第 38 卷第 4 期
2009 年 4 月

光 子 学 报
ACTA PHOTONICA SINICA

Vol. 38 No. 4
April 2009

Calculation of Linear and Nonlinear Intersubband Optical Absorption in Electric-field Biased Hyperbolic Quantum Wells*

TAN Peng, LI Bin, CHEN Li-bing
(*Department of Physics, Foshan University, Foshan Guangdong* 528000, *China*)

Abstract The analytic forms of the linear and the third-order nonlinear optical intersubband absorption coefficients were obtained for general asymmetric quantum well systems using the density matrix formalism. Based on the model, the linear and the third-order nonlinear intersubband optical absorptions in electric-field biased hyperbolic quantum wells were studied. The numerical results were presented for a typical $Al_xGa_{1-x}As$/ GaAs electric-field biased hyperbolic quantum wells. The results show that the contributors to the nonlinear absorption coefficient are due to the electric field, the shape of quantum well and the optical intensity.

Key words: Nonlinear optics; Hyperbolic quantum wells; Absorption coefficient
CLCN: O437 **Document Code**: A **Article ID**: 1004-4213(2009)04-805-4

0 Introduction

Recently nonlinear optical properties in semiconductor quantum wells systems, superlattices and nanostrctures are of considerable interest because of their potential contribution to the device application in far-infrared laser amplifiers, photo-detectors and high-speed electro-optical modulators[1-2]. In the recent work, much attention has been paid to the third-order nonlinear optical properties in various inversion symmetry quantum systems, which is because the third-order nonlinear susceptibility has a huge enhancement in low dimensional quantum systems compared with the bulk material[3-4]. The linear intersubband optical absorption in the conduction band of a GaAs quantum well had been studied experimentally without an electric field [5 and with an electric field.[6] The common theoretical prediction in the literatures[7-10] is that the asymmetric quantum wells display a very large dipole strength and third-order susceptibilities. This suggests that the intersubband optical transition in a quantum well may have very large optical nonlinearities.

In this paper, the linear and the third-order nonlinear optical absorption in $Al_xGa_{1-x}As$/GaAs electric-field biased hyperbolic quantum wells are investigated, with most emphasis is on the effects of the electric field and the variational rule of the optical absorption coefficients in hyperbolic quantum wells.

1 Theory

Fig. 1 shows the shapes of the hyperbolic quantum wells when the value of the parameter a is different.

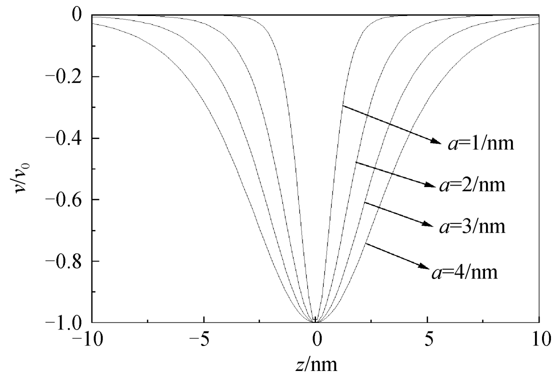


Fig. 1 The shape of the hyperbolic quantum wells when the value of the parameter a is different

Electrons in the hyperbolic quantum wells with an applied electric field can be described by the effective-mass Hamiltonian.

$$H=-\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+\left(-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial z^2}-\frac{V_0}{ch^2\frac{z}{a}}+eFz\right) \quad (1)$$

where a is a parametre which has a dimension of length. F is the intensity of the applied electric field, e is the electronic charge, m^* is the effective mass of electron.

So the Schrödinger equation can be written as following

$$H\varphi_{n,k}(\boldsymbol{r})=\varepsilon_{n,k}\varphi_{n,k}(\boldsymbol{r}) \quad (2)$$

where the eigenfunctions and the eigenvalues as follows

$$\varphi_{n,k}(\boldsymbol{r})=\psi_n(z)u(\boldsymbol{r})\exp(\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{r}')\quad(n=0,1,2\cdots) \quad (3)$$

---

*Supported by Natural Science Foundation of Guangdong Province(06029431)
Tel: 0757- 82981322 Email: tanpeng@fosu.edu.cn
Received date: 2008- 07- 16

$$\varepsilon_{n,k}=E_n+\frac{\hbar^2}{2m^*}|k'|^2 \quad (n=0,1,2\cdots) \tag{4}$$

Here $u(r)$ is the periodic part of the Bloch function, $\boldsymbol{k}'$ **and** $\boldsymbol{r}'$ are the wave vector in $x-y$ plane; $\psi_n(z)$ and E_n obey the following Schrödinger equation

$$H_0\psi_n(z)=E_n\psi_n(z) \tag{5}$$

$$H_0=-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial z^2}-\frac{V_0}{ch^2\frac{z}{a}}+eFz \tag{6}$$

As we know, Equation (5) can be solved exactly.

