Turk J Elec Engin, VOL.10, NO.1 2002, © TÜBİTAK

Hybrid SR ARQ Scheme Using Trellis Coded Modulation for Point-To-Multipoint Communication Over Nonstationary Channels

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Abstract

In this paper, a type-I hybrid SR ARQ scheme using trellis coded modulation (TCM) for point-to-multipoint communication was analyzed on nonstationary broadcast channels where the channel bit error rate (BER) varies over time. A nonstationary channel is modelled as a two-state Markov chain. In the numerical calculations, it is found that, for a given p_1 *and* $\bar{\varepsilon}$*, the values of the average burst length* $\bar{b}$ *have a very small influence on the throughput. Numerical results show that the throughput is better for values between* 10^{-6} *and* 10^{-4} *of average bit error rate* $\bar{\varepsilon}$ *if the errors occur in bursts. However, the scheme performs better for the stationary channel when the average bit error rate becomes large. For medium and high values of* $\bar{\varepsilon}$*, the throughput decreases when the number of receivers increases.*

Key Words: *Data transmission, multicast ARQ, nonstationary channel, throughput efficiency.*

1. Introduction

Forward error correction (FEC) and automatic repeat request (ARQ) are common techniques used to handle transmission errors when data are transmitted over noisy channels. In practical applications where feedback is possible, ARQ technique are often more preferable than FEC schemes because error detection requires much simpler decoding equipment and achieves a higher reliability than does error correction. When the channel is very noisy, the system throughput is smaller for ARQ techniques than FEC schemes because retransmissions will be requested too frequently. Hybrid ARQ schemes, which combine the concepts of FEC and ARQ can provide a high system throughput and maintain a high system reliability for communications over quite noisy channels.

Recently, applications of point-to-multipoint communications over broadcast links, such as file distribution, video text systems, mobile telephony and teleconferencing, have been observed to increase. This trend will continue in future communication environments, especially with the deployment of integrated services digital networks (ISDNs). Therefore, the volume of point-to-multipoint data traffic will be significant, and the need for efficient point-to-multipoint ARQ schemes will increase.

In constrast to the great deal of work done on point-to-point schemes, only a limited amount of work has been accomplished for point-to-multipoint schemes [1-15]. In [1-8], a linear block code is used for error detection only and these schemes are called basic multidestination schemes. Metzner [9] proposed a broadcast protocol where the replicas of the previously broadcasted frames are not retransmitted but the frames obtained by Reed-Solomon coding are retransmitted. Chandran and Lin [10] presented a multicast ARQ scheme based on hybrid ARQ schemes. The schemes proposed by Deng et al. [11] and Deng [12] use concatenated coding for error detection and error correction. The scheme in [12] is analyzed for a nonstationary broadcast channel. Taşpınar [13] proposed a multicast hybrid ARQ scheme with partial retransmission and code combining using convolutional codes and sequential decoding. Sakakibara and Kasahara [14] proposed a multicast hybrid ARQ scheme using MDS codes and GMD decoding. Shiozaki [15] presented an adaptive type-II hybrid broadcast ARQ scheme using BCH codes.

Conventional (block and convolutional) error-control codes are power efficient but bandwidth inefficient coding techniques. Trellis coded modulation (TCM), proposed by Ungerboeck [16], achieves both power and bandwidth efficiency modulation. In this paper, a type-I hybrid SR ARQ scheme using trellis coded modulation is described and is analyzed on a nonstationary broadcast channel. The paper is organized as follows. In Section 2, the nonstationary channel is described. In Section 3, operation of the scheme is given. In Section 4, the throughput efficiency analysis of the scheme is performed. In Section 5, numerical results obtained for the throughput efficiency are given. Section 6 contains conclusions.

