_j &< TOL_{\Delta TCR} \quad \text{if} \quad \Delta TCR_j \geq 0 \\ \alpha_j &< \alpha_j^{\min} \quad \text{if} \quad \Delta TCR_j < 0 \end{aligned} \tag{23}$$

where

$$\Delta TCR_j = TCR_j^{old} - TCR_j^{new} \tag{24}$$

is tested. $TOL_{\Delta TCR} > 0$ in (23) is the tolerance value for ΔTCR_j. If $\Delta TCR_j \geq 0$, then the first inequality in (23) is checked. If it is not satisfied, a new iteration is initiated with the newly determined active thermal generations and generation or pumping power of the pumped-storage unit. If $\Delta TCR_j < 0$, then the second inequality is checked. If it is not satisfied, the α_j value is decreased by a specific amount and new iteration is initiated with the last active thermal generations and generation or pumping power of the pumped-storage unit that gave $\Delta TCR_j > 0$. If the second inequality in (23) is satisfied, the last active thermal generations and generation or pumping power of the pumped-storage unit that gave $\Delta TCR_j > 0$ are taken as solution values.

2.1.1. Active Generation and Hydraulic Limits

Before determination of the pumped-storage unit's generation or pumping power in time interval j, the unit's generation or pumping power upper limit is determined by using the stored water amount in its reservoir at the end of time period $j-1$. If $j \in \mathbf{J}_{pump}$, the pumping rate upper bound is calculated as

$$q_{PH}(P_{PH,j}^{bound}) = \frac{V^{\max} - V_{j-1}}{t_j} \tag{25}$$

After that, the upper limit for the absolute value of pumping power is determined according to

$$\begin{aligned} &\text{if } q_{PH}(P_{PH,j}^{bound}) \geq q_{PH}^{\max} \quad \text{then} \quad P_{PH,j}^{limit} = P_{PH}^{\max} \\ &\text{if } q_{PH}(|P_{PH,j}^{bound}|) < q_{PH}^{\max} \quad \text{then} \quad P_{PH,j}^{limit} = |P_{PH,j}^{bound}| \end{aligned} \tag{26}$$

In a similar manner, if $j \in \mathbf{J}_{gen}$, the discharge rate upper bound is calculated as

$$q_{GH}(P_{GH,j}^{bound}) = \frac{V_{j-1} - V^{\min}}{t_j} \tag{27}$$

Again, the upper limit for the generation power is determined according to

$$\begin{aligned} &\text{if } q_{GH}(P_{GH,j}^{bound}) \geq q_{GH}^{\max} \quad \text{then} \quad P_{GH,j}^{limit} = P_{GH}^{\max} \\ &\text{if } q_{GH}(P_{GH,j}^{bound}) < q_{GH}^{\max} \quad \text{then} \quad P_{GH,j}^{limit} = P_{GH,j}^{bound} \end{aligned} \tag{28}$$

During the calculation procedure described in Section 2.1, if a bus to which a thermal generating unit exceeding one of its generating limits is connected buys active power from the other buses, that bus is considered as if it were a bus at which there is no active generation. The active generation at this bus is also taken equal to the exceeded limit value. If a bus to which a thermal generating unit exceeding one of its limits is connected never buys any active power from the other buses, the *sold active powers* by this bus is adjusted by changing its active generation incremental cost. The new incremental active generation cost is calculated according to

$$\lambda_{k,j}^{new} = \left[\beta_{k,j} \left(\frac{P_{load\ k,j} - \sum\limits_{d \in \left\{ \substack{\text{All buses to which} \\ \text{bus } k \text{ sells active power}} \right\}} TP_{dk,j} - P_k^{limit}}{P_k^{limit}} \right) + 1 \right] \lambda_k(P_k^{limit}), k \in N_s, 0 < \beta_{k,j} \leq 1 \tag{29}$$