Next the formulas of the linear and the third-order nonlinear optical absorption coefficients in electric-field biased hyperbolic quantum wells will be deduced[12]. Considering a monochromatic incident field $E(t)=\tilde{E}e^{-i\omega t}+\tilde{E}^*e^{i\omega t}$ is applied to the system, the evolution of the density matrix can be deduced by the time-dependent Schrödinger equation

$$\frac{\partial\rho_{ij}}{\partial t}=\frac{1}{i\hbar}[H_0-qzE(t),\rho]_{ij}-\Gamma_{ij}(\rho-\rho_0)_{ij} \tag{7}$$

where H_0 is the unperturbed Hamilitonian, ρ_0 is the unperturbed density matrix and Γ_{ij} is the relaxtion rate.

Equation (7) is solved using the usual iterative method and expanding ρ in powers as

$$\rho(t)=\sum_{n=0}^{\infty}\rho^{(n)}(t) \tag{8}$$

With

$$\frac{\partial\rho_{ij}^{(n+1)}}{\partial t}=\frac{1}{i\hbar}[H_0,\rho_{ij}^{(n+1)}]-\frac{E(t)}{i\hbar}[qz,\rho_{ij}^{(n)}]-i\hbar\Gamma_{ij}\rho_{ij}^{(n+1)} \tag{9}$$

The application of Equation (9) to $n=0$ can be obtain

$$\tilde{\rho}_{ij}^{(1)}=\frac{q\langle\varphi_i|z|\varphi_j\rangle(\rho_{ii}^{(0)}-\rho_{jj}^{(0)})}{(E_i-E_j)+\hbar\omega+i\hbar\Gamma_{ij}}=\frac{q\langle\varphi_i|z|\varphi_j\rangle(\rho_{ii}^{(0)}-\rho_{jj}^{(0)})\tilde{E}}{\hbar(\omega-\omega_{ji}+i\Gamma_{ij})} \tag{10}$$

The expressions of $\tilde{\rho}_{ij}^{(2)}$ and $\tilde{\rho}_{ij}^{(3)}$ can be obtained with the same method.

The electronic polarization $P^{(n)}(t)$ and susceptibility $x(t)$ caused by the optical field $E(t)$ can be expressed through the density matrix as

$$P^{(n)}(t)=\varepsilon_0\chi(\omega,I)\tilde{E}e^{-i\omega t}+\varepsilon_0\chi(-\omega,I)\tilde{E}^*e^{i\omega t}=\frac{1}{S}Tr(\rho^{(n)}qz) \tag{11}$$

where S is the area of the system, ε_0 is the permittivity of the free space and Tr denotes the trace or summation over the diagonal elements of the matrix ρM. The suscepertibility χ is related to the absorption coefficient $\alpha(\omega)$ by

$$\alpha(\omega,I)=\omega\sqrt{\frac{\mu}{\varepsilon_R}}\mathrm{Im}(\varepsilon_0\chi(\omega,I)) \tag{12}$$

where μ is the permeability of the system, ε_R is the real part of the permittivity, and $\chi(\omega)$ is the Fourier component of $\chi(t)$. From equation (7) to equation (12) the linear absorption coefficients $\alpha^{(1)}(\omega)$ and the third-order nonlinear absorption coefficients $\alpha^{(3)}(\omega,I)$ can be obtained

$$\alpha^{(1)}(\omega)=\omega\sqrt{\frac{\mu}{\varepsilon_R}}\frac{|\mu_{ij}|^2q^2\sigma_s\hbar\Gamma_{ij}}{\hbar^2(\omega-\omega_{ij})^2+(\hbar\Gamma_{ij})^2} \tag{13}$$

$$\alpha^{(3)}(\omega,I)=-\omega\sqrt{\frac{\mu}{\varepsilon_R}}\left(\frac{I}{2\varepsilon_0n_rc}\right)\{\sigma_sq^4|\mu_{ij}|^2\Gamma_{ij}[4|\mu_{ij}|^2\cdot(\omega_{ij}^2+\Gamma_{ii}^2)-|\mu_{ii}-\mu_{jj}|^2(3\omega_{ij}^2-4\omega\omega_{ij}+\omega^2-\Gamma_{ij}^2)]\}/\hbar^3(\omega_{ij}^2+\Gamma_{ii}^2)[(\omega_{ij}-\omega)^2+\Gamma_{ij}^2]^2 \tag{14}$$