2. Description of the Channel Model

The channel model considered in this paper is the two state Markov channel model as in Figure 1 [12]. State 0 is the quiet state where the bit error rate (BER) is ε_0. State 1 is the noisy state where the BER is $\varepsilon_1 >> \varepsilon_0$. p is the transition probability from state 0 to state 1 and p' is the transition probability from state 1 to itself. To simplify the treatment of the model, it is assumed that one time frame (i.e., one state transition period) in the model corresponds to the transmission of one data packet.

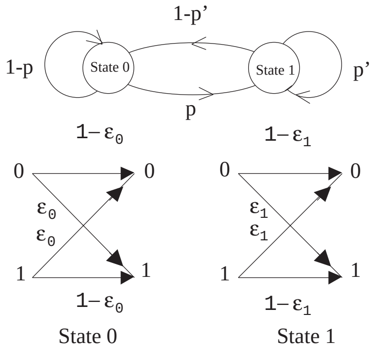

Figure 1. A two-state Markov chain nonstationary channel model

Let p_1 denote the probability of being in the noisy state or the duty cycle of the noisy bursts. This probability can be written as

$$p_1 = \frac{p}{1 - p' + p} \tag{1}$$

When the channel is dense, it has low duty cycle p_1 and high intensity, i.e., a large high-to-low bit error rate ratio $\rho=\varepsilon_1 / \varepsilon_0$. When the channel is diffuse, it has a large duty cycle and low intensity. The conditions for a burst channel are

$$\lim_{p_1 \to 0} \quad \varepsilon_0 = 0 \tag{2.1}$$

$$\lim_{p_1 \to 0} \quad \varepsilon_1 = 1/2 \tag{2.2}$$

and

$$\lim_{p_1 \to 1} \quad \varepsilon_0 = \bar{\varepsilon} \tag{3.1}$$

$$\lim_{p_1 \to 1} \quad \varepsilon_1 = \bar{\varepsilon} \tag{3.2}$$

where $\bar{\varepsilon}$ = (1-p$_1$).ε_0 + p$_1$.ε_1 is the average bit error rate of the channel. Conditions (2) represent the limiting case of a dense burst channel, i.e., p$_1$ $\to$0 and $\rho \to \infty$, while conditions (3) represent the limiting case of a diffuse burst channel, i.e., p$_1$ $\to$1 and $\rho \to 1$, which is equivalent to a binary symmetric channel (BSC). Equations (2.1)-(3.2) can be satisfied by the following constraint:

$$\varepsilon_0 = \bar{\varepsilon}.p_1 \tag{4}$$

For the burst noise channel (BNC), the average burst length, i.e., the average number of packets transmitted while the channel is in state 1, can be written as

$$\bar{b} = \frac{1}{(1 - p')} \tag{5}$$

The value of ε_1 is

$$\varepsilon_1 = \begin{cases} (\bar{\varepsilon}/p_1) - (1-p_1).\bar{\varepsilon} \quad , & \text{for} \quad p_1 > \frac{\bar{\varepsilon}+1/2-\sqrt{(0.5-\bar{\varepsilon})(0.5+3\bar{\varepsilon})}}{2.\bar{\varepsilon}} \\ \quad 1/2 \quad , & \text{otherwise} \end{cases} \tag{6}$$

The BNC is completely described by $\bar{\varepsilon}$, p$_1$ and $\bar{b}$, for if these parameters are known, p', p, ε_0 and ε_1 can be determined from (1) and from (4)-(6) [12].

3. Operation of the Scheme

The broadcast communication network consists of K+1 stations, one being the transmitter and the other K being receivers. The transmission path from the transmitter to the kth receiver is called the kth component channel of the broadcast channel, k=1,2,...,K. The component channels are assumed to produce independent noise processes and are modeled by the burst noise channels. The nonstationary broadcast channel is depicted in Figure 2.

