Is f_0 Distinguishable from f_2 by One-dimensional Projections of Angular Distributions in $J/\psi \to f_J \phi \to \pi\pi\bar{K}K$? *

ZHANG Yong-Jun[1,2;1)] ZOU Bing-Song[1;2)]
1 (Institute of High Energy Physics, CAS, Beijing 100039, China)
2 (Department of Basic Science, Liaoning Technical University, Liaoning 123000, China)

Abstract With helicity partial wave analysis formalism, we discuss how one can distinguish f_0 resonance from f_2 resonance in the process of $J/\psi \to f_J \phi(\theta_1, \phi_1)$ with $f_J \to \pi\pi(\theta_2, \phi_2)$ and $\phi \to \bar{K}K(\theta_3, \phi_3)$ by various projections of angular distributions. We find that f_0 and f_2 can give the same one-dimensional angular distributions for $I(\theta_1) \cup I(\phi_1) \cup I(\theta_2) \cup I(\phi_2)$, but cannot give the same $I(\theta_1) \cup I(\phi_1) \cup I(\theta_2) \cup I(\phi_2) \cup I(\phi_3)$. So it is necessary to consider all three decay vertices in order to distinguish f_0 from f_2 by one-dimensional projections of angular distributions.

Key words J/ψ decay, resonance, partial wave analysis

1 Introduction

Through the conservation laws of total angular momentum, parity, etc., the spin-parity property of an involved resonance affects angular distributions of final particles generated in J/ψ decay processes. In this paper, we use helicity partial wave analysis formalism to study in detail how one can distinguish resonance f_0 from f_2 by projections of various angular distributions in the $J/\psi \to f_J \phi \to \pi\pi\bar{K}K$ process which is now under investigation by BES experiment at the Beijing Electron Positron Collider (BEPC).

At BEPC, the J/ψ is generated in the way

$$e^+ e^- \to \gamma \to J/\psi. \tag{1}$$

Because the energy of e^+ and e^- is so high as 1.55GeV, the mass of e^+ and e^- is negligible so that the massless limit can be used. In the massless limit of the e^+ and e^-, the J/ψ can only be in two spin eigenstates, $|JM\rangle = |11\rangle$ and $|1-1\rangle$. And the probabilities in state of $|JM\rangle = |11\rangle$ and $|1-1\rangle$ are both $\frac{1}{2}$ because electron and positron at BEPC are not polarized.

In the process $J/\psi \to f_J \phi \to \pi\pi\bar{K}K$, there is only one final helicity state, $|h_\pi h_\pi h_{\bar{K}} h_K\rangle = |0000\rangle$. For J/ψ in its spin eigenstate of $|1M\rangle$, we denote its decay probability in process of $J/\psi \to \pi\pi\bar{K}K$ as $I(\Omega_1, \Omega_2, \Omega_3, 1M)$, where $\Omega_1(\theta_1, \phi_1)$ is the angle of f_J in the rest frame of J/ψ, $\Omega_2(\theta_2, \phi_2)$ is the angle of particle π in the rest frame of f_J and $\Omega_3(\theta_3, \phi_3)$ is the angle of K in the rest frame of ϕ. In experiment, the angular distribution is the average of all possible initial state according to their weight,

$$I(\Omega_1, \Omega_2, \Omega_3) = \frac{1}{2} I(\Omega_1, \Omega_2, \Omega_3, 11) + \frac{1}{2} I(\Omega_1, \Omega_2, \Omega_3, 1-1). \tag{2}$$

From the parity symmetry, we have relation of decay angular distribution between J/ψ spin eigenstates $|11\rangle$ and $|1-1\rangle$ as

$$I(\theta_1, \phi_1, \Omega_2, \Omega_3, 1-1) = I(\pi - \theta_1, \phi_1, \Omega_2, \Omega_3, 11). \tag{3}$$

Then formula (2) can be written as

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1) Yong-Jun.Zhang@cern.ch
2) zoubs@mail.ihep.ac.cn

$$I(\Omega_1,\Omega_2,\Omega_3) = \frac{1}{2}[I(\theta_1,\phi_1,\Omega_2,\Omega_3,11) + I(\pi-\theta_1,\phi_1,\Omega_2,\Omega_3,11)]. \quad (4)$$

So we only need to calculate the process with J/ψ initial spin state $|11\rangle$ in order to predict angular distributions. Due to $SO(2)$ symmetry of initial state, $I(\Omega_1,\Omega_2,\Omega_3)$ is ϕ_1 independent but may be dependent on ϕ_2 and ϕ_3.

In order to calculate angular distribution of each decay, we assume helicity partial wave analysis formalism as in Refs.[1,2],

$$M^J_{\lambda\nu}(\theta,\phi;M) = \langle\theta,\phi,\lambda\nu|M|JM\rangle \propto \langle\theta,\phi,\lambda\nu|JM\lambda\nu\rangle\langle JM\lambda\nu|M|JM\rangle \propto D^{J*}_{M\delta}(\phi,\theta,0)F^J_{\lambda\nu}, \quad (5)$$

where λ and ν are helicities of two final particles of each decay,

$$\delta = \lambda - \nu, \quad (6)$$

$$\langle\theta,\phi,\lambda\nu|JM\lambda\nu\rangle \propto D^{J*}_{M\delta}(\phi,\theta,0) = e^{iM\phi}d^J_{M,\delta}(\theta), \quad (7)$$

$$F^J_{\lambda\nu} \propto \langle JM\lambda\nu|M|JM\rangle, \quad (8)$$

$$F^J_{\lambda\mu} = \eta_J\eta_s\eta_\sigma(-1)^{J-s-\sigma}F^J_{-\lambda-\mu}. \quad (9)$$

Here, only $F^J_{\lambda\nu}$ are dynamically related, which should be either fitted to experimental data or calculated from some theories. In our following study, we just take them as parameters and perform remaining kinematics calculation to study various angular distributions in case of f_J to be f_0 or f_2.

