# Nucleon properties in the nuclear medium*

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**Abstract** We study the medium modifications of nucleon properties in nuclear matter and finite nuclei. The nucleons are described as nontopological solitons, which interact through the self-consistent exchange of scalar and vector mesons. The model adopted incorporates explicit quark degrees of freedom into nuclear many-body systems, and it can provide satisfactory results on the properties of nuclear matter and finite nuclei.

**Key words** Friedberg-Lee model, nuclear matter, finite nuclei

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## 1 Introduction

Many experimental evidences indicate that the properties of the nucleon bound in nuclei are significantly modified from those of a free nucleon. There are numerous theoretical works on the study of in-medium nucleon properties based on various models[1—4]. At present, we are still far away from describing nucleons and nuclei in terms of quarks and gluons using quantum chromodynamics (QCD). It is highly desirable to build models which could incorporate the quark degrees of freedom and respect the established theories based on hadronic degrees of freedom. One of these models is the quark-meson coupling (QMC) model[5], which takes into account the internal structure of nucleons in nuclear many-body systems. The QMC model has been extensively developed and applied with reasonable success to various nuclear phenomena[4, 6—12]. Another model is the quark mean-field (QMF) model[2], which takes the constituent quark model for the description of nucleon. The QMF model has been successfully used for the study of nuclear matter, finite nuclei, and hypernuclei[13—15]. The main advantage of these models is their simplicity and self-consistency in taking into account the quark degrees of freedom for nuclear many-body systems.

In this paper, we use the nontopological soliton bag model originally proposed by Friedberg and Lee[16, 17] for the description of nucleons in nuclear medium. In the Friedberg-Lee model, the nucleon is described as a bound state of three quarks in a nontopological soliton formed by a scalar field with nonlinear self-interactions. We develop a model to study the properties of nuclear matter and finite nuclei by describing the nuclear many-body system as a collection of nontopological soliton bags. The quarks inside the soliton bag couple not only to the scalar field that binds the quarks together into nucleons, but also to additional meson fields generated by the nuclear environment.

## 2 Model

we use the Friedberg-Lee model to describe nucleons. The Friedberg-Lee model in its simplest form is implemented through the effective Lagrangian density

$$\mathcal{L}=\bar{\psi}(\mathrm{i}\gamma_\mu\partial^\mu-m-g\phi)\psi+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-U(\phi)\,, \quad (1)$$

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where $\psi$ denotes the quark field. The quark mass $m$ is usually taken to be zero for $u$ and $d$ quarks. $\phi$ is a color-singlet scalar field which may be interpreted as the phenomenological representation of quantum excitations of the self-interacting gluon field. The self-interaction of the scalar soliton field is described by the potential

$$U(\phi)=\frac{a}{2!}\phi^2+\frac{b}{3!}\phi^3+\frac{c}{4!}\phi^4+B. \tag{2}$$

The constants $a$, $b$, and $c$ are fixed within a range so that $U(\phi)$ has a local minimum at $\phi=0$ and a global minimum at $\phi=\phi_v$. In the mean-field approximation, the soliton field is treated as a classical field which is a time-independent c-number field $\phi(r)$. The quark field operators are expanded in a complete orthogonal set of Dirac spinor functions as $\psi=\sum_{\rm k} c_{\rm k}\psi_{\rm k}$, where $c_{\rm k}$ are fermion annihilation operators. For a nucleon, the three valence quarks can be in the lowest Dirac state $\psi_0$, then $\phi$ and $\psi_0$ satisfy the coupled differential equations

$$(-{\rm i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+g\beta\phi)\psi_0=\epsilon_0\psi_0\,, \tag{3}$$

$$-\nabla^2\phi+\frac{\partial U(\phi)}{\partial\phi}=-3g\bar{\psi}_0\psi_0\,. \tag{4}$$

The total energy of the nucleon is given by

$$E=3\epsilon_0+4\pi\int{\rm d}r\ r^2\left[\frac{1}{2}\left(\frac{{\rm d}\phi}{{\rm d}r}\right)^2+U(\phi)\right]. \tag{5}$$