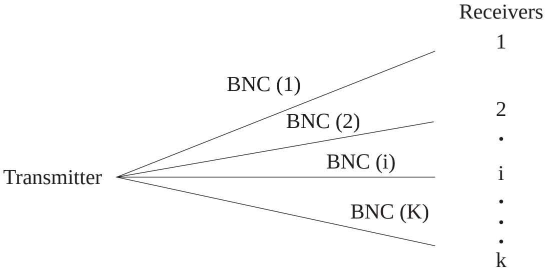

Figure 2. Channel model of the nonstationary broadcast channel.

This scheme employs an (n,k) error-detecting block code C_0 and a TCM code C_1. In this scheme, M-PSK modulation is used as a modulation scheme. A convolutional encoder with memory m and rate b/b+1 yields a (b+1)-bit output for every b information bit. Then the (b+1)-bit output is converted into signal points in an M-PSK signal constellation with $M=2^{b+1}$ points. A block diagram of the communication system is given in Figure 3. The operation of the system can be divided into transmitter operation and receiver operation.

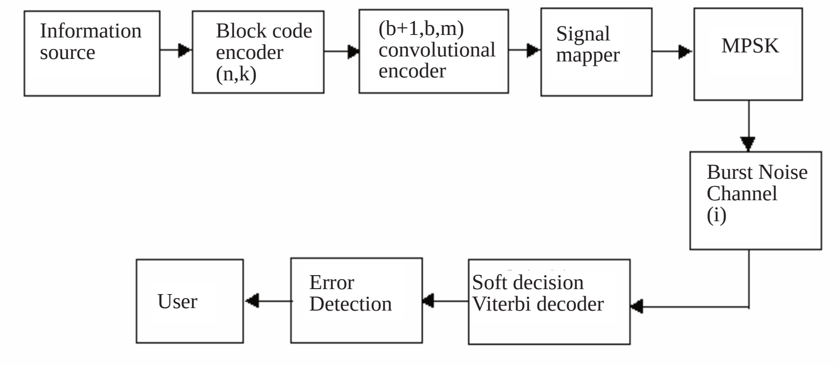

Figure 3. Block diagram of the communication system.

Transmitter operation: The transmitter transmits data packets continuously and constantly listens for acknowledgments of previous transmissions. Each transmitted packet carries a sequence number, and two consecutive packets always differ in sequence number by 1.

When a k-bit data sequence is ready for transmission, it is encoded into an n-bit codeword by C_0, which is an (n,k) block code for error detection. Then the n-bit codeword is encoded by the TCM encoder and a packet $X_1=(x_1,x_2,.......,x_L,x_{L+1},.....,x_{L+m})$ is obtained, where L=n/b is a positive integer. X_1 is transmitted to all receivers. If the transmitter has received positive acknowledgments (ACKs) from all receivers, it sends the next packet in the queue; otherwise it retransmits the packet X_1. Retransmissions are continued until decoding of the packet X_1 succeeds in all receivers.

Receiver operation: When a packet is received by receiver k, k=1,2,...,K, it is decoded by the TCM decoder with a soft-decision Viterbi algorithm. If the output of the Viterbi decoder is declared error-free by C_0, then its forward index f_r, the difference between the sequence number of the current received packet and the sequence number of the last accepted packet [11, 12], is computed. If $f_r=1$, it is accepted and delivered to user k; if $f_r < 1$, the received packet is regarded as a packet that was previously accepted and delivered and therefore it is discarded by receiver k; if $f_r > 1$, the decoded packet is stored in the receiver buffer at the proper position and will not be delivered to user k until all packets preceding it have been accepted and delivered to the user. In these cases, receiver k sends a positive ACK to the transmitter. If the received packet is not decoded error-free, the erroneous packet is discarded in receiver k and a negative acknowledgment (NACK) is sent to the transmitter to resend the erroneous packet.