2 Formulae for J/ψ → f$_0$ϕ with f$_0$ → ππ, ϕ→$\bar{K}$K

In this process, there are three subprocesses: J/ψ→$f_0\phi$, f_0→ππ and ϕ→$\bar{K}$K. Formula (5) will be used three times to these three subprocesses. Then we combine them to give the final result.

2.1 Subprocess J/ψ→f$_0$ϕ

In subprocess J/ψ→$f_0\phi$, the spin of (J/ψ, f_0, ϕ) is (1, 0, 1). the initial J/ψ spin state is $|JM\rangle = |11\rangle$ while the final particle (f_0, ϕ) helicity state may be $|\lambda\mu\rangle = |01\rangle$, $|00\rangle$, or $|0-1\rangle$. The corresponding decay amplitudes $M^J_{\lambda\nu}(\theta_1,\phi_1;M)$ are obtained according to the formula (5) as

$$M^1_{01}(\theta_1,\phi_1;1) \propto D^{1*}_{1-1}(\phi_1,\theta_1,0)F^1_{0,1}, \quad (10)$$

$$M^1_{00}(\theta_1,\phi_1;1) \propto D^{1*}_{10}(\phi_1,\theta_1,0)F^1_{0,0}, \quad (11)$$

$$M^1_{0-1}(\theta_1,\phi_1;1) \propto D^{1*}_{11}(\phi_1,\theta_1,0)F^1_{0,-1}. \quad (12)$$

The parity of (J/ψ, f_0, ϕ) is (−1, 1, −1). According to formula (9), we have

$$F^J_{\lambda\mu} = F^J_{-\lambda-\mu}. \quad (13)$$

Therefore there are only two independent dynamically related complex parameters,

$$F^1_{0,1} = F^1_{0,-1} = G_1 e^{i\zeta_1}, \quad (14)$$

$$F^1_{0,0} = G_2 e^{i\zeta_2}. \quad (15)$$

2.2 Subprocess f$_0$→ππ

Here the spin of (f_0, π, π) is (0,0,0). There is only one possible decay amplitude

$$M^0_{00}(\theta_2,\phi_2;0) \propto D^{0*}_{00}(\theta_2,\phi_2,0)F^0_{0,0} \propto 1. \quad (16)$$

2.3 Subprocess ϕ→$\bar{K}$K

In subprocess ϕ→$\bar{K}$K, the spin of (ϕ, $\bar{K}$, K) is (1, 0, 0), The possible initial helicity states of ϕ are $|JM\rangle = |11\rangle$, $|10\rangle$ and $|1-1\rangle$, while the final helicity state of $\bar{K}$K is $|\lambda\mu\rangle = |00\rangle$. The corresponding decay amplitudes are

$$M^1_{00}(\theta_3,\phi_3;1) = D^{1*}_{10}(\phi_3,\theta_3,0)F'^1_{0,0}, \quad (17)$$

$$M^1_{00}(\theta_3,\phi_3;0) = D^{1*}_{00}(\phi_3,\theta_3,0)F'^1_{0,0}, \quad (18)$$

$$M^1_{00}(\theta_3,\phi_3;-1) = D^{1*}_{-10}(\phi_3,\theta_3,0)F'^1_{0,0}. \quad (19)$$

There is only one dynamically related complex parameter, $F'^1_{0,0}$, which has no effect on $\bar{K}$K angular distribution and can be absorbed into the overall normalization constant.

2.4 Full amplitude for J/ψ→f$_0$ϕ→ππ$\bar{K}$K

Combining the three subprocess amplitudes for initial J/ψ spin state $|JM\rangle = |11\rangle$, we can write out the full amplitude for the process J/ψ→$f_0\phi$→ππ$\bar{K}$K as

$$\begin{aligned}A^0(\Omega_1,\Omega_2,\Omega_3,11) = {}& M^1_{01}(\theta_1,\phi_1;1)M^0_{00}(\theta_2,\phi_2;0)M^1_{00}(\theta_3,\phi_3;1) + \\ & M^1_{00}(\theta_1,\phi_1;1)M^0_{00}(\theta_2,\phi_2;0)M^1_{00}(\theta_3,\phi_3;0) + \\ & M^1_{0-1}(\theta_1,\phi_1;1)M^0_{00}(\theta_2,\phi_2;0)M^1_{00}(\theta_3,\phi_3;-1) \propto \\ & D^{1*}_{1,-1}(\phi_1,\theta_1,0)F^1_{0,1}D^{1*}_{1,0}(\phi_3,\theta_3,0) + \\ & D^{1*}_{1,0}(\phi_1,\theta_1,0)F^1_{0,0}D^{1*}_{0,0}(\phi_3,\theta_3,0) + \\ & D^{1*}_{1,1}(\phi_1,\theta_1,0)F^1_{0,-1}D^{1*}_{-1,0}(\phi_3,\theta_3,0) \propto \\ & d^1_{1,-1}(\theta_1)G_1e^{i\zeta_1}e^{i\phi_3}d^1_{1,0}(\theta_3) + \\ & d^1_{1,0}(\theta_1)G_2e^{i\zeta_2}d^1_{0,0}(\theta_3) + \\ & d^1_{1,1}(\theta_1)G_1e^{i\zeta_1}e^{-i\phi_3}d^1_{-1,0}(\theta_3).\end{aligned} \quad (20)$$

The corresponding decay probability is

$$I^0(\Omega_1,\Omega_2,\Omega_3,11) = |A^0(\Omega_1,\Omega_2,\Omega_3,11)|^2. \tag{21}$$

Then using formula (4), we can calculate the expected decay angular distribution $I(\Omega_1,\Omega_2,\Omega_3)$ for the process $J/\psi\to f_0\phi\to\pi\pi\bar{K}K$ at an electron-positron collider.

