$SU(3)$ simple group model and single top production at the $e^-\gamma$ colliders*

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Abstract In the framework of the $SU(3)$ simple group model, we consider the single top quark production process $e^-\gamma \to \nu_e b\bar{t}$. We find that the correction effects on the process mainly come from the terms of the tree-level Wqq' couplings. In the reasonable parameter space of the $SU(3)$ simple group model, the deviation of the total production cross section σ^{tot} from its SM value is larger than 5%, which might be detected in the future high energy linear e^+e^- collider (LC) experiments.

Key words $SU(3)$ simple group model, free parameter, cross section

PACS 12.60.-i, 13.66.Hk, 14.65.Ha

1 Introduction

The standard model (SM) provides an excellent effective field theory description of almost all particle physics experiments. But in the SM the Higgs boson mass suffers from an instability under radiative corrections. The naturalness argument suggests that the cutoff scale of the SM is not much above the electroweak scale: New physics will appear around TeV energies. Recently, the little Higgs model offers a very promising solution to the hierarchy problem in which the Higgs boson is naturally light as a result of nonlinearly realized symmetry[1]. The key feature of this model is that the Higgs boson is a pseudo-Goldstone boson of an approximate global symmetry which is spontaneously broken by a vacuum expectation value (VEV) at a scale of a few TeV and thus is naturally light. Little Higgs models can stabilize the little hierarchy between the electroweak scale and the 10 TeV scale at which strongly-coupled new physics is allowed by electroweak precision constraints.

Little Higgs models contain new gauge bosons, a heavy top-like quark, and new scalars, which cancel the quadratically divergent one-loop contributions to the Higgs boson mass from the SM gauge bosons, top quark, and Higgs self-interaction, respectively. Some of these new particles can generate characteristic signatures at the present and future collider experiments[2]. There are several variations of the little Higgs models, which differ in the assumed higher symmetry and in the representations of the scalar multiplets. According to the structure of the extend electroweak gauge group, the little Higgs models can be generally divided into two classes[3, 4]: product group models, in which the SM $SU(2)_L$ is embedded in a product gauge group, and simple group models, in which it is embedded in a large simple group. These two classes of models also exhibit an important difference in the implementation of the little Higgs mechanism in the fermion sector. The littlest Higgs model[5] and the $SU(3)$ simple group model[4] are the simplest examples of the product group models and the simple group models, respectively.

The $SU(3)$ simple group model consists of two σ model with a global symmetry $[SU(3)\times U(1)]^2$ and a gauge symmetry $SU(3)\times U(1)_X$. The global symmetry is spontaneously broken down to its subgroup $[SU(2)\times U(1)]^2$ by two vacuum condensates $\langle \Phi_{1,2}\rangle = (0, 0, f_{1,2})$, where $f_1 \sim f_2 \sim 1$ TeV. At the same time, the gauge boson symmetry is broken down to the SM gauge boson group $SU(2)\times U(1)$ by the gauge interactions. This breaking scenario gives to

Received 15 March 2007, Revised 8 May 2007
* Supported by National Natural Science Foundation of China (10775039) and Henan Institute of Science and Technology (06040)
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an $SU(2)_L$ doublet gauge bosons (Y^0, X^-) and a new neutral gauge boson Z'. Due to the gauged $SU(3)$ symmetry in the $SU(3)$ simple group model, all of the SM fermion representations have to be extended to transform as fundamental (or anti-fundamental) representations of $SU(3)$, which demands the existence of new heavy fermions in all three generations. The fermion sector of the $SU(3)$ simple group model can be constructed in two ways: universal and anomaly free, which might induce the different signatures at the high energy collider experiments. We find that the model has regions of parameter space for which TeV scale particles only couple very weakly to SM fields in tree level interactions. This allows them to hide from precision electroweak measurements while still canceling the divergences to the Higgs mass.

The top quark is by far the heaviest known fermion with a mass of the order of the electroweak scale $m_t = 172.7 \pm 2.9$ GeV[6]. Assuming this is not a coincidence, it is hoped that a detailed study of top quark couplings to other particles will be of utility in clarifying whether the SM provides the correct mechanism for electroweak symmetry-breaking, or whether new physics is responsible. It is therefore of interest to provide a general description of the top quark couplings, which might be modified due to the presence of new interactions or particles. There have been many papers studying the test of new physics via the top quark productions at high energy colliders in the literatures. For instance, supersymmetric corrections to top quark production at hadron colliders and electron(photon) linear colliders (LC) have been studied in Ref. [7]. Recently, there have been a lot of interests in studying single top quark productions in new physics models[8]. Future linear colliders are expected to be designed to function also as $\gamma\gamma$ or $e\gamma$ colliders with the photon beams generated by laser-scattering method, in these modes the flexibility in polarizing both the lepton and photon beams will allow unique opportunities to analyze the top quark properties and interactions. The aim of this paper is to consider the process $e^-\gamma \to \nu_e b\bar{t}$ in the context of the $SU(3)$ simple group model, and see whether the effects of this model on this process can be detected in the future LC experiments.