$TP_{dk,j}$ in (29) represents the active powers sold by bus k and they are taken as negative quantities in the summation. P_k^{limit} in (29) also denotes the exceeded active generation limit of the thermal unit connected to bus k. To change $\lambda_{k,j}^{new}$ by a small amount, a variable $\beta_{k,j}$ is used in (29). If the thermal unit connected to bus k exceeds its upper generation limit, the expression inside the inner bracket in (29) becomes positive and therefore $\lambda_{k,j}^{new}$ becomes higher than $\lambda_k(P_k^{limit})$. Because of this, the sold active powers by bus k decrease and consequently active generation of the thermal unit connected to bus k approaches down to its upper generation limit. If the thermal unit's generation becomes smaller than its lower limit, that expression inside the inner bracket in (29) becomes negative and therefore $\lambda_{k,j}^{new}$ becomes lower than $\lambda_k(P_k^{limit})$ in this case. Due to this, the sold active powers by bus k increase and active generation of the thermal unit connected to bus k approaches up to its lower active generation limit.

If the bus to which the pumped-storage unit operating in generation mode and also violating its *upper* generating limit is connected is a bus that *buys and sells* active power from and to the other buses, that bus also is considered as if it were a bus where there is no active generation and active generation of the pumped-storage unit is taken equal to its *upper* generation limit value. If the bus to which the pumped-storage unit operating in generation mode and also violating its upper generating limit is connected is a bus that *never buys* any active power from other buses, the sold active powers by the considered bus is adjusted by changing its active generation incremental cost according to

$$\lambda_{k,j}^{new} = \left[\beta_{k,j}\left(\frac{P_{load\ k,j} - \sum\limits_{d\in\left\{\substack{\text{All buses to which}\\ \text{bus } k \text{ sells active power}}\right\}} TP_{dk,j} - P_k^{limit}}{P_{k,j}^{limit}}\right) + 1\right]\lambda_H^{(\nu)}, k = GH, 0 < \beta_{k,j} \leq 1 \qquad (30)$$

where superscript ν denotes outer loop iteration number.

If the bus to which the pumped-storage unit operating *in pumping mode* and also violating its *upper* pumping limit is connected is a bus that *buys and sells* active power from and to the other buses, the unit's pumping power is just taken as equal to its upper pumping power. If the bus to which the pumped-storage unit operating in *pumping mode* and also violating its *upper* pumping limit is connected is a bus that *only buys* active power from the other buses, the bought active powers by the considered bus (pumping power) is adjusted by changing its active pumping incremental cost according to

$$\lambda_{k,j}^{new} = \left[\beta_{k,j}\left(\frac{P_{k,j}^{limit} + P_{load\ k,j} - \sum\limits_{d\in\left\{\substack{\text{All buses from which}\\ \text{bus } k \text{ buys active power}}\right\}} TP_{dk,j}}{P_{k,j}^{limit}}\right) + 1\right]\lambda_H^{(\nu)}, k = PH, 0 < \beta_{k,j} \leq 1 \qquad (31)$$

$TP_{dk,j}$ in (31) represents the active powers bought by bus k and they are taken as positive quantities in the summation. When the unit's absolute value of pumping power exceeds its upper limit, the expression inside the inner bracket becomes negative and so, $\lambda_{k,j}^{new} < \lambda_H^{(\nu)}$. That results in a decrease in bought active powers by the bus. Consequently the absolute value of pumping power approaches down is its upper limit.

If $j \in \mathbf{J}_{gen}$ and generation power of the pumped-storage unit does not exceed its upper limit, $\lambda_{GH,j}^{new} = \lambda_H^{(\nu)}$. If $j \in \mathbf{J}_{pump}$ and absolute value of pumping power of the pumped-storage unit does not exceed its upper limit, $\lambda_{PH,j}^{new} = \lambda_H^{(\nu)}$.