The mean-square charge radius of the proton is

$$\langle r^2\rangle=4\pi\int{\rm d}r r^4\left(u^2+v^2\right), \tag{6}$$

and the proton magnetic moment is

$$\mu_{\rm p}=\frac{8\pi}{3}\int{\rm d}r\ r^3uv\,. \tag{7}$$

The ratio of the axial-vector to vector coupling constants is given by

$$g_A/g_V=\frac{20\pi}{3}\int{\rm d}r\ r^2\left(u^2-\frac{1}{3}v^2\right). \tag{8}$$

We follow the work of Ref. [18] to take into account the recoil correction for nucleon properties. The nucleon mass is given by

$$M=\sqrt{E^2-\langle\boldsymbol{P}^2\rangle}\,, \tag{9}$$

where $\boldsymbol{P}$ is the total-momentum operator. The recoil corrected root-mean-squared (rms) radius is given by

$$r_{\rm c}=\sqrt{\left[1-\frac{2\epsilon_0}{E}+\frac{3\epsilon_0^2}{E^2}\right]\langle r^2\rangle+\frac{3}{2E^2}}\,. \tag{10}$$

The parameters in the Friedberg-Lee model, $a$, $b$, $c$, and $g$, are constrained by reproducing nucleon mass $M=939$ MeV and rms radius $r_{\rm c}=0.83$ fm. We take two sets of parameters which correspond to the two limiting cases. Set A: $a=0$, $b=-79.61$ fm$^{-1}$, $c=780$, $g=13.7$ is characterized by $a=0$, in which $U(\phi)$ has a inflection point at $\phi=0$. Set B: $a=69.945$ fm$^{-2}$, $b=-1600$ fm$^{-1}$, $c=12200$, $g=24.55$ is characterized by $B=0$. The parameter set A gives the proton magnetic moment $\mu_{\rm p}=2.80$ and the ratio of the axial-vector to vector coupling constants $g_A/g_V=0.87$, while the parameter set B predicts $\mu_{\rm p}=2.77$ and $g_A/g_V=0.90$. We note that the experimental values are $\mu_{\rm p}=2.79$ and $g_A/g_V=1.25$. It is shown that these two parameter sets in this simple model can give reasonable results for nucleon properties in free space.

To study nuclear matter properties, we describe the nuclear many-body system as a collection of non-topological soliton bags. The quarks inside the soliton bag interact through the self-consistent exchange of σ, ω, and ρ mesons, which are treated as classical fields in the mean-field approximation. We assume that the meson mean fields σ, ω, and ρ can be regarded as constants in uniform matter. The soliton field $\phi$, which serves to bind quarks together, does not participate in nucleon-nucleon interactions. With the presence of these additional meson fields in nuclear matter, the quark and soliton fields in a nucleon satisfy the coupled equations

$$\left(-{\rm i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+g\beta\phi+g^{\rm q}_{\sigma}\beta\sigma+g^{\rm q}_{\omega}\omega+g^{\rm q}_{\rho}\tau_3\rho\right)\psi_0=\widetilde{\epsilon}_0\psi_0\,, \tag{11}$$

$$-\nabla^2\phi+\frac{\partial U(\phi)}{\partial\phi}=-3g\bar{\psi}_0\psi_0\,, \tag{12}$$

where $g^{\rm q}_{\sigma}$, $g^{\rm q}_{\omega}$, and $g^{\rm q}_{\rho}$ are the coupling constants of the σ, ω, and ρ mesons with quarks, respectively. We solve the coupled equations and calculate the in-medium nucleon properties analogously to the case of free nucleons. The effective nucleon mass obtained is then given by

$$M^*(\sigma)=\sqrt{E^2-\langle\boldsymbol{P}^2\rangle}\,. \tag{13}$$

## 3 Nucleon properties in nuclear matter and finite nuclei

To perform a many-body calculation for nuclear matter, we start from the effective Lagrangian at the hadron level within the mean-field approximation

$$\mathcal{L}_{\rm RMF} = \bar{\psi}[{\rm i}\gamma_\mu \partial^\mu - M^*(\sigma) - g_\omega \gamma^0 \omega - g_\rho \gamma^0 \tau_3 \rho]\psi - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2 \ , \qquad (14)$$

where the nucleon-meson couplings are related to the quark-meson couplings as $g_\omega = 3g_\omega^{\rm q}$ and $g_\rho = g_\rho^{\rm q}$[13]. From the Lagrangian given by Eq. (14), we obtain the equations of motion for nucleons and mesons in nuclear matter. The quark-meson couplings $g_\sigma^{\rm q}$, $g_\omega^{\rm q}$, and $g_\rho^{\rm q}$ are determined by reproducing the nuclear matter equilibrium density (0.15 fm$^{-3}$), energy per nucleon ($-16$ MeV), and symmetry energy (35 MeV).