2 The process $e^-\gamma \to \nu_e b\bar{t}$ in the $SU(3)$ simple group model

The new charged gauge boson predicted by the $SU(3)$ simple group model get its mass from the f condensate, which breaks the extended gauge symmetry. At the leading order, the mass of the new charged gauge boson X^- can be written as[9]

$$M_X = \frac{gf}{\sqrt{2}} \approx 0.46 f\,. \tag{1}$$

The relative couplings of the charged gauge bosons W and X to the fermions can be unitive written as[9]

$$g_V^{We\nu} = -g_A^{We\nu} = \frac{ie}{2\sqrt{2}s_W}\left[1-\frac{1}{2}\delta_\nu^2\right], \tag{2}$$

$$g_V^{Xe\nu} = -g_A^{Xe\nu} = \frac{ie}{2\sqrt{2}s_W}\delta_\nu, \tag{3}$$

$$g_V^{Wtb} = -g_A^{Wtb} = \frac{ie}{2\sqrt{2}s_W}\left[1-\frac{1}{2}\delta_t^2\right], \tag{4}$$

$$g_V^{Xtb} = -g_A^{XTb} = \frac{ie}{2\sqrt{2}s_W}\delta_t, \tag{5}$$

with

$$\delta_t = \frac{v}{\sqrt{2}f}t_\beta\frac{x_\lambda^2-1}{x_\lambda^2+t_\beta^2}, \quad \delta_\nu = -\frac{v}{2ft_\beta}\,, \tag{6}$$

where $v = 246$ GeV is the electroweak scale, s_W represents the sine of the weak mixing angle, $f=\sqrt{f_1^2+f_2^2}$, $t_\beta = \tan\beta = f_2/f_1$, and $x_\lambda = \lambda_1/\lambda_2$, in which f_1 and f_2 are the vacuum condensate values of the two sigma-model fields Φ_1 and Φ_2, respectively. λ_1 and λ_2 are the Yukawa couplings parameters. We write the gauge boson-fermion couplings in the form of $i\gamma^\mu(g_V+g_A\gamma^5)$.

Compared with the process $e^-\gamma \to \nu_e b\bar{t}$ in the SM, this process in the $SU(3)$ simple group model receives additional contributions from the heavy boson X^-. Furthermore, the correction terms to the SM $We\nu_e$ and Wbt coupling can also produce corrections to this process. Certainly, these two classifications can be seen to overlap in the limit, in which the extra particles are heavy and decouple from the low energy description. The SM couplings between the ordinary particles take well defined and calculable values in the SM, any deviation from these values would indicate the presence of new physics.

In order to write a compact expression for the amplitudes, it is necessary to define the triple-boson couplings coefficient as:

$$\Gamma^{\alpha\beta\gamma}(p_1,p_2,p_3) = g^{\alpha\beta}(p_1-p_2)^\gamma + g^{\beta\gamma}(p_2-p_3)^\alpha + g^{\gamma\alpha}(p_3-p_1)^\beta\,, \tag{7}$$

with all motenta out-going.

The invariant production amplitudes of the process in the $SU(3)$ simple group model can be written as:

$$M = M_a + M_b + M_c + M_d\,, \tag{8}$$

with

$$M_a = \bar{u}(p_3)\gamma_\mu(1-\gamma_5)u(p_1)\{g_V^{We\nu}g_V^{Wtb}G(p_3-p_1,M_W) + g_V^{Xe\nu}g_V^{Xtb}G(p_3-p_1,M_X)\}g^{\mu\nu}\bar{u}(p_4)g^{\gamma b\bar{b}}\gamma_\rho \times G'(p_4-p_2,m_b)\gamma_\nu(1-\gamma_5)v(p_5)\varepsilon^\rho(p_2)\,, \tag{9}$$

$$M_{\rm b}=-\bar{u}(p_3)g_V^{\rm We\nu}\gamma_\mu(1-\gamma_5)u(p_1)\times \Gamma^{\mu\nu\rho}(p_3-p_1,-p_2,p_4+p_5)G(p_3-p_1,M_{\rm W})\times G(p_4+p_5,M_{\rm W})\bar{u}(p_4)g_V^{\rm Wtb}\gamma_\nu(1-\gamma_5)v(p_5)\varepsilon^\rho(p_2)\,, \tag{10}$$