After determination of the generation or pumping power of the pumped-storage unit in time period j, the water volume value in the pumped-storage unit reservoir at the end of time period j is calculated according to

$$\begin{aligned} V_j &= V_{j-1} - q_{GH}(P_{GH,j})t_j, \quad j \in \mathbf{J}_{gen} \\ V_j &= V_{j-1} + q_{PH}(|P_{PH,j}|)t_j, \quad j \in \mathbf{J}_{pump} \end{aligned} \tag{32}$$

2.2. Outer Loop Iteration Calculations

In order to start the optimization procedure, initial generations, $P_{GH,j}^{init}$ $j \in \mathbf{J}_{gen}$, $P_{PH,j}^{init}$ $j \in \mathbf{J}_{pump}$, $P_{n,j}^{init}$, $n \in N_s$, $j = 1, \ldots, j_{\max}$, except for the swing bus, which satisfy (3)-(5) are chosen. The chosen initial $P_{GH,j}^{init}$ $j \in \mathbf{J}_{gen}$, $P_{PH,j}^{init}$ $j \in \mathbf{J}_{pump}$ values cannot satisfy (8). In order to decrease the number of iterations in the outer loop, initial pseudo active generating/pumping incremental cost value for the pumped-storage unit, $\lambda_{GH}^{(1)} = \lambda_{PH}^{(1)} = \lambda_H^{(1)}$, is chosen in the following manner. First, the initial average incremental active generation cost for thermal units, λ_{avg}^{init}, is calculated according to

$$\lambda_{avg}^{init} = \frac{\sum_{j=1}^{j_{\max}} \sum_{n \in N_s} \lambda_{n,j}(P_{n,j}^{init})}{S\{N_s\} j_{\max}} \tag{33}$$

where $S\{N_s\}$ is the number of thermal units. As the pseudo incremental active generating/pumping cost takes higher values compared to λ_{avg}^{init}, the pumping active powers in time intervals where the system active load is low start to increase, at the same time the generating active powers in time intervals where the system active load is high start to decrease. Because of this, the net water amount spent by the pumped-storage unit starts to take more and more negative values. In the reverse situation, as the pseudo incremental active generating/pumping cost takes lower values compared to λ_{avg}^{init}, the pumping active powers in time intervals where the system active load is low start to decrease, at the same time the generating active powers in time intervals where the system active load is high start to increase. So, the net water amount spent by the pumped-storage unit starts to take more and more positive values. The purpose here is to find the critical pseudo incremental active generating/pumping cost value for which the net water amount spent by the pumped-storage unit becomes zero or close to zero within the chosen tolerance. To accomplish that, a $\lambda_H^{(1)}$ value being close to λ_{avg}^{init} is chosen, and active thermal generations, pumping or generation active powers of the pumped-storage unit in each time interval are calculated as described in Section 2.1. At this point, the net water amount spent by the pumped-storage unit, $q_{net\ spent}^{(1)}$, is calculated according to

$$q_{net\ spent}^{(1)} = \sum_{j \in \mathbf{J}_{gen}} q_{GH}(P_{GH,j}^{(1)})t_j - \sum_{j \in \mathbf{J}_{pump}} q_{PH}(|P_{PH,j}^{(1)}|)t_j \tag{34}$$

where $P_{GH,j}^{(1)}$ $j \in \mathbf{J}_{gen}$ and $P_{PH,j}^{(1)}$ $j \in \mathbf{J}_{pump}$ in (34) are the pumped-storage hydraulic unit's generation and pumping power values that are obtained at the end of the first outer loop iteration ($\nu = 1$). After that the condition $q_{net\ spent}^{(1)} < 0$ is checked. If this condition is not met, a new pseudo active generating/pumping incremental cost value, which is *higher* than the previous one, is chosen and the above calculations are performed again. This procedure is repeated until $q^{(1)}_{net\ spent} < 0$ is satisfied. The pseudo active generating/pumping incremental cost value of the pumped-storage unit for the second outer loop iteration ($\nu = 2$) is chosen as $\lambda^{(2)}_H < \lambda^{(1)}_H$ such that

