Belief Propagation Decoding of Low-density Parity-check Codes for Atmospheric Turbulent Optical PPM Communication Systems*

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Abstract: BP decoding of LDPC code for weak atmospheric turbulent optical PPM systems was studied, and corresponding decoding algorithm was introduced according to the system model. Through theoretical analysis and extensive computer simulations, the result shows that the LDPC coded weak atmospheric turbulent optical PPM system has better BER than the uncoded weak atmospheric turbulent optical PPM system, but the BER performance of the system increases with the increase of slot.

Key words: Atmospheric turbulence optical communication; Belief Propagation (BP); Pulse Position Modulation (PPM); Low-Density Parity-Check (LDPC) Code; Lognormal

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0 Introduction

Pulse Position Modulation (PPM) is widely used in atmospheric turbulent optical communication system due to its high energy efficiency, sensitivity and power radiation efficiency. But the performance of the system can be severely affected by atmospheric turbulence, particularly over ranges of the order of 1 km or longer. However, error-control coding can be applied to improve the error performance on such channels[1]. Consequently, combining PPM with error-control coding was applied to optical communication system and has widely been researched. But error-control coding is main RS codes and Turbo codes[2-5]. While adopting Belief Propagation (BP) decoding whose process uses iterative decoding and makes full use of channel information, the performance of Low-Density Parity - Check (LDPC) Code is almost close to the Shannon limit. Consequently, LDPC code can become an excellent error-control coding. Therefore, the performance research of combining PPM with LDPC code in atmospheric optical communication system has a certain value. Document[6-7] has researched in this field, while the type of optical and electric detector was an avalanche photodiode (APD) and only analyzed the performance of 4-PPM.

In this paper, we considered an intensity-modulated direct-detection (IM/DD) system with PPM and a point detector. We researched BP decoding of LDPC codes for weak atmosphere turbulence optical PPM channel. Here the optical atmosphere channel was considered as a stationary ergodic channel with lognormal intensity fading[8-10]. The BER performance using BP decoding of LDPC coded PPM versus the information bit energy-to-noise ratio E_b/N_0 is analyzed and simulated at the conditions of different atmospheric turbulence intensities and different M-PPM.

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1 Weak atmospheric turbulent channel model with PPM

1.1 PPM system model

Binary information bits are encoded by LDPC endcoder, modulated using M-PPM, and transmitted across a weak atmospheric turbulent communication system. For each PPM symbol, the received M soft outputs are produced, one soft output for each slot. For convenience, we assume the length of each codeword is divisible by $\log_2^M$. Consequently, the codeword with length N acquired through encoding is grouped into $\log_2^M$-bit blocks and fed to the PPM modulator, which transforms each block into a pulse in one of M locations. After PPM modulated, each block could be transmitted as a PPM symbol having M slots and each codeword could be transmitted as $N/\log_2^M$ PPM symbols.

Assuming $A=\log_2^M$, let $X=(x_1, x_2, \ldots, x_A)$ be the binary information bits of the j th PPM block and the corresponding pulse location is $d=1+\sum_{i=1}^{A} x_i 2^{A-i} (1\leqslant d\leqslant M)$. Let $Y=(y_1, y_2, \ldots, y_M)$ be the soft outputs from the j th PPM symbol at the receiving end.

1.2 Weak atmospheric turbulent channel model

Under weak turbulence intensity ($\sigma_x < 0.3$)[11], link model of atmosphere optical communication is characterized as

$$y_k = I_k z_k + n_k \quad I_k > 0,\ 1 \leqslant k \leqslant M \tag{1}$$

where $z_k \in \{0,1\}$ is the kth slot signal from the j th PPM symbol, y_k is the received soft output signal, and n_k is the additive white Gaussian noise with zero mean and variance $N_0/2$. We assume that the scintillation is an ergodic and time-invariant random process. I_K denotes the instantaneous intensity gain induced by atmospheric scintillation. I_K follows the lognormal distribution with mean $-2\sigma_x^2$ (σ_x^2 is logamplitude fluctuation variance), variance $4\sigma_x^2$, and p. d. f.

$$f_I(I) = \frac{1}{2I\sqrt{2\pi\sigma_x^2}} \exp\left\{-\frac{[\ln(I)+2\sigma_x^2]^2}{8\sigma_x^2}\right\} \tag{2}$$

Assume that perfect channel side information (CSI) denoted by I is aware and is a constant in time interval of sending each slot of the j th PPM symbol. Considering PPM, lognormal fading channel becomes an equivalent memoryless AWGN channel with discrete-time binary input $\{0,I\}$ and continuous output[8] in each slot of the$j$ th PPM symbol.