We define

$$I(\theta) = \frac{dI}{\sin\theta d\theta}, \quad I(\phi) = \frac{dI}{d\phi},$$

where

$$I = \int d\Omega_1 \int d\Omega_2 \int d\Omega_3 I(\Omega_1,\Omega_2,\Omega_3). \tag{22}$$

Then we use MATHEMATIC software to calculate the one-dimensional projections of the angular distributions as

$$I^0(\theta_1) \propto 1 + \frac{G_1^2 - G_2^2}{3G_1^2 + G_2^2}\cos(2\theta_1), \tag{23}$$

$$I^0(\theta_2) \propto 1, \tag{24}$$

$$I^0(\theta_3) \propto 1 + \frac{-G_1^2 + G_2^2}{G_1^2 + G_2^2}\cos(2\theta_3), \tag{25}$$

$$I^0(\phi_2) \propto 1, \tag{26}$$

$$I^0(\phi_3) \propto 1 + \frac{-G_1^2}{2G_1^2 + G_2^2}\cos(2\phi_3). \tag{27}$$

3 Formulae for $J/\psi\to f_2\phi$ with $f_2\to\pi\pi$, $\phi\to\bar{K}K$

Formulae for this process can be obtained by the same procedure as for the process $J/\psi\to f_0\phi$ with $f_0\to\pi\pi$ and $\phi\to\bar{K}K$.

3.1 Subprocess $J/\psi\to f_2\phi$

In subprocess $J/\psi\to f_2\phi$, the spin of (J/ψ, f_2, ϕ) is (1,2,1). The initial J/ψ spin state is $|JM\rangle = |11\rangle$ while the final particle (f_2, ϕ) helicity state may be

$$|\lambda\mu\rangle = |21\rangle,\ |11\rangle,\ |01\rangle,\ |10\rangle,\ |00\rangle,\ |-10\rangle,\ |0-1\rangle,\ |-1-1\rangle,\ |-2-1\rangle. \tag{28}$$

Corresponding to these helicity states, the decay amplitudes are

$$M^1_{21}(\theta_1,\phi_1;1) = D^{1*}_{11}(\phi_1,\theta_1,0)F^1_{2,1}, \tag{29}$$

$$M^1_{11}(\theta_1,\phi_1;1) = D^{1*}_{10}(\phi_1,\theta_1,0)F^1_{1,1}, \tag{30}$$

$$M^1_{01}(\theta_1,\phi_1;1) = D^{1*}_{1-1}(\phi_1,\theta_1,0)F^1_{0,1}, \tag{31}$$

$$M^1_{10}(\theta_1,\phi_1;1) = D^{1*}_{11}(\phi_1,\theta_1,0)F^1_{1,0}, \tag{32}$$

$$M^1_{00}(\theta_1,\phi_1;1) = D^{1*}_{10}(\phi_1,\theta_1,0)F^1_{0,0}, \tag{33}$$

$$M^1_{-10}(\theta_1,\phi_1;1) = D^{1*}_{1-1}(\phi_1,\theta_1,0)F^1_{-1,0}, \tag{34}$$

$$M^1_{0-1}(\theta_1,\phi_1;1) = D^{1*}_{11}(\phi_1,\theta_1,0)F^1_{0,-1}, \tag{35}$$

$$M^1_{-1-1}(\theta_1,\phi_1;1) = D^{1*}_{10}(\phi_1,\theta_1,0)F^1_{-1,-1}, \tag{36}$$

$$M^1_{-2-1}(\theta_1,\phi_1;1) = D^{1*}_{1-1}(\phi_1,\theta_1,0)F^1_{-2,-1}. \tag{37}$$

The parity of (J/ψ, f_2, ϕ) is (−1, 1, −1). According to formula (9), we have

$$F^J_{\lambda\mu} = F^J_{-\lambda-\mu}. \tag{38}$$

Therefore there are only five independent dynamical complex parameters,

$$F^1_{0,1} = F^1_{0,-1} = H_1 e^{i\eta_1}, \tag{39}$$

$$F^1_{1,0} = F^1_{-1,0} = H_2 e^{i\eta_2}, \tag{40}$$

$$F^1_{2,1} = F^1_{-2,-1} = H_3 e^{i\eta_3}, \tag{41}$$

$$F^1_{1,1} = F^1_{-1,-1} = H_4 e^{i\eta_4}, \tag{42}$$

$$F^1_{00} = H_5 e^{i\eta_5}. \tag{43}$$

3.2 Subprocess $f_2\to\pi\pi$

In subprocess $f_2\to\pi\pi$, the spin of (f_2, π, π) is (2, 0, 0). There are five possible decay amplitudes:

$$M^2_{00}(\theta_2,\phi_2;2) = D^{2*}_{20}(\theta_2,\phi_2,0)F^2_{0,0}, \tag{44}$$

$$M^2_{00}(\theta_2,\phi_2;1) = D^{2*}_{10}(\theta_2,\phi_2,0)F^2_{0,0}, \tag{45}$$

$$M^2_{00}(\theta_2,\phi_2;0) = D^{2*}_{00}(\theta_2,\phi_2,0)F^2_{0,0}, \tag{46}$$

$$M^2_{00}(\theta_2,\phi_2;-1) = D^{2*}_{-10}(\theta_2,\phi_2,0)F^2_{0,0}, \tag{47}$$

$$M^2_{00}(\theta_2,\phi_2;-2) = D^{2*}_{-20}(\theta_2,\phi_2,0)F^2_{0,0}. \tag{48}$$

The common factor $F^2_{0,0}$ can be absorbed into the overall normalization factor.