where $\mu_{ij}=|\langle\varphi_i|z|\varphi_j\rangle|$, σ_s is the density of the electrons in the quantum well, I is the optical intensity per unit area, n_r is the refractive index, c is the speed of light in free space. From equation (13) and equation (14) the total absorption coefficient of the intersubband $\alpha(\omega,I)$ is given by

$$\alpha(\omega,I)=\alpha^{(1)}(\omega)+\alpha^{(3)}(\omega,I) \tag{15}$$

2 Results and discussion

In the following, the absorption coefficient $\alpha(\omega,I)$ obtained in Equation (15) is calculated numerically for various optical intensity I and applied electric field F. The parameters are as follows[11]

$\mu=4\pi\times10^{-7}$ H · m^{-1}, $n_r=3.2$, $\sigma_s=5\times10^{24}$ m^{-3}, $\frac{1}{\Gamma_{ij}}=0.14$ ps, $m^*=0.067m_e$.

From equation (14) we can know the linear intersubband optical absorption coefficient doesn't depend on optical intensity I. Fig. 2 shows that the peak of the linear optical absorption coefficient shifts upward both in energy and in magnitude with an increasing applied electric field F. Upward shifts in energy can be explained by the quantum confined Stark effects and the results were observed by Harwit and Harris[12].

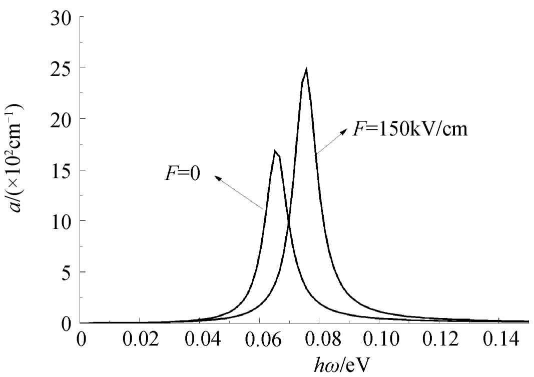


Fig. 2 The linear intersubband optical absorption coefficient $\alpha^{(1)}$ when parameter $a=2$ but applied electric field F is different

Fig. 3 shows that the absorption coefficient reduce by half at $I=1.0$ MW/cm^2 and the effect of the electric field is to shift the total absorption peak to a higher energy. This can be explained by the equation (14).

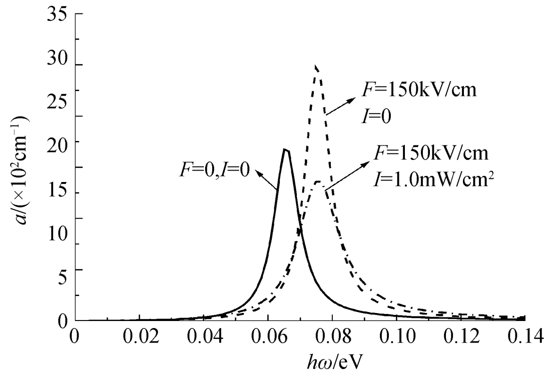


Fig. 3 The intersubband optical absorption coefficient $\alpha(\omega, I)$ when parameter $a=2$ but applied field F and optical intensity I are different

Fig. 4 shows the intersubband absorption coefficient $\alpha(\omega, I)$ when a and I are different but F is a constant, the peak of the linear absorption coefficient shifts upward in energy and shifted downward in magnitude with an increasing a. A very important feature is that the smaller parameter a is, the bigger the peak is. When parameter a increase, the peak will move to the right of the curve. This is because when parameter a increase, the well width will increase and the confinement in the hyperbolic quantum wells will become weaker. Therefore the excitonic effect on the third-order nonlinearoptical absorption becomes more and more weak[10].

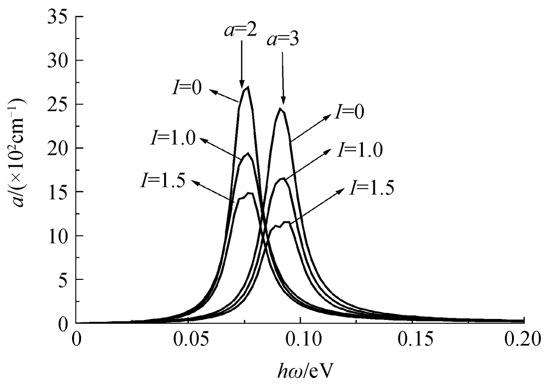


Fig. 4 The absorption coefficient α when $F=150$ kV/cm but paramete a and optical intensity I are different

From Fig. 4, we also can see that when parameter a and applied electric field F are a constant the absorption coefficient is reduced with an increase optical intensity I and the strong absorption saturation will occur with optical intensity I increasing. These results accord with the experimental results of the absorption coefficient in the quantum wires[1]. The intersubband optical saturation is sensitively related to the electron dynamic processes including electron relaxation and electron tunneling.

