

# Measuring $D^0\text{-}\bar{D}^0$ mixing in $D \rightarrow f_0(980)K^*$ and more<sup>\*</sup>

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**Abstract** We investigate the  $D^0\text{-}\bar{D}^0$  mixing through the doubly Cabibbo suppressed (DCS) channel  $D^0 \rightarrow f_0(980)K^{*0}$  and its charge conjugate channel, in which the  $K^{*0}$  meson is reconstructed in both  $K^+\pi^-$  and  $K_S\pi^0$  final state. Although the decay  $D^0 \rightarrow f_0(980)K^*$  has a small branching ratio, the final state mesons are relatively easy to identify. The  $f_0(980)$  meson can be replaced by the  $S$ -wave  $\pi^+\pi^-$  state, or a longitudinally polarized vector meson  $\rho^0$ . All mixing parameters, including the mass difference and decay width difference, can be extracted by studying the time-dependent decay width of these channels. We show that the method is valid in all regions for mixing parameters and it does not depend on the strong phase difference.

**Key words**  $D^0\text{-}\bar{D}^0$  mixing,  $D^0 \rightarrow f_0(980)K^*$  decay

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## 1 Introduction

The neutral  $D^0$  meson mixes with its  $CP$ -conjugate  $\bar{D}^0$  through box diagrams in the standard model (SM). In the box diagrams, the  $b$  quark coupling is small and the  $s$  and  $d$  quark have small masses. Thus the mixing of  $D^0\text{-}\bar{D}^0$  in the SM is very small, measurements on the mixing parameters serve as a probe to detect new physics scenarios. In the presence of mixtures, the interference in transition amplitudes of  $D$  and  $\bar{D}^0$  is possible. Experimentalists can study the time-dependent decay widths to extract the mixing parameters. Recently, the BaBar, Belle and CDF collaborations have reported their measurements on  $D^0\text{-}\bar{D}^0$  mixing<sup>[1–5]</sup>. Meanwhile, a number of theoretical methods are proposed to measure the mixing parameters<sup>[6–13]</sup>.

In this note, we propose a new method to extract the mixing parameters in  $D^0\text{-}\bar{D}^0$  mixing: using the doubly Cabibbo suppressed (DCS) channel  $D^0 \rightarrow MK^{*0}$  ( $M$  denotes a  $f_0(980)$  meson, a non-resonant  $S$ -wave  $\pi^+\pi^-$  state, or a longitudinally polarized  $\rho^0$  or  $\omega$  meson) and its  $CP$ -conjugate channel. The decay  $D^0 \rightarrow MK^{*0}$  can proceed in term of the DCS amplitude or the amplitude from the mixing followed by a Cabibbo allowed decay  $\bar{D}^0 \rightarrow MK^{*0}$ . The two amplitudes interfere with each other. If the  $K^{*0}(\bar{K}^{*0})$

meson is reconstructed in both  $K^+\pi^- (K^-\pi^+)$  and  $K_S\pi^0$  final state, the mixing parameters can be determined through the time-dependent studies on the decay widths.

## 2 $D^0\text{-}\bar{D}^0$ mixing in $D \rightarrow K^*f_0$

At present, there is no direct experimental measurement on  $D^0 \rightarrow f_0(980)\bar{K}^{*0}$ . The branching ratio for  $D^0 \rightarrow f_0(980)K_S$  is given by<sup>[14]</sup>:

$$\mathcal{BR}(D^0 \rightarrow f_0(980)K_S) = (1.36_{-0.22}^{+0.30}) \times 10^{-3}, \quad (1)$$

where  $f_0(980)$  is identified in the  $\pi^+\pi^-$  final state. This value can be used to estimate the decay rate of  $D^0 \rightarrow f_0(980)\bar{K}^{*0}$ , together with the following results<sup>[14]</sup>:

$$\begin{aligned} \mathcal{BR}(D^0 \rightarrow \pi^0 K_S) &= (1.14 \pm 0.12)\%, \\ \mathcal{BR}(D^0 \rightarrow \pi^0 \bar{K}_{K^-\pi^+}^{*0}) &= (1.91 \pm 0.24)\%, \\ \mathcal{BR}(D^0 \rightarrow \pi^0 \bar{K}_{K_S\pi^0}^{*0}) &= (6.3_{-1.5}^{+1.8}) \times 10^{-3}, \end{aligned} \quad (2)$$

where the subscript denotes the daughter mesons to reconstruct the  $\bar{K}^{*0}$  meson. Under the factorization assumption, the decay amplitudes are proportional to  $D^0 \rightarrow \pi^0(f_0)$  form factor times the  $K_0(K^{*0})$  decay constants. From Eqs. (1) and (2), we expect a somewhat