3.3 Full amplitude for $J/\psi\to f_2\phi\to\pi\pi\bar{K}K$

The subprocess $\phi\to\bar{K}K$ is the same as in subsection 2.3. Combining the three subprocess amplitudes, for initial J/ψ spin state $|JM\rangle = |11\rangle$, we get the full amplitude for $J/\psi\to f_2\phi\to\pi\pi\bar{K}K$ as

$$\begin{aligned} A^2(\Omega_1,\Omega_2,\Omega_3,11) = {} & M^1_{21}(\theta_1,\phi_1;1)M^2_{00}(\theta_2,\phi_2;2)M^1_{00}(\theta_3,\phi_3;1) + \\ & M^1_{11}(\theta_1,\phi_1;1)M^2_{00}(\theta_2,\phi_2;1)M^1_{00}(\theta_3,\phi_3;1) + \\ & M^1_{01}(\theta_1,\phi_1;1)M^2_{00}(\theta_2,\phi_2;0)M^1_{00}(\theta_3,\phi_3;1) + \\ & M^1_{10}(\theta_1,\phi_1;1)M^2_{00}(\theta_2,\phi_2;1)M^1_{00}(\theta_3,\phi_3;0) + \\ & M^1_{00}(\theta_1,\phi_1;1)M^2_{00}(\theta_2,\phi_2;0)M^1_{00}(\theta_3,\phi_3;0) + \\ & M^1_{-10}(\theta_1,\phi_1;1)M^2_{00}(\theta_2,\phi_2;-1)M^1_{00}(\theta_3,\phi_3;0) + \\ & M^1_{0-1}(\theta_1,\phi_1;1)M^2_{00}(\theta_2,\phi_2;0)M^1_{00}(\theta_3,\phi_3;-1) + \\ & M^1_{-1-1}(\theta_1,\phi_1;1)M^2_{00}(\theta_2,\phi_2;-1)M^1_{00}(\theta_3,\phi_3;-1) + \\ & M^1_{-2-1}(\theta_1,\phi_1;1)M^2_{00}(\theta_2,\phi_2;-2)M^1_{00}(\theta_3,\phi_3;-1) \end{aligned} \tag{49}$$

The corresponding decay probability is

$$I^2(\Omega_1,\Omega_2,\Omega_3,11) = |A^2(\Omega_1,\Omega_2,\Omega_3,11)|^2. \quad (50)$$

Then using formula (4), we can calculate the expected decay angular distribution $I(\Omega_1,\Omega_2,\Omega_3)$ of process $J/\psi \to f_2\phi \to \pi\pi\bar{K}K$ at an electron-positron collider. With the definition of (22), we have

$$I^2(\theta_1) \propto 1 + \frac{H_1^2 + H_2^2 + H_3^2 - 2H_4^2 - H_5^2}{3H_1^2 + 3H_2^2 + 3H_3^2 + 2H_4^2 + H_5^2}\cos(2\theta_1), \quad (51)$$

$$I^2(\theta_2) \propto 1 + \frac{12(2H_1^2 - H_3^2 + H_5^2)}{22H_1^2 + 12H_2^2 + 9H_3^2 + 12H_4^2 + 11H_5^2}\cos(2\theta_2), \quad (52)$$

$$+ \frac{3(6H_1^2 - 4H_2^2 + H_3^2 - 4H_4^2 + 3H_5^2)}{22H_1^2 + 12H_2^2 + 9H_3^2 + 12H_4^2 + 11H_5^2}\cos(4\theta_2), \quad (53)$$

$$I^2(\theta_3) \propto 1 + \frac{-H_1^2 + 2H_2^2 - H_3^2 - H_4^2 + H_5^2}{H_1^2 + 2H_2^2 + H_3^2 + H_4^2 + H_5^2}\cos(2\theta_3), \quad (54)$$

$$I^2(\phi_2) \propto 1 + \frac{-\left(H_2^2 + \sqrt{\frac{2}{3}}H_1H_3\cos(\eta_1 - \eta_3)\right)}{2H_1^2 + 2H_2^2 + 2H_3^2 + 2H_4^2 + H_5^2}\cos(2\phi_2), \quad (55)$$

$$I^2(\phi_3) \propto 1 + \frac{-H_1^2}{2H_1^2 + 2H_2^2 + 2H_3^2 + 2H_4^2 + H_5^2}\cos(2\phi_3). \quad (56)$$

4 Is f_0 distinguishable from f_2 by one-dimensional angular distributions?

In this section, first we show that f_0 cannot be distinguished from f_2 by any single one-dimensional angular distribution. Then we show that even with combined one-dimensional angular distributions for first two-decay angles $I(\theta_1)\cup I(\theta_2)\cup I(\phi_2)$, the f_0 is still ont distinguishable from f_2; but with more combined noe-dimensional angular distributions including the third decay vertex $I(\theta_1)\cup I(\theta_2)\cup I(\theta_3)\cup I(\phi_2)\cup I(\phi_3)$, the f_0 is definitely distinguishable from f_2.