4. Throughput Efficiency Analysis

For point-to-multipoint ARQ schemes we define the throughput efficiency as the average number of information bits per transmitted modulation symbol. In the throughput efficiency analysis, it is assumed that the feedback channels are noiseless and that the buffer size of receivers is infinite. Let Y_k, k=1,2,....,K be the number of transmission and retransmissions of a packet required until it is accepted by receiver k. The average number of transmissions of a given information packet to be decoded error-free and delivered to users by all K receivers is given as follows [11 ,12]:

$$\begin{aligned} E[T] &= \sum_{i=1}^{\infty} i.P\{T=i\} = \sum_{i=1}^{\infty} i.P\left\{\max_k [Y_k]=i\right\} \\ &= \sum_{i=0}^{\infty}\left[1-P\{Y_k \leq i\}^K\right] \end{aligned} \tag{7}$$

where i is the number of transmission and retransmissions of a packet. The last term is obtained by using a formula for finding the average of the maximum of several independent identically distributed integer-valued random variables [17].

Let $S(\varepsilon)$ and $F(\varepsilon)$ denote the events "decoding success" and "decoding failure" respectively, when decoding of a given packet in receiver k. The probability of $P\{S(\varepsilon)\}$ can be written as

$$P\{S(\varepsilon)\} \geq \quad (1-P(E))^L \tag{8}$$

where P(E) is the probability of an error event of Viterbi decoding, which is upper bounded as

$$P(E) \leq \quad \sum_{j=0}^{\infty} a_{d_j}.P(d_j) \tag{9}$$

where a_{d_j} is the distance spectra of X_i and $P(d_j)$ is the probability that a wrong path at distance d_j is selected as the transmitted packet.

If a received packet contains uncorrectable but detectable errors after its decoding and this packet has not been accepted by the receiver before, it is called an outstanding error packet (OEP). It is supposed that an initial OEP has been received. Then the kth receiver's decoding status can be modeled by the simple Markov chain as in [12]. In the Markov chain there are three states: 0, 1 and s. States 0 and 1 correspond to the reception of an OEP, while the BNC(k) (the transmission path from the transmitter to the kth receiver) was in states 0 and 1, respectively. State s corresponds to successful decoding. By arranging the states in the order: 0, 1, s, the transition probabilities of the Markov chain can be obtained as a one-step transition probability matrix:

$$P = \begin{bmatrix} P_{00} & P_{01} & P_{0s} \\ P_{10} & P_{11} & P_{1s} \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

where

$$P_{00} = [1 - q_{01}(N)].[1 - P\{S\,(\varepsilon_0)\}] \tag{11a}$$

$$P_{01} = [q_{01}(N)].[1 - P\{S\,(\varepsilon_1)\}] \tag{11b}$$

$$P_{0s} = 1 - P_{00} - P_{01} \tag{11c}$$

$$P_{10} = [1 - q_{11}(N)].[1 - P\{S\,(\varepsilon_0)\}] \tag{11d}$$

$$P_{11} = [q_{11}(N)].[1 - P\{S\,(\varepsilon_1)\}] \tag{11e}$$

$$P_{1s} = 1 - P_{10} - P_{11} \tag{11f}$$

In the above transition probabilities, q_{01}(N) and q_{11}(N) are the N-step BNC(k) transition probabilities given by [12]

$$q_{01}(N) = \frac{p}{1+p-p'} - \frac{p}{1+p-p'}.\,(p'-p)^N \tag{12a}$$

$$q_{11}(N) = \frac{p}{1+p-p'} + \frac{(1-p')}{1+p-p'}.\,(p'-p)^N \tag{12b}$$

where N is the number of packets that can be transmitted during one round- trip delay period of the BNC(k).

The probability P$\{Y_k \le i\}$ in Eq. (7) is obtained as follows:

$$P\{Y_k \le i\} = \begin{cases} P_c & \text{, for } i = 1 \\ P_c + (1-p_1).[1 - P\{S(\varepsilon_0)\}].P_{0s}(i-1) \\ \quad +p_1.[1 - P\{S(\varepsilon_1)\}].P_{1s}(i-1) \\ & \text{, for } i > 1 \end{cases} \tag{13}$$

where P_c is the average probability that a received packet will be decoded correctly, and is given by

$$P_c = (1-p_1).P\{S(\varepsilon_0)\} + p_1.P\{S(\varepsilon_1)\} \tag{14}$$

In (13), p_{0s}(i-1) and p_{1s}(i-1) are the (i-1)-step transition probabilities from state 0 to step s and from state 1 to state s, respectively. These probabilities can be obtained from P^{i-1}=[p_{ij}(i-1)].