$$q^{(2)}_{net\ spent} = \left[\sum_{j \in \mathbf{J}_{gen}} q_{GH}(P^{(2)}_{GH,j})t_j - \sum_{j \in \mathbf{J}_{pump}} q_{PH}(|P^{(2)}_{PH,j}|)t_j \right] > 0 \quad (35)$$

is satisfied. The matter here is to choose two initial $\lambda^{(1)}_H$, $\lambda^{(2)}_H < \lambda^{(1)}_H$ values being close λ^{init}_{avg}, giving also negative and positive total net spent water amounts so that the pseudo incremental active generating/pumping cost values in the subsequent outer loop iterations could be determined *by means of linear interpolation*. Determination of the pseudo active generating/pumping incremental cost values by means of linear interpolation is illustrated in Figure 1.

In the proposed algorithm, during the first three outer loop iterations ($\nu = 1, 2, 3$), reservoir volume constraints are not controlled. This is done in order to determine suitable new $\lambda^{(\nu)}_H$ values in the subsequent outer loop iterations so that the net spent water amount by the pumped-storage unit converges to zero in fewer outer loop iterations. Controlling of reservoir volume constraints and stopping criterion is initiated after the third outer loop iteration.

$$|q^{(\nu)}_{net\ spent}| < TOL_{q_{TOT}}, \quad \nu \geq 4 \quad (36)$$

$TOL_{q_{TOT}}$ in (36) represents the tolerance value for the absolute value of the net spent water amount by the pumped-storage unit. If the stopping criterion given in (36) is not satisfied, the new pseudo active generating/pumping incremental cost value for the pumped-storage hydraulic unit is determined by using linear interpolation as illustrated in Figure 1. Outer loop iterations are repeated until the stopping criterion is satisfied.

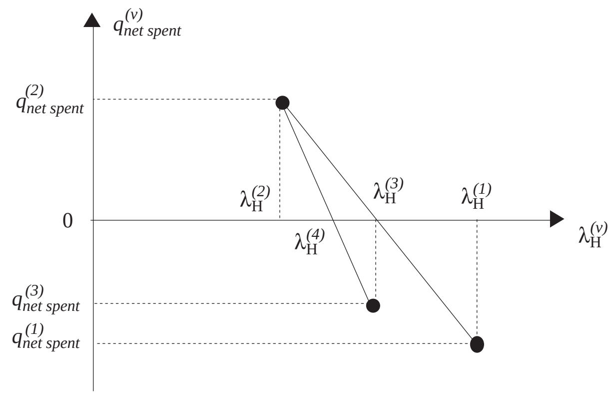

Figure 1. Determination of new $\lambda^{(\nu)}_H$ values by means of linear interpolation.

3. Example

In order to demonstrate the proposed solution algorithm, an example power system, whose one-line diagram is shown in Figure 2, was chosen. A twenty-four-hours operation cycle having six equal time intervals is considered. Equivalent π circuit pu parameters of the transmission lines are also shown in the figure. Generation units that are connected to buses 1, 4, 7, 9 and 11 are taken as thermal units. The pumped-storage unit is connected to bus 6. Thermal units' cost rate curves are taken as quadratic polynomials