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larger branching fraction for the  $D^0 \rightarrow f_0(980)\bar{K}^{*0}$  decay channel than  $\mathcal{BR}(D^0 \rightarrow f_0(980)K_S)$ . Since the sum of masses of  $f_0(980)$  and  $K^{*0}$  is close to the mass of  $D^0$  meson, this decay channel will be suppressed by the phase space. To obtain some information on the phase space suppression, we can compare the two decays  $D^0 \rightarrow \phi K_S$  and  $D^0 \rightarrow \phi\bar{K}^{*0}$ <sup>[14]</sup>:

$$\begin{aligned}\mathcal{BR}(D^0 \rightarrow \phi_{K^+K^-} K_S) &= (2.10 \pm 0.16) \times 10^{-3}, \\ \mathcal{BR}(D^0 \rightarrow \phi_{K^+K^-} \bar{K}_{K^-\pi^+}^{*0}) &= (1.01 \pm 0.20) \times 10^{-4}.\end{aligned}\quad (3)$$

The branching ratio of  $D^0 \rightarrow \phi\bar{K}^{*0}$  is about 5% of that of  $D^0 \rightarrow \phi K_S$ . Assume the same suppression for  $D^0 \rightarrow f_0(980)\bar{K}^{*0}$ , we expect this decay has a branching ratio of  $\mathcal{O}(10^{-5})$  which can be studied by the future experiments. In our method, one can also replace the  $f_0(980)$  by the non-resonant  $\pi^+\pi^-$  final state where  $\pi^+\pi^-$  is restricted to be an  $S$ -wave state. The three-body (non-resonant) decay  $\bar{D}^0 \rightarrow \pi^+\pi^-\bar{K}^{*0}$  also possesses a sizable branching fraction<sup>[14]</sup>:

$$\mathcal{BR}(D^0 \rightarrow \pi^+\pi^-\bar{K}_{K^-\pi^+}^{*0}) = (9.7 \pm 2.1) \times 10^{-3}.\quad (4)$$

In the following, we will take  $D \rightarrow f_0(980)K^*$  as an example and use  $f_0$  to denote  $f_0(980)$  for convenience.

For  $D^0 \rightarrow f_0K^{*0}(\bar{K}^{*0})$ , the transition in quark level is either  $c \rightarrow s\bar{d}u$  or  $c \rightarrow d\bar{s}u$ . The former transition is proportional to  $V_{cs}V_{ud}^* \sim 1$  and the latter is suppressed by the CKM matrix elements:  $V_{cd}V_{us}^* \sim 0.04$ . Since there is only one amplitude for each decay in the SM, the direct  $CP$  asymmetries are 0 and the amplitudes satisfy the relation:  $A(D^0 \rightarrow f_0K^{*0}) = A(\bar{D}^0 \rightarrow f_0\bar{K}^{*0})$  and  $A(D^0 \rightarrow f_0\bar{K}^{*0}) = A(\bar{D}^0 \rightarrow f_0K^{*0}) \equiv A_{f_0K^*}$ . Neglecting direct  $CP$  asymmetries, one often defines the two parameters  $r_{f_0K^*}$  and  $\delta_{f_0K^*}$  by:

$$-r_{f_0K^*} e^{-i\delta_{f_0K^*}} \equiv \frac{A(D^0 \rightarrow f_0K^{*0})}{A(D^0 \rightarrow f_0\bar{K}^{*0})} = \frac{A(\bar{D}^0 \rightarrow f_0\bar{K}^{*0})}{A(\bar{D}^0 \rightarrow f_0K^{*0})}.\quad (5)$$