$$M_{\rm c}=\bar{u}(p_4)\gamma_\mu(1-\gamma_5)v(p_5)\{g_V^{\rm Wtb}g_V^{\rm We\nu}G(p_4+p_5,M_{\rm W})+ g_V^{\rm Xtb}g_V^{\rm Xe\nu}G(p_4+p_5,M_{\rm X})\}g^{\mu\nu}\bar{u}(p_3)\gamma_\nu(1-\gamma_5)\times G'(p_1+p_2,m_{\rm e})g^{\gamma{\rm e\bar{e}}}\gamma_\rho u(p_1)\varepsilon^\rho(p_2)\,, \tag{11}$$

$$M_{\rm d}=\bar{u}(p_3)\gamma_\mu(1-\gamma_5)u(p_1)\{g_V^{\rm We\nu}g_V^{\rm Wtb}G(p_3-p_1,M_{\rm W})+ g_V^{\rm Xe\nu}g_V^{\rm Xtb}G(p_3-p_1,M_{\rm X})\}g^{\mu\nu}\bar{u}(p_4)\gamma_\nu(1-\gamma_5)\times G'(p_2-p_5,m_{\rm t})g^{\gamma{\rm t\bar{t}}}\gamma_\rho v(p_5)\varepsilon^\rho(p_2)\,, \tag{12}$$

where $G(p,m)=1/(p^2-m^2)$ denotes the propagator of the charged gauge boson, $G'(p,m)=(p\cdot\gamma+m)/(p^2-m^2)$ denotes the propagator of the fermions.

The hard photon beam of the eγ collider can be obtained from laser backscattering at the $\rm e^+e^-$ linear collider. Let $\hat{s}$ and s be the center-of-mass energies of the eγ and $\rm e^+e^-$ systems, respectively. After calculating the cross section $\sigma(\hat{s})$ for the subprocess $\rm e^-\gamma\to\nu_e b\bar{t}$, the total cross section at the $\rm e^+e^-$ linear collider can be obtained by folding $\sigma(\hat{s})$ with the photon distribution function that is given in Ref. [10]:

$$\sigma({\rm tot})=\int_{(M_{\rm t}+M_{\rm b})^2/s}^{x_{\rm max}}{\rm d}x\sigma(\hat{s})f_\gamma(x)\,, \tag{13}$$

where

$$f_\gamma(x)=\frac{1}{D(\xi)}\left[1-x+\frac{1}{1-x}-\frac{4x}{\xi(1-x)}+\frac{4x^2}{\xi^2(1-x)^2}\right], \tag{14}$$

with

$$D(\xi)=\left(1-\frac{4}{\xi}-\frac{8}{\xi^2}\right)\ln(1+\xi)+\frac{1}{2}+\frac{8}{\xi}-\frac{1}{2(1+\xi)^2}\,. \tag{15}$$

In the above equation, $\xi=4E_{\rm e}\omega_0/m_{\rm e}^2$ in which $m_{\rm e}$ and $E_{\rm e}$ stand, respectively, for the incident electron mass and energy, ω_0 stands for the laser photon energy, and $x=\omega/E_{\rm e}$ stands for the fraction of energy of the incident electron carried by the backscattered photon. f_γ vanishes for $x>x_{\rm max}=\omega_{\rm max}/E_{\rm e}=\xi/(1+\xi)$. In order to avoid the creation of $\rm e^+e^-$ pairs by the interaction of the incident and backscattered photons, we require $\omega_0x_{\rm max}\leqslant m_{\rm e}^2/E_{\rm e}$, which implies that $\xi\leqslant 2+2\sqrt{2}\simeq 4.8$. For the choice of $\xi=4.8$, we obtain

$$x_{\rm max}\approx 0.83,\qquad D(\xi_{\rm max})\approx 1.8\,. \tag{16}$$

For simplicity, we have ignored the possible polarization for the electron and photon beams.

With the above production amplitudes, we can obtain the production cross section directly. In the calculation of the cross section, instead of calculating the square of the amplitudes analytically, we calculate the amplitudes numerically by using the method of Ref. [11] which can greatly simplify our calculation.

3 The numerical results and discussions

In our numerical results, we take the input parameters as $M_{\rm t}=172.7$ GeV[6], $\alpha_{\rm e}=1/128.8$, $M_Z=91.187$ GeV, $s_{\rm W}^2$=0.2315 and $m_{\rm W}$=80.45 GeV[12]. The value of the relative correction parameter is insensitive to the degree of the electron and positron polarization and the c.m. energy $\sqrt{s}$. Therefore, we do not consider the polarization of the initial states and take $\sqrt{s}$=500 GeV in our numerical calculation. Except for these SM input parameters, the contributions of the $SU(3)$ simple group model to single top quark production are dependent on the free parameters (f, x_λ, t_β). Considering the constraints of the electroweak precision data on these free parameters, we will assume $f\geqslant 1$ TeV, $x_\lambda>1$, and $t_\beta>1$ for the $SU(3)$ simple group model[13]. The relative correction of the $SU(3)$ simple group model to the cross section of single top production is in the expression of the relative correction parameter $R=\delta\sigma/\sigma^{\rm SM}$ with $\delta\sigma=|\sigma^{\rm tot}-\sigma^{\rm SM}|$ and $\sigma^{\rm SM}$ is the tree-level cross section of $\rm e^-\gamma\to\nu_e b\bar{t}$ production predicted by the SM.