For $J/\psi \to f_J\phi \to \pi\pi\bar{K}K$ with $J = 0, 2$, one-dimensional angular distributions can be written in a general form as

$$I^J(\theta_1) \propto 1 + h^J_{\theta_1}\cos(2\theta_1), \quad (57)$$

$$I^J(\theta_2) \propto 1 + h^J_{\theta_2}\cos(2\theta_2) + h^{J'}_{\theta_2}\cos(4\theta_2), \quad (58)$$

$$I^J(\theta_3) \propto 1 + h^J_{\theta_3}\cos(2\theta_3), \quad (59)$$

$$I^J(\phi_2) \propto 1 + h^J_{\phi_2}\cos(2\phi_2), \quad (60)$$

$$I^J(\phi_3) \propto 1 + h^J_{\phi_3}\cos(2\phi_3). \quad (61)$$

For f_0, by comparing (23)—(27) with (57)—(61), we have

$$h^0_{\theta_1} = \frac{G_1^2 - G_2^2}{3G_1^2 + G_2^2}, \quad (62)$$

$$h^0_{\theta_2} = 0, \quad (63)$$

$$h^{0'}_{\theta_2} = 0, \quad (64)$$

$$h^0_{\theta_3} = \frac{-G_1^2 + G_2^2}{G_1^2 + G_2^2}, \quad (65)$$

$$h^0_{\phi_2} = 0, \quad (66)$$

$$h^0_{\phi_3} = \frac{-G_1^2}{2G_1^2 + G_2^2}, \quad (67)$$

which limit the range of each h^0 parameter as

$$h^0_{\theta_1} \in \left(-1, \frac{1}{3}\right), \quad (68)$$

$$h^0_{\theta_2} = 0, \quad (69)$$

$$h^{0'}_{\theta_2} = 0, \quad (70)$$

$$h^0_{\theta_3} \in (-1, 1), \quad (71)$$

$$h^0_{\phi_2} = 0, \quad (72)$$

$$h^0_{\phi_3} \in \left(-\frac{1}{2}, 0\right). \quad (73)$$

For f_2, by comparing (51)—(56) with (57)—(61), we have

$$h^2_{\theta_1} = \frac{H_1^2 + H_2^2 + H_3^2 - 2H_4^2 - H_5^2}{3H_1^2 + 3H_2^2 + 3H_3^2 + 2H_4^2 + H_5^2}, \quad (74)$$

$$h^2_{\theta_2} = \frac{12(2H_1^2 - H_3^2 + H_5^2)}{22H_1^2 + 12H_2^2 + 9H_3^2 + 12H_4^2 + 11H_5^2}, \quad (75)$$

$$h^{2'}_{\theta_2} = \frac{3(6H_1^2 - 4H_2^2 + H_3^2 - 4H_4^2 + 3H_5^2)}{22H_1^2 + 12H_2^2 + 9H_3^2 + 12H_4^2 + 11H_5^2}, \quad (76)$$

$$h^2_{\theta_3} = \frac{-H_1^2 + 2H_2^2 - H_3^2 - H_4^2 + H_5^2}{H_1^2 + 2H_2^2 + H_3^2 + H_4^2 + H_5^2}, \quad (77)$$

$$h^2_{\phi_2} = \frac{-\left(H_2^2 + \sqrt{\frac{2}{3}}H_1H_3\cos(\eta_1 - \eta_3)\right)}{2H_1^2 + 2H_2^2 + 2H_3^2 + 2H_4^2 + H_5^2}, \quad (78)$$

$$h^2_{\phi_3} = \frac{-H_1^2}{2H_1^2 + 2H_2^2 + 2H_3^2 + 2H_4^2 + H_5^2}. \quad (79)$$

which limit the range of each h^2 parameter as

$$h^2_{\theta_1} \in \left(-1, \frac{1}{3}\right), \tag{80}$$

$$h^2_{\theta_2} \in \left(-\frac{4}{3}, \frac{12}{11}\right), \tag{81}$$

$$h^{2'}_{\theta_2} \in \left(-1, \frac{9}{11}\right), \tag{82}$$

$$h^2_{\theta_3} \in (-1, 1), \tag{83}$$

$$h^2_{\phi_2} \in \left(-\frac{1}{2}, \frac{1}{2\sqrt{6}}\right), \tag{84}$$

$$h^2_{\phi_3} \in \left(-\frac{1}{2}, 0\right), \tag{85}$$

By comparing Eqs. (68)—(73) with (80)—(85), we have the following relations

$$h^0_{\theta_1} \subset h^2_{\theta_1}, \tag{86}$$

$$h^0_{\theta_2} \subset h^2_{\theta_2}, \tag{87}$$

$$h^{0'}_{\theta_2} \subset h^{2'}_{\theta_2}, \tag{88}$$

$$h^0_{\theta_3} \subset h^2_{\theta_3}, \tag{89}$$

$$h^0_{\phi_2} \subset h^2_{\phi_2}, \tag{90}$$

$$h^0_{\phi_3} \subset h^2_{\phi_3}. \tag{91}$$

These relations mean that any single one-dimensional distribution for the f_0 case can be simulated by f_2; so f_0 is not distinguishable from f_2 by any single one-dimensional angular distribution. But most one-dimensional angular distributions of f_2 cannot be simulated by f_0. Hence f_2 is usually distinguishable from f_0.