3 Conclusion

In conclusion, from the above results it can be seen that the shape of the well will change with parameter a and the linear and the nonlinear absorption coefficients will change with parameter a, applied field F and optical intensity I and the strong absorption saturation will occur when I increases. The contributions to the nonlinear absorption coefficient are due to the parameter a, the optical intensity I and the applied electric field F. So this study may have potential application for practical devices such as ultra fast optical switches and so on. We also hope this paper will be helpful in the experimental study of the influence of the quantum well's shape and the applied electric field on the third-order nonlinear optical properties.

References

[1] CHEN R, LIN D L Enhancement of the third-order nonlinear optical susceptibility in Si quantum wires [J]. *Phys Rev (B)*, 1993, **48**(16):11879-11882.

[2] TAN Peng, GUO Kang-xian, LU Hong. Optical rectification in electric-field biased hyperbolic quantum wells [J]. *Acta Photonica Sinica*, 2007, **36**(5):812-815.

[3] ZHANG Li. Effect of applied electric field on linear and nonlinear intersubband refractive index changes in semi-perabolic quantum wells [J]. *J Infrared Millim Waves*, 2005, **24**(5):378-385.

[4] HERBERT L E, WEISS B L. The optical properties of hyperbolic quantum-well strcture [J]. *J Appl Phys*, 1991, **70**(2):1054-1058.

[5] TAN Peng, LUO Shi-yu. Linear and nonlinear refracitive index change in square tangent quantum well [J]. *Acta Photonica Sinica*, 2007, **36**(7): 1253-1256.

[6] WANG G H, GUO K X. Second-order nonlinear optical susceptibility of a special asymmetric quantum [J]. *Acta Photonica Sinica*, 2001, **30**(11):1314-1317.

[7] ROSENCHER E, BOIS Ph. Model system for optical nonlinearities: Asymmetric quantum well [J]. *Phys Rev* (B), 1991, **44**(20):11315-11327.

[8] GUO K X, CHEN C Y. Electro-optic effect in electric-field biased parabolic quantum wells [J]. *Acta Photonica Sinica*, 1998, **27**(6):494-498.

[9] GURNICK M K, De-TEMPLE T A. Synthetic nonlinear semiconductors[J]. *IEEE J Quantum Electron* 1983, **QE-19**(5):791-794.

[10] GUO K X, CHEN C Y. Extition effects on the nonlinear optical absorption in hyperbolic quantum well [J]. *Acta Photonica Sinica*, 2000, **29**(6):501-505.

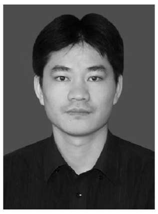
[11] TAN Peng, LI Bin. Calculation of linear and nonlinear intersuband optical absorption in the Pöschl-Teller well [J]. *Acta Photonica Sinica*, 2003, **32**(7): 815-818.

[12] AHN D, CHUNG S L. Calculation of linear and nonlinear intersubband optical absorption in a quantum model with an applied electric field [J]. *IEEE J Quantum Electron*, 1987, **QE-23**(12):2196-2204.

加偏置电场的双曲线量子阱中线性与三阶非线性光学吸收系数的计算

谭鹏，李斌，陈立冰
（佛山科学技术学院 理学院 物理系，广东佛山 528000）
收稿日期：2008-07-16

摘　要：利用紧致密度矩阵近似方法，研究了加偏置电场双曲线量子阱中的线性与三阶非线性光学吸收系数．得到了该系统中的线性与三阶非线性光学吸收系数的解析表达式．分析了势阱的形状、外加电场的大小以及入射光场的强度对吸收系数的影响规律．文章以典型的 $Al_xGa_{1-x}As$/GaAs 双曲线量子阱为例作了数值计算．结果表明：随着势阱宽度的增加，系统的吸收系数将减小；随着外加电场的增加，系统的非对称性增加，系统的吸收系数将增加；随着外加光场强度的增加，系统的吸收系数将减小，并且当光强增加到一定值时会出现明显的饱和吸收现象，这一结论为进一步的实验研究提供了相应的理论依据．
关键词：非线性光学；双曲线量子阱；吸收系数


TAN Peng was born in 1974. He obtained his M. S degree in 2001. His current research interests focus on the optical nonlinearities in low-dimensional semiconductors structures.