In Fig. 1, we show the ratio of the effective nucleon mass in nuclear matter to that in free space as a function of nuclear matter density. It is shown that the effective nucleon mass decreases with increasing density. We show in Fig. 2 the ratio of the nucleon rms radius in nuclear matter to that in free space as a function of nuclear matter density. It is very interesting to see the expansion of the nucleon size in medium. We find that the nucleon rms radius significantly increases at normal nuclear matter density.

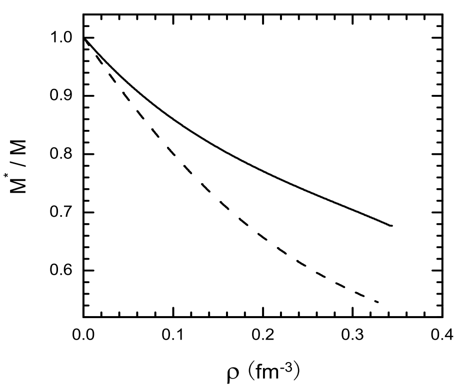


Fig. 1. The ratio of the effective nucleon mass in nuclear matter to that in free space, $M^*/M$, as a function of nuclear matter density $\rho$. The results with the parameter set A are shown by the solid curve, while those with the parameter set B by the dashed curve.

We extend the present model to study the properties of finite nuclei and the modification of nucleon properties in a nucleus. The nucleus is described as a collection of nontopological soliton bags which interact through the self-consistent exchange of σ, ω, and ρ mesons. We use the local density approximation in which the meson mean fields are replaced by their value at the center of the nucleon and the spatial

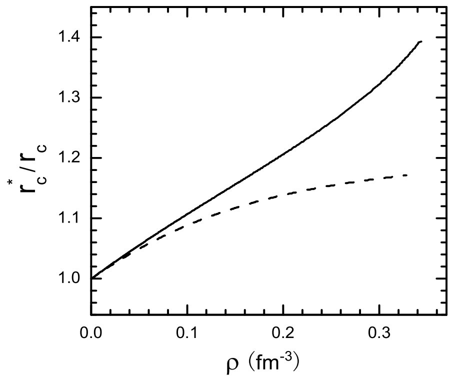


Fig. 2. The ratio of the nucleon rms radius in nuclear matter to that in free space, $r_{\rm c}^*/r_{\rm c}$, as a function of nuclear matter density $\rho$. The curves are labeled as in Fig. 1.

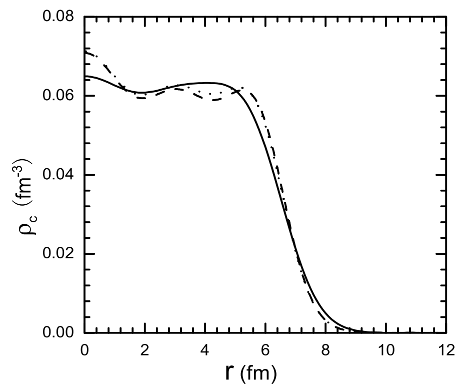


Fig. 3. The calculated charge density distributions for $^{208}$Pb compared with the experimental data (solid curve). The results with the parameter set A (B) are shown by the dashed (dotted) curve.

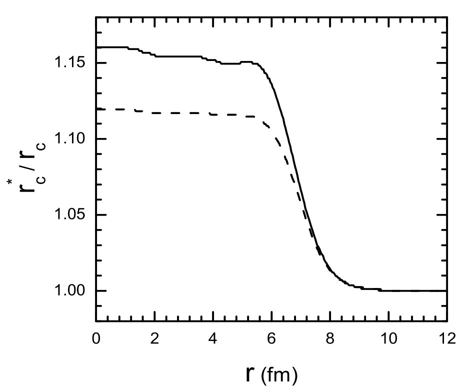


Fig. 4. The ratio of the proton rms radius in $^{208}$Pb to that in free space as a function of radial coordinate $r$. The results with the parameter set A are shown by the solid curve, while those with the parameter set B by the dashed curve.