Then how about combined one-dimensional angular distributions: if several one-dimensional angular distributions are taken into account, is f_0 distinguishable from f_2? In order to answer this question, we should construct an equation set from combined one-dimensional angular distributions for f_0 and f_2, and check whether it has a solution. If no solution, it means that the f_0 cannot be simulated by f_2 and is distinguishable.

We first study a simple case in which the ϕ decay process is not considered. In this case there are only two decay vertices described by angles of θ_1, θ_2, ϕ_2. Corresponding to the combined one-dimensional angular distribution $I(\theta_1) \cup I(\theta_2) \cup I(\phi_2)$, the equation set is

$$\begin{cases} h^0_{\theta_1} = h^2_{\theta_1} \\ h^0_{\theta_2} = h^2_{\theta_2} \\ h^{0'}_{\theta_2} = h^{2'}_{\theta_2} \\ h^0_{\phi_2} = h^2_{\phi_2}, \end{cases} \tag{92}$$

which is found to have solution satisfying

$$\begin{cases} h^0_{\theta_1} = h^2_{\theta_1} \in \left(-\frac{1}{3}, \frac{1}{3}\right) \\ h^0_{\theta_2} = h^2_{\theta_2} = 0 \\ h^{0'}_{\theta_2} = h^{2'}_{\theta_2} = 0 \\ h^0_{\phi_2} = h^2_{\phi_2} = 0. \end{cases} \tag{93}$$

It means that f_0 may still be indistinguishable from f_2.

If the ϕ decay process is considered, we will have an additional decay vertex described by angles θ_3, ϕ_3. Then we can construct a bigger combined one-dimensional angular distribution as $I(\theta_1) \cup I(\theta_2) \cup I(\theta_3) \cup I(\phi_2) \cup I(\phi_3)$ which leads to an equation set as

$$\begin{cases} h^0_{\theta_1} = h^2_{\theta_1} \\ h^0_{\theta_2} = h^2_{\theta_2} \\ h^{0'}_{\theta_2} = h^{2'}_{\theta_2} \\ h^0_{\theta_3} = h^2_{\theta_3} \\ h^0_{\phi_2} = h^2_{\phi_2} \\ h^0_{\phi_3} = h^2_{\phi_3}, \end{cases} \tag{94}$$

which is found with Eqs. (62)—(67) and Eqs. (74)—(79) to have no solution. Thus there must be a method to distinguish resonance f_0 from f_2 if we take all these one-dimensional angular distributions into account.

But what is the method? We will answer this question in the next section.

5 How to Distinguish f_0 From f_2?

By comparing the formulae (68)—(73) with (80)—(85), it is easy to see that if $h^J_{\theta_2}$, $h^{J'}_{\theta_2}$ and $h^J_{\phi_2}$ do not satisfy the following conditions:

$$\begin{cases} h^J_{\theta_2} = 0 \\ h^{J'}_{\theta_2} = 0 \\ h^J_{\phi_2} = 0, \end{cases} \tag{95}$$

the resonance is definitely f_2. Otherwise, it may be both f_0 and f_2. If it is f_2, these conditions give the constraint on H^2 parameters of resonance f_2 decay process as

$$\begin{cases} 2H_1^2 - H_3^2 + H_5^2 = 0 \\ 6H_1^2 - 4H_2^2 + H_3^2 - 4H_4^2 + 3H_5^2 = 0 \\ H_2^2 + \sqrt{\frac{2}{3}} H_1 H_3 \cos(\eta_1 - \eta_3) = 0, \end{cases} \tag{96}$$

which leads to the following relations

$$\begin{cases} H_5^2 = -2H_1^2 + H_3^2 \Rightarrow H_3^2 > 2H_1^2 \\ H_4^2 = -H_2^2 + H_3^2 \Rightarrow H_3^2 > H_2^2, \\ H_2^2 \leqslant \sqrt{\frac{2}{3}}\,|H_1 H_3|. \end{cases} \tag{97}$$

These relations then put constraints on parameters of $h^2_{\theta_1}$, $h^2_{\theta_3}$, $h^2_{\phi_3}$. Here we use $h^{2c}_{\theta_1}$, $h^{2c}_{\theta_3}$, $h^{2c}_{\phi_3}$ to represent $h^2_{\theta_1}$, $h^2_{\theta_3}$, $h^2_{\phi_3}$ under the constraint of (97). For $h^2_{\theta_1}$, we have

$$\begin{cases} h^2_{\theta_1} = \dfrac{H_1^2 + H_2^2 + H_3^2 - 2H_4^2 - H_5^2}{3H_1^2 + 3H_2^2 + 3H_3^2 + 2H_4^2 + H_5^2} = \\ \dfrac{3H_1^2 + 3H_2^2 - 2H_3^2}{H_1^2 + H_2^2 + 6H_3^2} = \\ \dfrac{10}{3}\dfrac{H_1^2 + H_2^2}{H_1^2 + H_2^2 + 6H_3^2} - \dfrac{1}{3} > -\dfrac{1}{3} \\ h^2_{\theta_1} = \dfrac{10}{3}\dfrac{H_1^2 + H_2^2}{H_1^2 + H_2^2 + 6H_3^2} - \dfrac{1}{3} = \\ \dfrac{10}{3}\dfrac{H_1^2 + H_2^2}{H_1^2 + H_2^2 + 2H_3^2 + 4H_3^2} - \dfrac{1}{3} < \\ \dfrac{10}{3}\dfrac{H_1^2 + H_2^2}{5H_1^2 + 5H_2^2} - \dfrac{1}{3} = \dfrac{1}{3} \end{cases}$$

$$\Rightarrow h^{2c}_{\theta_1} \in \left(-\frac{1}{3}, \frac{1}{3}\right). \tag{98}$$