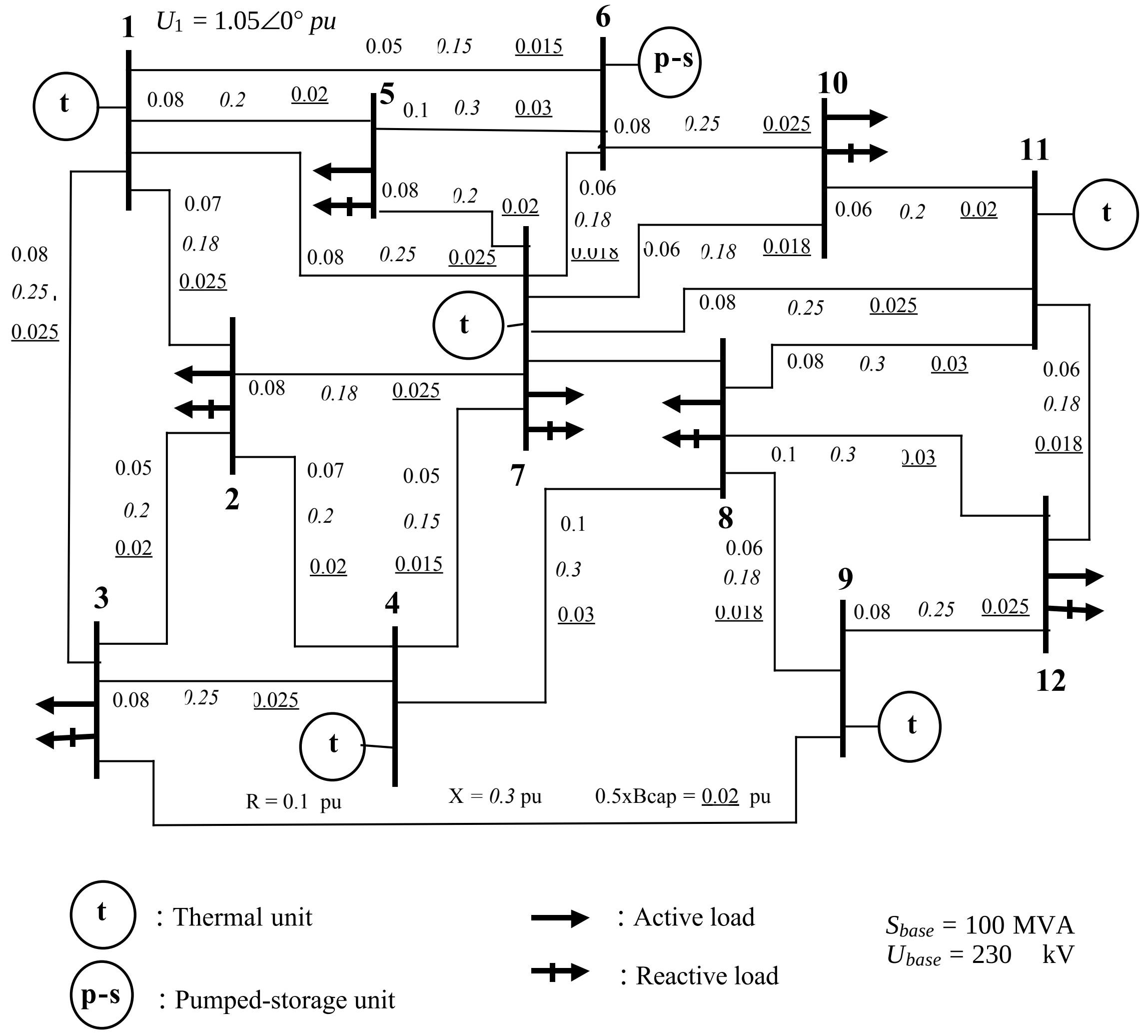

Figure 2. One line diagram of the example power system

$$F_n(P_n) = a_n + b_n P_n + c_n P_n^2 \quad (R/h), \quad n = 1, 4, 7, 9, 11 \tag{37}$$

where a_n, b_n, and c_n coefficients and those units' active generation limits are given in Table 1. The thermal units connected to bus 9 and 11 are chosen as inefficient units with respect to the other thermal units. If these thermal units' generations drop below their minimum generation limits during the optimization procedure, they are taken as equal to zero and those units are operated as synchronous compensators. Their active power consumptions when they are operated as synchronous compensators are assumed to be zero. These units also are taken as fast starting units (for instance, gas-fired units).