Consequently, $y_d = I_d + n_d$, $y_k = n_k (k \neq d, 1 \leqslant k \leqslant M, 1 \leqslant d \leqslant M)$. The received soft output signal y_k from the k th slot of the$j$ th PPM symbol is given by additive Gaussian noise alone. The conditional distribution $P(y_k \mid z_k = 0)$ follows Gaussian distribution with zero mean, variance $N_0/2$, and p. d. f.

$$P(y \mid z=0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{y^2}{N_0}\right] \tag{3}$$

The received soft output signal y_d from the d th slot of the j th PPM symbol is given by additive Gaussian noise and instantaneous intensity gain. The conditional distribution $P(y_d \mid z_d = 1)$ follows Gaussian distribution with mean I_d, variance $N_0/2$, and p. d. f.

$$P(y \mid z=1) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(y-I)^2}{N_0}\right] \tag{4}$$

Assume that each slot is "0" or "1" in identical probability, then the posterior probability is characterized as

$$p(z_l = 0 \mid y_l) = \frac{1}{1+\exp\left[\dfrac{2y_l I_l - I_l^2}{N_0}\right]} \tag{5a}$$

$$p(z_l = 1 \mid y_l) = \frac{1}{1+\exp\left[\dfrac{I_l^2 - 2y_l I_l}{N_0}\right]} \quad (1 \leqslant l \leqslant M) \tag{5b}$$

We assume $P(c;T) = P$(the l th slot of the j th PPM symbol is $c \mid Y$). Consequently[5-7]

$$P(c;T) = \sum_{X=(x_1,x_2\cdots,x_A)x_l=c} P(X \mid Y) \quad c \in \{0,1\} \tag{6}$$

In Eq. (6), we need to compute the probability that a given M-PPM symbol $X = (x_1, x_2, \ldots, x_A)$ was transmitted, given its corresponding set of M soft counts, $Y = (y_1, y_2, \cdots, y_M)$. Assuming $p(.)$ denotes the pdf of a continuous random variable or vector, and $P(.)$ denotes the probability mass function of a discrete random variable or vector[5]. We acquire

$$P(X \mid Y) = \frac{p(Y \mid X)P(X)}{p(Y)} = \frac{p(Y \mid X)P(X)}{\sum_{i=1}^{M} p(Y \mid X_i)P(X_i)} \tag{7}$$

Owing to $P(X_i) = \frac{1}{M}$ for all i, consequently,

$$P(X \mid Y) == \frac{p(Y \mid X)}{\sum_{i=1}^{M} p(Y \mid X_i)} \tag{8}$$

where in Eq. (7), we have summed over all possible A- bit symbols, i. e. , $X_1 = (0, \ldots, 0)$, $X_2 = (0, \ldots, 0, 1)$, and so on. Each of these symbols is a priori equiprobable.

Each M-PPM symbol brings about one optical intensity slot and $M-1$ non-optical intensity slots. Considering independence of each soft output[5], we may separate $p(Y \mid X)$ as the product of $M-1$ non-optical pdfs and one optical pdf.

$$P(Y \mid X) = p_S(y_d)\prod_{i=1 i\neq d}^{M} p_n(y_i) = \frac{p_S(y_d)}{p_n(y_d)}\prod_{i=1}^{M} p_n(y_i) \tag{9}$$

where the pdf of the soft output of each slot is expressed as $p_S(.)$ or $p_n(.)$, according to whether it is a optical slot or non-optical slot, respectively. The corresponding pulse location of the$j$ th PPM symbol is d, and where we have used X as either an A-bit vector or as an index equal to the number it represents in binary.

Consequently, $p_s(y_i) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(y_i - I_i)^2}{N_0}\right]$ $p_n(y_i) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{y_i^2}{N_0}\right]$. Assuming the likelihood function for the i th slot of the j th PPM symbol are denoted by $L_i^j = \frac{p_s(y_i)}{p_n(y_i)}$ (where, the soft counts y_i are from the j th PPM symbol). By above channel model, we acquire $L_i^j = \exp\left[\frac{2y_i I_i - I_i^2}{N_0}\right]$ and plugging Eq. (9) into Eq. (8), it follows that

$$P(X \mid Y) = \frac{L_d^j \prod_{i=1}^{M} P_n(y_i)}{\sum_{i=1}^{M}\left[L_i^j \prod_{i=1}^{M} P_n(y_i)\right]} = \frac{L_d^j}{\sum_{i=1}^{M} L_i^j} \tag{10}$$

So acquire a posterior probability

$$P(c;T) = \frac{\sum_{X=(x_1,x_2,\cdots,x_A)x_l=c} L_d^j}{\sum_{i=1}^{M} L_i^j} \tag{11}$$