For $h^2_{\theta_3}$, we have

$$\begin{aligned} h^2_{\theta_3} &= \frac{-H_1^2 + 2H_2^2 - H_3^2 - H_4^2 + H_5^2}{H_1^2 + 2H_2^2 + H_3^2 + H_4^2 + H_5^2} = \\ &\frac{-3H_1^2 + 3H_2^2 - H_3^2}{-H_1^2 + H_2^2 + 3H_3^2} = \\ &3 - \frac{10H_3^2}{3H_3^2 + H_2^2 - H_1^2} \leqslant \\ &3 - \frac{10H_3^2}{3H_3^2 + \frac{\sqrt{6}}{3}|H_1||H_3| - H_1^2} = \\ &3 - \frac{10}{\left(\frac{|H_1|}{|H_3|}\right)^2 - \frac{\sqrt{6}}{3}\frac{|H_1|}{|H_3|} + 1} = \\ &3 - \frac{2}{\frac{19}{6} - \left(\frac{|H_1|}{|H_3|} - \frac{1}{\sqrt{6}}\right)^2} \leqslant \\ &-\frac{3}{19}, \end{aligned} \tag{99}$$

hence

$$h^{2c}_{\theta_3} \in \left(-1, -\frac{3}{19}\right]. \tag{100}$$

For $h^2_{\phi_3}$, we have

$$\begin{cases} h^2_{\phi_3} = \dfrac{-H_1^2}{2H_1^2 + 2H_2^2 + 2H_3^2 + 2H_4^2 + H_5^2 +} < 0 \\ h^2_{\phi_3} = \dfrac{-H_1^2}{2H_1^2 + 2H_2^2 + 2H_3^2 + 2H_4^2 + H_5^2 +} = \\ \dfrac{H_1^2}{5H_3^2} > -\dfrac{H_1^2}{10H_1^2} = -\dfrac{1}{10} \end{cases}$$

$$\Rightarrow h^{2c}_{\phi_3} \in \left(-\frac{1}{10}, 0\right). \tag{101}$$

If $h^{2c}_{\theta_1}$, $h^{2c}_{\theta_3}$, $h^{2c}_{\phi_3}$ and $h^0_{\theta_1}$, $h^0_{\theta_3}$, $h^0_{\phi_3}$ do not overlap, then the resonances f_0 and f_2 can be distinguished. But in fact they have overlaps:

$$h^{2c}_{\theta_1} \cap h^0_{\theta_1} \in \left(-\frac{1}{3}, \frac{1}{3}\right), \tag{102}$$

$$h^{2c}_{\theta_3} \cap h^0_{\theta_3} \in \left(-1, -\frac{3}{19}\right], \tag{103}$$

$$h^{2c}_{\phi_3} \cap h^0_{\phi_3} \in \left(-\frac{1}{10}, 0\right). \tag{104}$$

So it seems that the f_0 and f_2 are still not surely distinguishable. However, in such case, if the resonance is f_0, the corresponding parameter h^0 must satisfy the constraint

$$h^{0c}_{\theta_1} \in \left(-\frac{1}{3}, \frac{1}{3}\right), \tag{105}$$

which leads to

$$h^0_{\theta_1} = \frac{G_1^2 - G_2^2}{3G_1^2 + G_2^2} > -\frac{1}{3} \Rightarrow 3G_1^2 - 3G_2^2 > -3G_1^2 - G_1^2 \Rightarrow 3G_1^2 > G_2^2. \tag{106}$$

The above constraint on G_1 and G_2 puts further constraint on $h^0_{\phi_3}$. If we use $h^{0c}_{\theta_3}$, $h^{0c}_{\phi_3}$ to represent $h^0_{\theta_3}$, $h^0_{\phi_3}$ under the constraint of (106), we have

$$\begin{cases} h^{0c}_{\theta_3} = \dfrac{-G_1^2 + G_2^2}{G_1^2 + G_2^2} = 1 - 2\dfrac{G_1^2}{G_1^2 + G_2^2} > -1 \\ h^{0c}_{\theta_3} = 1 - 2\dfrac{G_1^2}{G_1^2 + G_2^2} < 1 - 2\dfrac{G_1^2}{G_1^2 + 3G_1^2} = \dfrac{1}{2} \end{cases}$$

$$\Rightarrow h^{0c}_{\theta_3} \in \left(-1, \frac{1}{2}\right). \tag{107}$$

and

$$\begin{cases} h^{0c}_{\phi_3} = \dfrac{-G_1^2}{2G_1^2 + G_2^2} > -\dfrac{1}{2} \\ h^{0c}_{\phi_3} = \dfrac{-G_1^2}{2G_1^2 + G_2^2} < \dfrac{-G_1^2}{2G_1^2 + 3G_1^2} = \dfrac{-1}{2+3} = -\dfrac{1}{5} \end{cases}$$

$$\Rightarrow h^{0c}_{\phi_3} \in \left(-\frac{1}{2}, -\frac{1}{5}\right). \tag{108}$$

From the relations of (101) and (108), we know that $h^{2c}_{\phi_3}$ and $h^{0c}_{\phi_3}$ do not overlap so that the resonances f_0 and f_2 can be surely distinguished: under the constraint of

(95) and (106), if $h^J_{\phi_3} \in \left(-\frac{1}{2}, -\frac{1}{5}\right)$, the resonance must be f_0; otherwise the resonance must be f_2 and there must be $h^J_{\phi_3} \in \left(-\frac{1}{10}, 0\right)$. The $h^{0c}_{\theta_3}$ from Eq. (107) still has overlap with $h^{2c}_{\theta_3}$ from Eq. (100), hence $I(\theta_3)$ cannot guarantee f_0 and f_2 to be distinguishable.