Table 1. Cost rate curve coefficients and generation limits for thermal units.

n	a_n	b_n	c_n	$P_n^{\max}$ (MW)	$P_n^{\min}$ (MW)
1	527	7.48	0.001495	350	50
4	561	7.92	0.001562	180	45
7	310	7.85	0.00194	175	40
9	476	9.52	0.00436	100	5
11	460	9.40	0.00397	100	3

The pumped-storage unit's discharge and pumping rate curves are taken as follows

$$q_{GH}(P_{GH}) = \begin{cases} 200 + 2.0P_{GH} & (\text{acre} - ft/h) \quad \text{if} \quad 0 < P_{GH} \leq 130\ MW \\ 0 & (\text{acre} - ft/h) \quad \text{if} \quad P_{GH} = 0\ MW \end{cases} \tag{38}$$

$$q_{GH}(|P_{PH}|) = \begin{cases} 200 + \frac{4}{3}|P_{PH}| & (\text{acre} - ft/h) \quad \text{if} \quad 0 < |P_{PH}| \leq 130\ MW \\ 0 & (\text{acre} - ft/h) \quad \text{if} \quad |P_{PH}| = 0\ MW \end{cases} \tag{39}$$

The coefficients of the discharge rate and the pumping rate curves are chosen in such a manner that the total efficiency of the pumped-storage unit is 0.67 [3]. The pumped-storage unit's reservoir storage limits, starting and ending water volumes are taken as $V^{\min} = 5000$ *acre-ft*, $V^{\max} = 15000$ *acre-ft*, $V^{start} = V^{end} = 10000$ *acre-ft*.

Table 2. Per unit active and reactive load schedule for the example power system.

Interval #, (j)	Load (pu)	Bus #							Total load (pu)
		2	3	5	7	8	10	12	
1	P	0.20	0.50	0.30	0.25	0.40	0.15	0.20	2.00
	Q	0.15	0.40	0.24	0.20	0.30	0.12	0.15	1.56
2	P	0.80	1.10	0.90	1.00	0.70	0.60	0.90	6.00
	Q	0.60	0.85	0.70	0.75	0.52	0.45	0.70	4.57
3	P	1.00	1.20	0.80	1.10	0.90	1.05	0.95	7.00
	Q	0.75	0.90	0.60	0.85	0.70	0.80	0.75	5.35
4	P	0.80	1.10	0.90	1.00	0.70	0.60	0.90	6.00
	Q	0.60	0.85	0.70	0.75	0.52	0.45	0.70	4.57
5	P	0.40	0.60	0.25	0.50	0.30	0.45	0.50	3.00
	Q	0.30	0.45	0.20	0.40	0.24	0.35	0.40	2.34
6	P	0.20	0.50	0.30	0.25	0.40	0.15	0.20	2.00
	Q	0.15	0.40	0.24	0.20	0.30	0.12	0.15	1.56

The per unit active and reactive load schedule for the example power system is given in Table 2. Chosen initial active and reactive generations are shown in Table 3. For the sake of comparison, the reactive generation and reactive pumping power of the pumped-storage unit is taken as 0 $MVAr$. It is also seen in Table 3 that the pumped-storage unit is taken as in generation mode in all six time intervals initially (see column 6 in Table 3).

We solved the dispatch problem where the pumped-storage unit is off-line at first. In this solution, the tolerance values in (23) were taken as $\alpha_j^{\min} = 0.5, j = 1, \ldots, 6, TOL_{\Delta TCR} = 0.01\ R$. The total number of load flow calculations performed during all six time intervals was 35. Active generations and thermal cost values in each time interval in the solution point are given in Table 4. Thermal units connected to buses

9 and 11, which are expensive units, generate active power only in the time interval where the peak load occurs. They operate as synchronous compensators in the other time intervals. The total thermal cost was found be 125268.54 R in this case.

Table 3. Per unit initial active and reactive generation values for the example power system.