Under the assumption of  $CPT$  invariance, mass eigenstates of neutral  $D$  meson system are given by:

$$|D_1\rangle = p|D^0\rangle + q|\bar{D}^0\rangle, \quad |D_2\rangle = p|D^0\rangle - q|\bar{D}^0\rangle,\quad (6)$$

where  $p$  and  $q$  satisfy the normalization condition  $|p|^2 + |q|^2 = 1$ .  $\phi$  is defined as the phase of  $q/p$ :  $\phi = \arg(q/p)$ . In the SM, the phase  $\phi$  is very small and thus the large value for this parameter will definitely imply the presence of new physics.  $D_1$  and  $D_2$  have different masses and different decay widths. The differences are defined as:  $\Delta M = M_1 - M_2$  and  $\Delta\Gamma = \Gamma_1 - \Gamma_2$ . For convenience, experimentalists also use the two parameters  $x \equiv \frac{\Delta M}{\Gamma}$ ,  $y \equiv \frac{\Delta\Gamma}{2\Gamma}$ , where  $\Gamma$  is the averaged decay width of the  $D$ -mesons.

In the limit of  $x \ll 1$ ,  $y \ll 1$  and  $\Gamma t \ll 1$ , the time-dependent decay amplitudes squared of  $D^0 \rightarrow f$  in which there is a purely  $D^0$  state at  $t = 0$  is given

by:

$$|A(D^0(t) \rightarrow f)|^2 = e^{-\Gamma t} [X_f + Y_f \Gamma t + Z_f (\Gamma t)^2 + \dots],\quad (7)$$

where the expansion is only at  $\mathcal{O}(\Gamma t)^2$  accuracy. It is similar for  $|A(\bar{D}^0(t) \rightarrow \bar{f})|^2$ :

$$|A(\bar{D}^0(t) \rightarrow \bar{f})|^2 = e^{-\Gamma t} [\bar{X}_f + \bar{Y}_f \Gamma t + \bar{Z}_f (\Gamma t)^2 + \dots].\quad (8)$$

In the following, we will take  $f = f_0K^{*0}$  and  $\bar{f} = f_0\bar{K}^{*0}$ . If the  $K^{*0}(\bar{K}^{*0})$  meson is reconstructed in  $K^\pm\pi^\mp$  mode, the coefficients in the time-dependent decay amplitudes squared are given by:

$$X_{f_0K^*} = \bar{X}_{f_0K^*} = |A_{f_0K^*}|^2 r_{f_0K^*}^2,\quad (9)$$

$$\begin{aligned}Y_{f_0K^*} &= \left|\frac{q}{p}\right| |A_{f_0K^*}|^2 r_{f_0K^*}^2 \times \\ &\quad (y'_{f_0K^*} \cos\phi - x'_{f_0K^*} \sin\phi),\end{aligned}\quad (10)$$

$$\begin{aligned}\bar{Y}_{f_0K^*} &= \left|\frac{p}{q}\right| |A_{f_0K^*}|^2 r_{f_0K^*}^2 \times \\ &\quad (y'_{f_0K^*} \cos\phi + x'_{f_0K^*} \sin\phi),\end{aligned}\quad (11)$$

$$Z_{f_0K^*} = \left|\frac{q}{p}\right|^2 |A_{f_0K^*}|^2 \frac{x^2 + y^2}{2},\quad (12)$$

$$\bar{Z}_{f_0K^*} = \left|\frac{p}{q}\right|^2 |A_{f_0K^*}|^2 \frac{x^2 + y^2}{2}.\quad (13)$$

$x'$  and  $y'$  are linear combinations of  $x$  and  $y$ :

$$\begin{aligned}x'_{f_0K^*} &= (x \cos\delta_{f_0K^*} + y \sin\delta_{f_0K^*}), \\ y'_{f_0K^*} &= (y \cos\delta_{f_0K^*} - x \sin\delta_{f_0K^*}).\end{aligned}\quad (14)$$