6 Summary and discussions

In summary, in the decay process of $J/\psi \to f_J \phi \to \pi\pi\bar{K}K$, the one-dimensional projections of angular distributions have the following general form:

$$I(\theta_1) = \frac{dI}{d\sin\theta_1} \propto 1 + h^J_{\theta_1}\cos(2\theta_1) \quad (109)$$

$$I(\theta_2) = \frac{dI}{d\sin\theta_2} \propto 1 + h^J_{\theta_2}\cos(2\theta_2) + h^{J'}_{\theta_2}\cos(4\theta_2) \quad (110)$$

$$I(\theta_3) = \frac{dI}{d\sin\theta_3} \propto 1 + h^J_{\theta_3}\cos(2\theta_3) \quad (111)$$

$$I(\phi_2) = \frac{dI}{d\phi_2} \propto 1 + h^J_{\phi_2}\cos(2\phi_2) \quad (112)$$

$$I(\phi_3) = \frac{dI}{d\phi_3} \propto 1 + h^J_{\phi_3}\cos(2\phi_3). \quad (113)$$

The resonance must be f_0 if h^J parameters satisfy the condition

$$\begin{cases} h^J_{\theta_2} = 0 \\ h^{J'}_{\theta_2} = 0 \\ h^J_{\phi_2} = 0 \\ h^J_{\theta_1} \in \left(-1, -\frac{1}{3}\right) \end{cases} \quad (114)$$

or

$$\begin{cases} h^J_{\theta_2} = 0 \\ h^{J'}_{\theta_2} = 0 \\ h^J_{\phi_2} = 0 \\ h^J_{\theta_1} \in \left(-\frac{1}{3}, \frac{1}{3}\right) \\ h^J_{\phi_3} \in \left(-\frac{1}{2}, -\frac{1}{5}\right). \end{cases} \quad (115)$$

Otherwise, h^J parameters must satisfy one of the following 4 conditions and the resonance must be f_2,

$$h^J_{\theta_2} \neq 0, \quad (116)$$

$$h^{J'}_{\theta_2} \neq 0, \quad (117)$$

$$h^J_{\phi_2} \neq 0, \quad (118)$$

and

$$\begin{cases} h^J_{\theta_2} = 0 \\ h^{J'}_{\theta_2} = 0 \\ h^J_{\phi_2} = 0 \\ h^J_{\theta_1} \in \left(-\frac{1}{3}, \frac{1}{3}\right) \\ h^J_{\phi_3} \in \left(-\frac{1}{10}, 0\right). \end{cases} \quad (119)$$

These results indicate that resonances f_0 and f_2 can give the same one-dimensional angular distributions for $I(\theta_1) \cup I(\phi_1) \cup I(\theta_2) \cup I(\phi_2)$, but cannot give the same $I(\theta_1) \cup I(\phi_1) \cup I(\theta_2) \cup I(\phi_2) \cup I(\phi_3)$. So it is necessary to consider all three decay vertices in order to distinguish f_0 from f_2 by one-dimensional projections of angular distributions.

However, with two-dimensional projections of angular distributions $I(\theta_2, \phi_2)$ or $I(\theta_1, \theta_2)$ which inclued the correlation between various angles, one may distinguish f_0 from f_2 without considering information from the third decay vertex. Both moment analysis method[3,4] and full amplitude fitting method[5,6,7] include the information of angle correlations, hence they can be used to distinguish f_2 from f_2 without considering information from the third decay vertex although the additional information from the third decay vertex will give a more clear distinction. These methods are more powerful than using the simple one-dimensional projections, but the latter gives more intuitive evidence.

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$J/\psi \to f_J \phi \to \pi\pi K^+ K^-$ 反应的各种一维角分布是否足以鉴别 f_0 和 f_2?*

张永军[1,2;1)] 邹冰松[1;2)]

1(中国科学院高能物理研究所 北京 100039)

2(辽宁技术大学基础科学系 辽宁 123000)

摘要 基于螺旋度分波分析公式,我们探讨在 $J/\psi \to f_J\phi(\theta_1,\phi_1)$、$f_J \to \pi\pi(\theta_2,\phi_2)$、$\phi \to K^+K^-(\theta_3,\phi_3)$级联衰变过程中是否可以通过各种一维角分布投影鉴别出 f_0 和 f_2 共振态. 结果表明,f_0 和 f_2 可以同时给出完全相同的$(\theta_1,\phi_1,\theta_2,\phi_2)$一维投影,但不能同时给出完全相同的$(\theta_1,\phi_1,\theta_2,\phi_2,\phi_3)$一维投影. 因此,要想保证从角分布的一维投影鉴别出 f_0 和 f_2,必须同时考虑所有三个衰变顶角的角分布信息.

关键词 J/ψ 衰变 共振态 分波分析

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1) Yong-Jun. Zhang@cern.ch

2) Zoubs@mail.ihep.ac.cn