Interval #, (j)	Generation (pu)	Bus #				
		4	6	7	9	11
1	P	0.450	0.02	0.400	0.200	0.200
	Q	0.080	0.00	0.080	0.080	0.080
2	P	1.500	0.02	1.400	0.450	0.350
	Q	0.700	0.00	0.700	0.700	0.700
3	P	1.700	0.02	1.700	0.550	0.450
	Q	1.000	0.00	1.000	1.000	1.000
4	P	1.500	0.02	1.400	0.450	0.350
	Q	0.700	0.00	0.700	0.700	0.700
5	P	0.700	0.02	0.600	0.350	0.300
	Q	0.200	0.00	0.200	0.200	0.200
6	P	0.45	0.02	0.400	0.200	0.200
	Q	0.080	0.00	0.080	0.080	0.080

The same dispatch problem was resolved with the pumped-storage unit being on-line and with the tolerance values $\alpha_j^{\min} = 0.5, j = 1, \ldots, 6,\ TOL_{\Delta TCR} = 0.01\ R,\ TOL_{q_{TOT}} = 0.5$ *acre-ft*. In this solution, by considering the initial active generations given in Table 3 and using (32), the λ_{avg}^{int} value was calculated to be 8.99 R/h. Regarding this value, $\lambda_H^{(1)} = 8.7\ R/h$ was chosen and $q_{net\ spent}^{(1)} = -5279.644 < 0$ *acre-ft* was calculated at the end of the first outer loop iteration. For the second outer loop iteration, $\lambda_H^{(2)} = 8.5 < 8.7$ R/h was selected and $q_{net\ spent}^{(2)} = 1629.426 > 0$ *acre-ft* was calculated. After the second outer loop iteration, all the subsequent $\lambda_H^{(\nu)}$ values were calculated by means of linear interpolation. The solution point was reached at the end of the ninth outer loop iteration. The total number of load flow calculations performed during nine outer loop iterations was found to be 481. The total solution time was measured at 126.72 seconds on a Pentium 100 PC.

Plots for $\lambda_H^{(\nu)}$ and $q_{net\ spent}^{(\nu)}$ with respect to outer loop iterations are shown in Figures 3 and 4 respectively. The effect of the pseudo incremental generating/pumping cost value on the net spent water amount by the pumped-storage unit and convergence of the net spent amount to zero due to that effect is clearly seen in these figures. The solution point active thermal generations, generating and pumping powers of the pumped-storage unit and cost values are given in Table 5. It is seen in this table that expensive units that are connected to buses 9 and 11 operate as synchronous compensators during all time intervals. The pumped-storage unit operates in generation mode in time intervals 2,3 and 4 where the system load values are high. The pumped-storage unit's active generation becomes maximum in time interval 3 where the peak load occurs in the system. The pumped-storage unit operates in pumping mode in time interval 1, 5 and 6 where the system load values are low. The pumped-storage unit's pumping power value becomes maximum in time periods 1 and 6 where the system load is at its minimum value.

Table 4. The solution point active generations and cost values in each interval when the pumped-storage unit is *off-line.*

Interval # (j)	Generation (pu)	Bus # 1	4	7	9	11	F_{Tj} (R)
1	P	1.14861	0.45000	0.43710	-	-	11933.155
2	P	2.83083	1.80000	1.75000	-	-	26178.564
3	P	2.93690	1.80000	1.75000	0.50967	0.38748	33743.444
4	P	2.83083	1.80000	1.75000	-	-	26178.564
5	P	1.48463	0.70147	0.89816	-	-	15301.658
6	P	1.14861	0.45000	0.43710	-	-	11933.155
						F_T	**125268.540**

Table 5. Solution point active thermal generations, active generating and pumping powers of the pumped storage unit and thermal cost values.

Interval # (j)	Generation (pu)	Bus # 1	4	6	7	9	11	F_{Tj} (R)
1	P	1.988461	0.470554	-1.156974	0.776395	-	-	15767.134
2	P	2.470681	1.800000	0.336229	1.750000	-	-	24986.800
3	P	2.668229	1.800000	1.300000	1.750000	-	-	25638.572
4	P	2.470681	1.800000	0.336229	1.750000	-	-	24986.800
5	P	2.000972	0.758582	-0.644416	0.994979	-	-	17458.542
6	P	1.988461	0.470554	-1.156947	0.776395	-	-	15767.134
							F_T	**124604.982**

The total cost becomes 124604.98 R when the pumped-storage unit is on-line. Thus, the inclusions of the pumped-storage unit with 0.67 total efficiency saves 663.56 R daily for the given daily load schedule.