Table 1. The binding energy per nucleon $E/A$ and the rms charge radius $R_c$ for $^{40}$Ca, $^{90}$Zr, and $^{208}$Pb.

| | $E/A$/MeV | | | $R_c$/fm | | |
|---|---|---|---|---|---|---|
| | Set A | Set B | Expt. | Set A | Set B | Expt. |
| $^{40}$Ca | 8.53 | 7.66 | 8.55 | 3.42 | 3.46 | 3.45 |
| $^{90}$Zr | 8.36 | 7.81 | 8.71 | 4.26 | 4.27 | 4.26 |
| $^{208}$Pb | 7.48 | 7.13 | 7.87 | 5.52 | 5.50 | 5.50 |

variation of the mean fields over the small nucleon volume are neglected. We solve the coupled equations of nucleons and meson mean fields self-consistently with the effective nucleon mass obtained at the quark level. In Table 1, the calculated binding energies per nucleon and rms charge radii are compared with the experimental values[19]. We plot in Fig. 3 the resulting charge density distributions for $^{208}$Pb, and compare with the experimental values[20]. It is also possible to investigate the modification of nucleon properties in finite nuclei. In Fig. 4 we show the ratios of the proton rms radius in $^{208}$Pb to those in free space as functions of the radius $r$.

## 4 Conclusion

We have proposed a model for the description of nuclear matter and finite nuclei based on the non-topological soliton bag model. The present model enables us to investigate the medium modification of nucleon properties. We have performed the numerical calculations for nuclear matter and finite nuclei. We find that the properties of the nucleon are significantly modified in nuclear medium. The present model incorporates explicit quark degrees of freedom into nuclear many-body systems. It is notable that the quark structure of the nucleon plays a crucial role in the description of nuclear matter and finite nuclei.

## References


1 Celenza L S, Rosenthal A, Shakin C M. Phys. Rev. C, 1985, **31**: 232

2 Toki H, Meyer U, Faessler A et al. Phys. Rev. C, 1998, **58**: 3749

3 Barnea N, Walhout T S. Nucl. Phys. A, 2000, **677**: 367

4 Saito K, Tsushima K, Thomas A W. Prog. Part. Nucl. Phys., 2007, **58**: 1

5 Guichon P A M. Phys. Lett. B, 1988, **200**: 235

6 Saito K, Tsushima K, Thomas A W. Nucl. Phys. A, 1996, **609**: 339

7 Guichon P A M, Saito K, Rodionov E et al. Nucl. Phys. A, 1996, **601**: 349

8 Guichon P A M, Saito K, Thomas A W. Austral. J. Phys., 1997, **50**: 115

9 Muller H, Jennings B K. Nucl. Phys. A, 1997, **626**: 966

10 Muller H, Jennings B K. Nucl. Phys. A, 1998, **640**: 55

11 Pal S, Hanauske M, Zakout I et al. Phys. Rev. C, 1999, **60**: 015802

12 Panda P K, Menezes D P, Providencia C. Phys. Rev. C, 2004, **69**: 025207

13 SHEN H, Toki H. Phys. Rev. C, 2000, **61**: 045205

14 SHEN H, Toki H. Nucl. Phys. A, 2002, **707**: 469

15 SHEN H, Toki H. Phys. Rev. C, 2005, **71**: 065208

16 Friedberg R, Lee T D. Phys. Rev. D, 1977, **15**: 1694; D, 1977, **16**: 1096; D, 1978, **18**: 2623

17 Goldflam R, Wilets L. Phys. Rev. D, 1982, **25**: 1951

18 Dethier J L, Goldflam R, Henley E M et al. Phys. Rev. D, 1983, **27**: 2191

19 Sugahara Y, Toki H. Nucl. Phys. A, 1994, **579**: 557

20 Vries H de, Jager C W de, Vries C de. At. Data Nucl. Data Tables, 1987, **36**: 495