2 Probability domain BP decoding

We present BP decoding of LDPC codes for weak atmosphere turbulenc optical M-PPM channel with the known CSI. A binary (N, K) LDPC code is a linear block code described by a sparse $W \times N$ parity-check H. Let $W(n)$ denote the set of check nodes connected to symbol node n, i. e. , the positions of 1s in the n th column of the parity-check matrix H, and let $N(w)$ denote the set of symbol nodes that participate in the w th parity-check equation, i. e. , the positions of 1s in the w th row of H. Moreover, $N(w)\backslash n$ represents the set $N(w)$, excluding the nth symbol node, and similarly, $W(n)\backslash w$ represents the set $W(n)$, excluding the wth check node[12].

Additionally, q_{wn}^{x}, $x \in \{0, 1\}$, denotes the message that symbol node n sends to check node w indicating the probability of symbol n being 0 or 1, based on all the checks involving n except w. Similarly, r_{wn}^{x}, $x \in \{0,1\}$, denotes the message that the w th check node sends to the n th symbol node indicating the probability of symbol n being 0 or 1, based on all the symbols checked by w except n. Finally, $Y = [y_1, \cdots, y_N]$ denotes the received codeword corresponding to the transmitted codeword $X = [x_1, \cdots, x_N]$[12].

The probability-domain BP decoding process is as follows.

1) Initialization: For every n, use a posterior probability in Eq. (11) as the a priori probability under the case of using M-PPM, this is done by initializing q_{wn}^{0} and q_{wn}^{1} for all w, n for which $H_{wn}=1$.

2) Parity node updates[13]: Assume $\delta q_{wn} = q_{wn}^{0} - q_{wn}^{1}$ and acquire

$$\delta r_{wn} = r_{wn}^{0} - r_{wn}^{1} = \prod_{n' \in N(w)\backslash n} \delta q_{wn'} \tag{12}$$

Consequently, updating

$$r_{wn}^{x} = \frac{1 + (-1)^{x} \delta r_{wn}}{2} \quad x \in \{0,1\} \tag{13}$$

3) Variable node updates: Updating

$$q_{wn}^{x} = \alpha_{wn} p_{n}^{x} \prod_{w' \in W(n)\backslash w} r_{w'n}^{x} \quad x \in \{0,1\} \tag{14}$$

where the constants α_{wn} are chosen to ensure $q_{wn}^{0} + q_{wn}^{1} = 1$. p_n^0 and p_n^1 are posterior probabilities under the case of variable node 0 and 1, respectively.

4) Posterior probabilies: For every n, compute the a posterior probability

$$q_{n}^{x} = \alpha_{n} p_{n}^{x} \prod_{w \in W(n)} r_{wn}^{x} \quad x \in \{0,1\} \tag{15}$$

where the constants α_n are chosen to ensure $q_n^0 + q_n^1 = 1$.

5) Tentative estimate: For every n, set

$$\hat{x}_n = \begin{cases} 1 & q_n^1 > 0.5 \\ 0 & q_n^1 < 0.5 \end{cases} .$$

If the condition $H \cdot \hat{x}' = 0 \pmod 2$ is satisfied or the maximum number of iterations is reached, the decoding algorithm is terminated. Otherwise, the algorithm goes back to step 2).

3 Performance theoretical analysis

We carried out plenty of simulations according to the link model described in section 1 to test the BER performance using BP decoding of LDPC coded M-PPM versus E_b/N_0. For performance comparison of BP decoding of LDPC coded M-PPM with different parameters, we considered the two different codeword lengths ($n = 96$ and $n = 504$ bits) and the two different code rates ($R=0.5$ and $R = 1/3$) under two different atmosphere turbulence intensities ($\sigma_x = 0.1$ and $\sigma_x = 0.2$) and three different slots ($M=2$, $M=8$ and $M=64$). Fig. 1 shows the simulation results.

From Fig. 1, we can get five conclusions as follows.