The relative correction R is plotted in Fig. 1 as a function of the free parameter t_β for $f=1$ TeV and different values of the mixing parameter x_λ. From Fig. 1, we can see that the $SU(3)$ simple group model has negative contributions to single top production. If we assume $f=1$ TeV, $x_\lambda\geqslant 3$, and $1\leqslant\beta\leqslant 5$, the absolute value of the relative correction R is larger than 5% in most of the parameter spaces preferred by the electroweak precision data. For $x_\lambda<3$, the absolute value of relative parameter R is smaller than 5% in most of the parameter spaces in the $SU(3)$ simple group model.

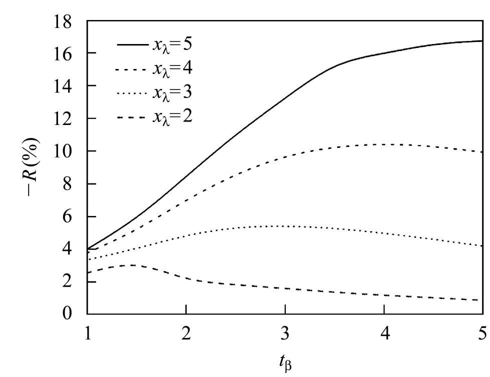

Fig. 1. The relative correction parameter R as a function of the free parameter t_β for $f=1$ TeV and different values of the mixing parameter x_λ.

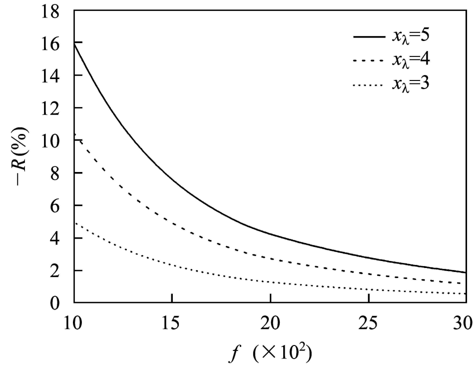

Fig. 2. The relative correction parameter R as a function of the scale parameter for $t_\beta = 4$ three values of the mixing parameters x_λ.

In general, the contributions of the little Higgs model to the observables are dependent on the factor $1/f^2$. To see the f on the dependence of the contributions of the $SU(3)$ simple group model to the cross section of single top production, we plot R as a function of the scalar parameter f for $t_\beta = 4$ three values of the mixing parameters x_λ in Fig. 2. One can see from Fig. 2 that the absolute value R drops sharply with the value of scalar parameter f increasing. Thus, the contributions of the $SU(3)$ simple group model to single top production decouple for large value of the scale parameter f, which is consistent with the conclusions for the corrections of the little Higgs model to other observables[14]. However, for $t_\beta \geqslant 4$, $x_\lambda \geqslant 4$, and 1 TeV$< f <$1.5 TeV, the absolute value of the relative correction parameter R is generally larger than 5%. The corresponding statistical uncertainty at the 95% C.L. is 2% for $\sqrt{s} = 500$ GeV[15]. Thus, such relative correction of the $SU(3)$ simple group model to the cross section of $e^-\gamma \to \nu_e b\bar{t}$ might be clearly detected in the future LC experiments.

The little Higgs model, which can solve the hierarchy problem, is a promising alternative new physics model. All of the little Higgs model predict the existence of the new heavy gauge bosons and generate corrections to the SM tree-level Wqq' couplings. Thus, the little Higgs model has effects on single top production at the $e^-\gamma$ colliders and studying the little Higgs model effects on single top production is very interesting and needed.

Little Higgs models can be generally divided in two classes: product group models and simple group models. The littlest Higgs model and the $SU(3)$ simple group model are the simplest examples of the two class models. The contribution of the littlest Higgs model to the process $e^-\gamma \to \nu_e b\bar{t}$ is smaller than 2% in all of the parameter spaces[16], which cannot be detected in the future LC experiments. However, Our numerical results show that, in sizable regions of the parameter space in the $SU(3)$ simple group model, the absolute value of the relative correction $\delta\sigma/\sigma^{SM}$ is larger than 5%, which is comparable to the future LC measurement precision. Thus, The process $e^-\gamma \to \nu_e b\bar{t}$ provides a feasible window to determine the structure of the extended electroweak gauge group and test the little Higgs mechanism in the gauge sector.

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