As a result, the throughput efficiency of the scheme is given by

$$\eta = .\frac{b}{E[T]}.\frac{k/b}{L+m} \tag{15}$$

Usually, (k/b)/(L+m) ≅ 1, and $\eta \cong b/E[T]$.

5. Numerical Results

In this section, numerical results of the throughput are presented for the rate 2/3, four state trellis coded 8-PSK modulation. For numerical calculations, the information message length is assumed to be k=200 bits and n-k=16 bits are attached to the information message for error detection. In the numerical calculations, it is found that, for a given p_1 and $\bar{\varepsilon}$, the values of the average burst length $\bar{b}$ have a very small influence on the throughput. Figure 4 shows the throughput as a function of the average bit error rate $\bar{\varepsilon}$, for respectively the values $p_1=0.05$ (dense channel), $p_1=0.25$ (diffuse channel) and $p_1=1$ (stationary channel), for N=5, K=20, and $\bar{b}=10$. For the values between 10^{-6} and 10^{-4} of $\bar{\varepsilon}$, the dense channel and the diffuse channel yield better throughput efficiency than the stationary channel. However, the scheme performs better for the stationary channel when the average bit error rate becomes large. Figure 5 shows the effects of the number of receivers on the throughput for N=50, $\bar{b}=10$, and $p_1=0.1$. It is seen from this figure that the throughput decreases when the number of receivers increases.

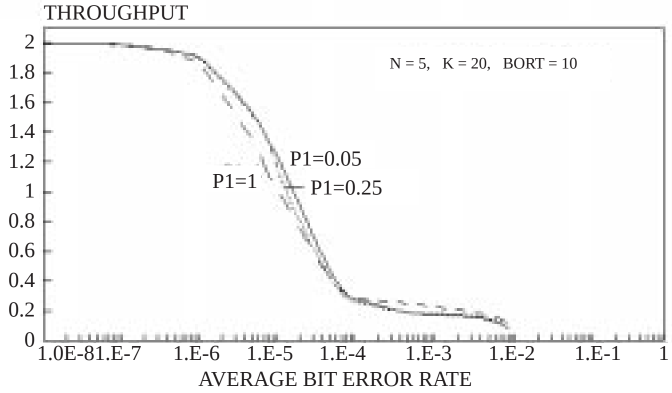

Figure 4. Throughput efficiency of the hybrid SR ARQ scheme for various values of p_1.

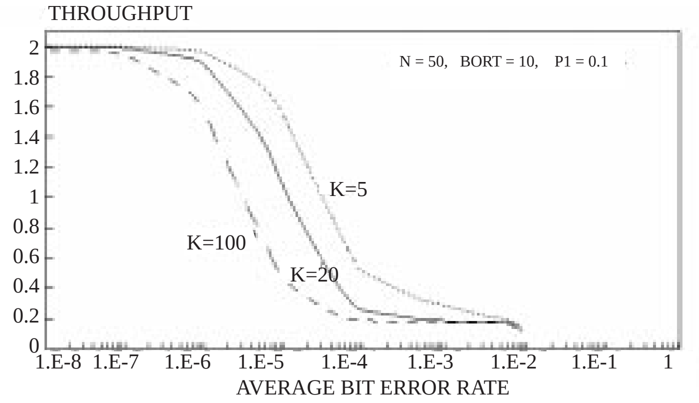

Figure 5. Throughput efficiency of the hybrid SR ARQ scheme for various values of K.