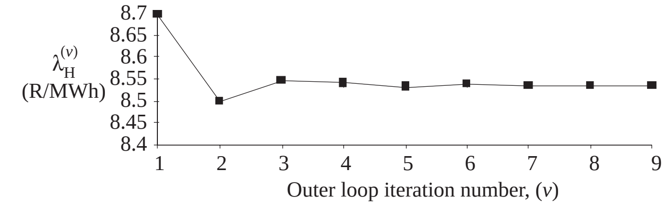

Figure 3. Change in $\lambda_H^{(\nu)}$ values with respect to outer loop iterations

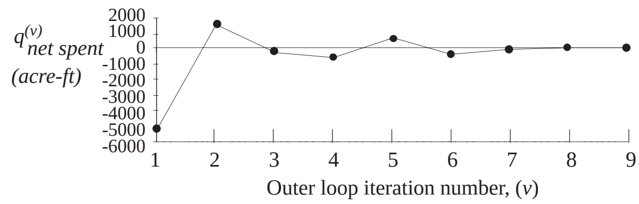

Figure 4. Change in $q_{net\ spent}^{(\nu)}$ values with respect to outer loop iterations.

The stored water amount that is calculated at the end of the operation cycle was found to be $V_6^{(9)} = 9999.799$ *acre-ft.*

The same dispatch problem where the pumped-storage unit is on-line was solved by means of another solution technique based on genetic algorithm [4]. In this solution, the number of populations and the number of iterations were chosen as 150 and 100 respectively. The solution giving the minimum total thermal cost as 124916 *R* was obtained at the end of the 98th iteration. The number of load flow calculations performed to reach the solution is 90000. This number is more than 180 times the total number of load flow calculations performed in the proposed solution technique. The total thermal cost is also 311 *R* higher than the one obtained by using the proposed dispatch technique. The active thermal generations, active generating and pumping powers of the pumped storage unit in the solution point are given in Table 6. The stored water amount calculated at the end of the operation period was 9998.4 *acre-ft*.

Table 6. The solution point active thermal generations, active generating and pumping powers of the pumped storage unit that are obtained by using a solution technique based on genetic algorithm

Interval #	Generation	Bus #					
(j)	(pu)	1	4	6	7	9	11
1	P	1.69440	0.74180	-1.01400	0.64510	-	-
2	P	3.11500	1.52270	0.30000	1.46000	-	-
3	P	3.19810	1.67520	1.23900	1.46410	-	-
4	P	2.76790	1.62050	0.42300	1.56640	-	-
5	P	2.07910	1.05940	-1.19400	1.19170	-	-
6	P	1.44150	0.64730	-0.73200	0.69450	-	-

4. Conclusion

We propose an active dispatch technique using pseudo spot price of electricity for power systems that include thermal units and a pumped-storage hydraulic unit. The dispatch technique consists of two nested iterative parts. In the first part, active thermal generations, generating or pumping active powers of the pumped-storage unit in all time intervals are determined by adjusting the active bought powers by buses. The active bought powers by buses are adjusted according to the relative difference between the incremental active generation cost of buses and the pseudo spot price of the active bought powers. In the second part, after determination of active thermal generations, generating or pumping active powers of the pumped-storage unit in all time intervals, the net water amount spent by the pumped storage unit is calculated. If the absolute value of the net spent water amount is not less than its tolerance value, a new pseudo active generating/pumping incremental cost value for the next outer loop iteration (if it is equal to or greater than three) is calculated by means of linear interpolation.

The demonstration of the proposed dispatch technique on a chosen example power system is also given. The same dispatch problem is solved by using another solution technique based on a genetic algorithm. Results obtained from both solutions are compared.

To our knowledge, the proposed solution technique given in this paper *has not been used* previously to solve the pumped-storage hydraulic unit scheduling problem.

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