The amplitude squared  $|A_{f_0K^*}|^2$  can be easily measured using the time-integrated rate for Cabibbo favored mode  $D^0 \rightarrow f_0\bar{K}^{*0}$ . In  $D^0 \rightarrow f_0\bar{K}^{*0}$ , the  $Y$  and  $Z$  terms come from the mixing followed by the DCS decay  $\bar{D}^0 \rightarrow f_0\bar{K}^{*0}$  which are at least suppressed by  $r_{f_0K^*} \sim 0.04$ . Neglecting the power suppressed terms, only the first term  $X = |A_{f_0K^*}|^2$  survives and the time-integrated rate is given by:

$$\int_0^\infty |A(D^0(t) \rightarrow f_0\bar{K}^{*0})|^2 dt \approx \frac{|A_{f_0K^*}|^2}{\Gamma}.\quad (15)$$

With  $|A_{f_0K^*}|^2$  determined, the ratio  $r_{f_0K^*}$  is easily determined by combing the measurements on  $X_{f_0K^*}$  and  $|A_{f_0K^*}|^2$ . Furthermore,  $|q/p|$  and  $x^2 + y^2$  are determined using  $Z_{f_0K^*}$  and  $\bar{Z}_{f_0K^*}$  in Eqs. (12) and (13) where terms proportional to  $r_{f_0K^*}^2$  are neglected.

Using the decay widths given in Eq. (7), Eq. (8) and Eq. (15), the four parameters are determined while the other ones ( $\phi$ ,  $\delta_{f_0K^*}$ ,  $x/y$ ) still remain unknown. The decay  $D^0 \rightarrow f_0K^{*0}$  where the  $K^{*0}$  is identified by the  $K_S\pi^0$  final state could provide more measurements which are helpful to extract the other parameters. The amplitudes receive two parts of con-

tributions:

$$A(D^0 \rightarrow f_0 K_{K_S\pi^0}^*) = A(\bar{D}^0 \rightarrow f_0 K_{K_S\pi^0}^*) = A_{f_0 K^*} [1 - r_{f_0 K^*} e^{-i\delta_{f_0 K^*}}] \equiv A_{f_0 K_S\pi}, \quad (16)$$

where the appropriate reconstruction factors with a  $K^{*0}$  identified by the  $K_S\pi^0$  are assumed in the experimental measurements. The coefficients in time-dependent decay widths are given by:

$$X_{f_0 K_S\pi} = \bar{X}_{f_0 K_S\pi} = |A_{f_0 K_S\pi}|^2, \quad (17)$$

$$Y_{f_0 K_S\pi} = -\left|\frac{q}{p}\right| |A_{f_0 K_S\pi}|^2 (-x \sin \phi + y \cos \phi), \quad (18)$$

$$\bar{Y}_{f_0 K_S\pi} = -\left|\frac{p}{q}\right| |A_{f_0 K_S\pi}|^2 (x \sin \phi + y \cos \phi), \quad (19)$$

where we have added the subscript  $K_S\pi$  for the coefficients. With these measurements, the mixing parameters,  $\tan^2 \phi$  and  $(x/y)^2$ , are solved by:

$$\tan^2 \phi = \frac{2f^2 - \mathcal{F}_{f_0 K_S\pi} - \sqrt{\mathcal{F}_{f_0 K_S\pi}^2 - 4f^2 Y_{f_0 K_S\pi}^{(+2)}}}{\mathcal{F}_{f_0 K_S\pi} + \sqrt{\mathcal{F}_{f_0 K_S\pi}^2 - 4f^2 Y_{f_0 K_S\pi}^{(+2)}}, \quad (20)$$

$$\frac{x^2}{y^2} = \frac{\mathcal{F}_{f_0 K_S\pi} - 2Y_{f_0 K_S\pi}^{(+2)} + \sqrt{\mathcal{F}_{f_0 K_S\pi}^2 - 4f^2 Y_{f_0 K_S\pi}^{(+2)}}}{2Y_{f_0 K_S\pi}^{(+2)}}, \quad (21)$$

where  $\cos 2\phi$  is chosen positive (very small  $\phi$  in SM). For convenience, the two coefficients have been reexpressed by:

$$Y_{f_0 K_S\pi}^{(+)} = \frac{\bar{Y}_{f_0 K_S\pi} |q|^2 + Y_{f_0 K_S\pi} |p|^2}{2X_{f_0 K_S\pi} |q||p|} = -y \cos \phi, \quad (22)$$

$$Y_{f_0 K_S\pi}^{(-)} = \frac{\bar{Y}_{f_0 K_S\pi} |q|^2 - Y_{f_0 K_S\pi} |p|^2}{2X_{f_0 K_S\pi} |q||p|} = -x \sin \phi, \quad (23)$$

and  $\mathcal{F}_{f_0 K_S\pi} = f^2 + Y_{f_0 K_S\pi}^{(+2)} - Y_{f_0 K_S\pi}^{(-2)}$ . In the limit of  $\phi \rightarrow 0$ , the measured value of  $Y_{f_0 K_S\pi}^{(-)}$  is very small and thus we can expand the above solutions in Eq. (20) and Eq. (21) into:

$$\tan^2 \phi = \frac{Y_{f_0 K_S\pi}^{(-2)}}{f^2 - Y_{f_0 K_S\pi}^{(+2)}} + \dots, \quad (24)$$

$$\frac{x^2}{y^2} = \frac{f^2 - Y_{f_0 K_S\pi}^{(+2)}}{Y_{f_0 K_S\pi}^{(+2)}} - \frac{Y_{f_0 K_S\pi}^{(-2)} f^2}{Y_{f_0 K_S\pi}^{(+2)} (f^2 - Y_{f_0 K_S\pi}^{(+2)})} + \dots, \quad (25)$$

where higher powers of  $Y_{f_0 K_S\pi}^{(-)}$  were neglected. As we can see from Eq. (25), there are two advantages in

our method: the ratio of  $x^2$  and  $y^2$  is finite even for tiny  $\phi$ ; our method does not need the strong phase difference  $\delta$  which is unknown at all.

### 3 Discussions and conclusions

The authors in Ref. [13] propose to extract the mixing parameters using the time-dependent study of  $D^0 \rightarrow K^{*0}\pi^0$ . The four parameters,  $|A_{f_0(\pi^0)K^*}|^2$ ,  $|A_{f_0(\pi^0)K^*}|^2$ ,  $|p/q|$  and  $f^2 = x^2 + y^2$ , are determined in the similar way but the other parameters are extracted in different ways. In  $D^0 \rightarrow K_S\pi^0\pi^0$  decays, the final state contains two neutral pion. It requires four photons to reconstruct them which is a rather difficult job. Thus they suggest to use the normalization of  $D^0 \rightarrow K_S\pi^0\pi^0$  to give a constraint on the strong phase difference. In the present method, the normalization constraint is not used and we utilize measurements on the time-dependent decay width of  $D^0 \rightarrow f_0 K_{K_S\pi^0}^*$  instead. Although the  $D^0 \rightarrow f_0 K^*$  has a small branching ratio, the advantage is that the  $f_0$  meson is easier to re-construct than  $\pi^0$  on the experimental side.

In summary, we have studied the  $D^0\text{-}\bar{D}^0$  mixing in  $D \rightarrow f_0 K^*$  decay, where the  $K^{*0}$  meson is reconstructed in both  $K^+\pi^-$  and  $K_S\pi^0$  final state. The method can be directly generalized to the three-body decay  $D \rightarrow \pi^+\pi^-K^*$  where the  $\pi^+\pi^-$  is an  $S$ -wave state. The  $f_0$  meson can also be replaced by a longitudinally polarized  $\rho^0$  meson. All parameters in  $D^0\text{-}\bar{D}^0$  mixing can be extracted by studying the time-dependent decay widths of these channel. The extraction is valid in all regions for mixing parameters and it does not depend on the strong phase difference defined in Eq. (5). The future experimental studies on  $D \rightarrow f_0 K^*$  decay, together with the  $D \rightarrow \rho^0 K^*$  decay in which the vector meson is longitudinally polarized and the three-body decay  $D \rightarrow \pi^+\pi^-K^*$ , can provide another alternative method to extract the mixing parameters.

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