Firstly, for a given codeword length, the performance of LDPC code increases as code rate decreases. Given some redundancies are added into the information bits to resist all kinds of interference arising in the transmission process and the ability of anti-interruption increases with increasing

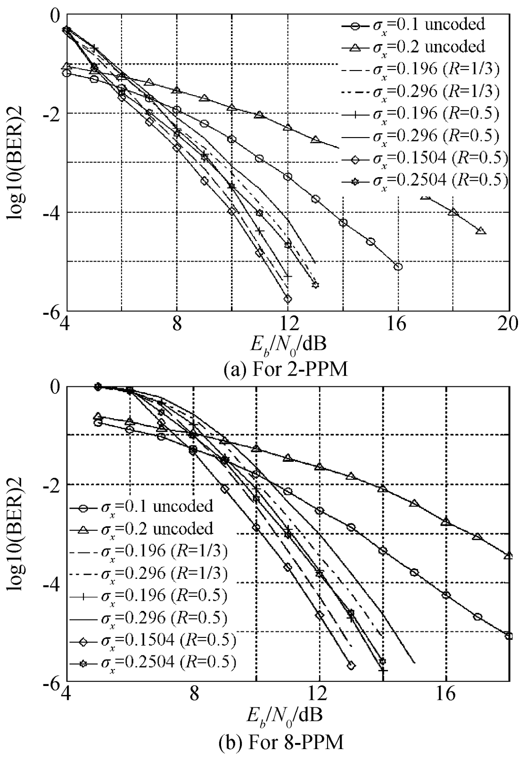
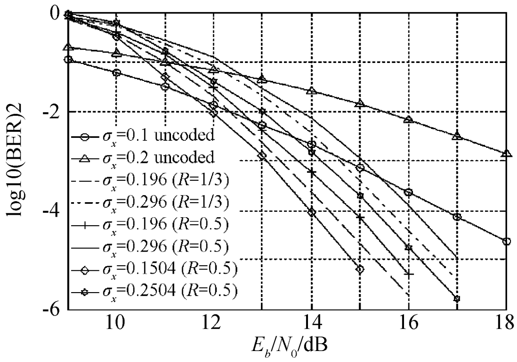


Fig. 1 BER versus E_b/N_0 for 2-PPM, 8-PPM, and 64-PPM

of redundancy degree. In addition, under a given codeword length, the rate of LDPC code decreases with the number of the check bits of the codeword,



(c) For 64-PPM

namely redundancy degree increases. Consequently, the performance of LDPC code increases as the rate of LDPC code decreases.

Secondly, for a given code rate, the performance of LDPC code increases as codeword length increases. In the tanner graph of LDPC code, the minimum circle length degrades the performance of LDPC code. The longer the minimum circle is, the better the performance of LDPC code is. Therefore by increasing the codeword length, we can increase the length of the minimum circle and consequently improve the performance of LDPC code.

Thirdly, as the turbulence intensity is increased, greater coding gain is achieved. Such as at the BER $=10^{-4}$ and using 2-PPM, the coding gain of LDPC code with codeword length 504 bits over the uncoded system is 3.6 dB when $\sigma_x=0.1$ and 7.1 dB when $\sigma_x=0.2$, respectively.

Moreover, the negative impact of scintillation is considerably evident in the uncoded system. Such as at $\sigma_x=0.1$ and using 8-PPM, the uncoded system has BER$=2.8\times10^{-3}$ and LDPC code with codeword length 504 bits has BER $=2.2\times10^{-5}$ when $E_b/N_0=12$ dB. Consequently, LDPC code is necessary to improve transmission quality of communication system.

Finally, the BER performance of system increases as slot M increases. Such as at $\sigma_x=0.1$, BER$=10^{-5}$ and using LDPC code with codeword length 504 bits , the coding gain of LDPC coded 2-PPM over LDPC coded 8-PPM and LDPC coded 64-PPM is 1.2 dB and 3.6 dB, respectively. Given right demodulation probability of each PPM symbol is $1/M$. Consequently, wrong demodulation probability of each PPM symbol increases as slot M increases.

4 Conclusions

In this paper, BP decoding of LDPC code for weak atmosphere turbulence optical M-PPM channel is researched. The BER performance of the system is simulated and analyzed for different slot M and different atmosphere turbulence, respectively. Simulation results show that LDPC code acquires more coding gain in more strong turbulent channel, and the BER performance of system increases as slot M increases. Therefore, using of LDPC code can obtain very high coding gain and good transmission performance in atmosphere turbulence optical channel. And we should choose appropriate slot M according to practical channel condition.

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大气湍流光 PPM 通信系统中 LDPC 码的置信传播译码

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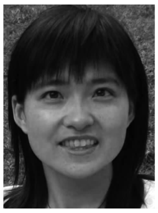
摘　要：对大气弱湍流光 PPM 通信系统中 LDPC 码的 BP 解码算法进行了研究，提出了相应系统模型下的解码算法. 通过理论分析和大量计算机仿真，结果表明：采用 LDPC 的大气弱湍流光 PPM 通信系统比未编码的大气弱湍流光 PPM 通信系统的性能好，但是系统的 BER 性能随着 PPM 的时隙数的增加而增大.

关键词：大气湍流光通信；置信传播；脉冲位置调制；低密度校验码；对数正态



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