6. Conclusion

In this paper, a type-I hybrid SR ARQ scheme using trellis coded modulation was analyzed on a nonstationary broadcast channel. Numerical results show that this scheme is robust against burst channel errors and is able to provide useful throughput for values of $\bar{\varepsilon}$ between 10^{-6} and 10^{-4}. However, the scheme performs better for the stationary channel when the average bit error rate becomes large. For medium and high values of $\bar{\varepsilon}$, the throughput decreases when the number of receivers increases. Comparison of this scheme with the scheme in [12] shows that this scheme performs better than the scheme in [12] at high average bit error rates, whereas the scheme in [12] performs better than the proposed scheme at low average bit error rates.

References

[1] S.B. Calo and M.C. Easton, "A broadcast protocol for file transfers to multiple sites*", IEEE Trans. Communications*, vol. 29, pp. 670-678, Nov 1981.

[2] K. Mase, T. Takenake, H. Yamamato, and M. Shinohara," Go-back-N ARQ schemes for point-to-multipoint satellite communications", *IEEE Trans. Communications*, vol. 31, pp. 583-590, Apr. 1983.

[3] I. S. Gopal and J. M. Jaffe, "Point-to-multipoint communication over broadcast links", *IEEE Trans. Communications*, vol. 32, pp. 1034 - 1044, Sept. 1984.

[4] K. Sabnani and M. Schwartz, "Multidestination protocols for satellite communications", *IEEE Trans. Communications*, vol. 33, pp. 232-239, March 1985.

[5] D. Towsley, "An analysis of a point-to-multipoint channel using a Go-back-N error control protocol", *IEEE Trans. Communications*, vol. 33, pp. 282 - 285, March 1985.

[6] Y.J. Cho and C.K. Un, "Continuous multidestination ARQ schemes for high error rate channels", *Electron. Letters*, vol. 24, pp. 694-695, 1988.

[7] T. H. Lee, "Some continuous multidestination ARQ schemes for high error rate conditions", *Electron. Letters*, vol. 26, pp.1686-1687, 1990.

[8] T.H. Lee, "Improving performance of multidestination ARQ schemes under high error rate conditions", *Electron. Letters*, vol. 27, pp. 293-294, 1991.

[9] J.J Metzner, "An improved broadcast retransmission protocol", *IEEE Trans. Communications*, vol. 32, pp. 679-683, June 1984.

[10] S.R. Chandran and S. Lin, "Selective- repeat-ARQ schemes for broadcast links", *IEEE Trans. Communications*, vol.40, pp. 12-19, Jan. 1992.

[11] R.H. Deng, X.Y. Zhang, and Y.K. Tham, "New point-to-multipoint communication protocols", *IEE Proceedigs - I*, vol.136, pp.312 -316, Aug. 1989.

[12] R.H. Deng, "Hybrid ARQ schemes for point-to-multipoint communication over nonstationary broadcast channels", *IEEE Trans. Communications*, vol. 41, pp. 1379-1387, Sept. 1993.

[13] Necmi Taşpınar, "Hybrid SR ARQ using sequential decoding with partial retransmission and code combining for point-to-multipoint communications", *Elektrik Turkish Journal of Electrical Engineering & Computer Sciences* , vol. 3, no. 1, pp. 1-6, 1995.

[14] K. Sakakibara and M. Kasahara, "A multicast hybrid ARQ scheme using MDS codes and GMD decoding", *IEEE Trans. Communications*, vol. 43, pp. 2933- 2940, Dec. 1995.

[15] A. Shiozaki, "Adaptive type- II hybrid broadcast ARQ system", *IEEE Trans. Communications*, vol. 44, pp. 420-422, Apr. 1996.

[16] G. Ungerboeck, "Channel coding with multilevel/phase signals", *IEEE Trans. Information Theory*, vol. 28, pp. 55-67, Jan. 1982.

[17] M.A. David, *Order statistics*, New York, Wiley, 1970.