Abelian-Higgs phase of $SU(2)$ QCD and glueball energy*

JIA Duojie(贾多杰)[1)]

(Institute of Theoretical Physics, College of Physics and Electronic Engineering,
Northwest Normal University, Lanzhou 730070, China)
(Department of Physics, North Carolina State University, Raleigh, NC, 27695-8202, USA)

Abstract It is shown that $SU(2)$ QCD admits an dual Abelian-Higgs phase, with a Higgs vacuum of a type-II superconductor. This is done by using a connection decomposition for the gluon field and the random-direction approximation. Using a bag picture with soft wall, we presented a calculational procedure for the glueball energy based on the recent proof for wall-vortices [Nucl. Phys. B 741(2006)1].

Key words QCD vacuum, Abelian-Higgs phase, bag, glueball

PACS 12.38.-t, 11.15.Tk, 12.38.Aw

1 Introduction

Recently, the multi-vortices, of the Abrikosov-Nielsen-Olesen type, were found to be wall vortices for the Abelian-Higgs (AH) model[1]. Such a multi-vortex is a bag object with a wall tension $T_{\rm W}$ and a thickness that separates an internal region with energy density $\Delta\varepsilon$ and an external region with energy density 0. This provides a novel mechanism for bag objects formation in a field-theoretical framework.

In our previous work[2], an dual AH model has been derived from Yang-Mills (YM) theory and the dual superconductor vacuum was then investigated. In this paper, we show that the $SU(2)$ QCD admits an dual Abelian-Higgs phase, with a type-II superconductor Higgs vacuum. This is done by applying a connection decomposition[3—5] to the gluon field and the random-phase approximation for the field in the QCD vacuum state. Based on the bag picture of hadrons that bag is built by wall-vortices, a calculational procedure for the glueball energy is presented for the $SU(2)$ QCD.

Our study is also inspired by the natural emergence of the partial "electric-magnetic" duality as well as a gauge-invariant scalar kernel $Z(\phi)$ both in the reformulated YM theory[5, 6] and in the effective confining model of QCD suggested by 't Hooft[7]. In the latter, $Z(\phi)$ assumes the role of the vacuum medium factor, quite similar to the dia-chromoelectric constant in the dia-chromoelectric soliton (DCS) model[8, 9]. Now that the bag object can arise in the AH model as a many-vortex soliton, namely, wall vortices[1], it is interesting to investigate the QCD origin of the dual AH model, the dual and relativistic version of the Ginzburg-Landau theory for superconductors.

2 The duality in $SU(2)$ QCD and hadronic picture

We begin with the $SU(2)$ YM theory, reformulated by a reparameterization called connection decomposition (CD)[3—5]. The gluon field $\boldsymbol{A}_\mu$ (the arrow denotes the three color indices $a=1,2,3$, along the generators τ^a) is decomposed into[3, 4] $\boldsymbol{A}_\mu = A_\mu\hat{n}+g^{-1}\partial_\mu\hat{n}\times\hat{n}+\boldsymbol{b}_\mu$, in which $\boldsymbol{b}_\mu$ can be further decomposed into $\boldsymbol{b}_\mu = g^{-1}[\phi_1\partial_\mu\hat{n}+\phi_2\partial_\mu\hat{n}\times\hat{n}]$[5] if one considers only the transverse degrees of freedom. Here, A_μ is an Abelian potential and $\hat{n}$ is an unit iso-vector. As a result, one has the Faddeev-Niemi decomposition[5]

$$\boldsymbol{A}_\mu = A_\mu\hat{n}+\boldsymbol{C}_\mu+g^{-1}\phi_1\partial_\mu\hat{n}+g^{-1}\phi_2\partial_\mu\hat{n}\times\hat{n} \qquad (1)$$

Received 17 July 2007, Revised 25 February 2008

* Supported by National Natural Science Foundation of China (10547009) and Research Backbone Fostering Program of Knowledge and S&T Innovation Project of NWNU (KJCXGC 03-41)

1) E-mail: jduojie@yahoo.com

with $\boldsymbol{C}_\mu := g^{-1}\partial_\mu \hat{n}\times\hat{n}$ the non-Abelian magnetic potential. Here, Eq. (1) has been written in such a form that ϕ is dimensionless. The Abelian magnetic field $H_{\mu\nu}/g = \hat{n}\cdot(\partial_\mu\hat{n}\times\partial_\nu\hat{n})/g$ can be defined via explicitly calculating the non-Abelian magnetic field tensor $\boldsymbol{C}_{\mu\nu} = -g^{-1}H_{\mu\nu}\hat{n}$ corresponding to $\boldsymbol{C}_\mu$. We note that the covariance of $\boldsymbol{b}_\mu$ under the gauge rotation $U(\alpha\hat{n}) = \exp(\mathrm{i}\alpha n^a\tau^a)$ (n^a is the a-component of $\hat{n}$) yields the transformation $\phi\to\phi\mathrm{e}^{-\mathrm{i}\alpha}$ for the complex field $\phi := \phi_1+\mathrm{i}\phi_2$, showing that it forms a charged scalar field. This idea of CD is closely associated with the Abelian projection[10], and can be generalized to the spinorial-decomposition case[11, 12].

With (1), the YM Lagrangian becomes[6]

$$\mathcal{L}^{\mathrm{YM}} = -\frac{1}{4}\left[F_{\mu\nu} - \frac{Z(\phi)}{g}H_{\mu\nu}\right]^2 - \frac{1}{4g^2}\left\{(n_{\mu\nu}-\mathrm{i}H_{\mu\nu})(\nabla^\mu\phi)^\dagger\nabla^\nu\phi + \mathrm{h.c}\right\}, \quad (2)$$

where $F_{\mu\nu} := \partial_\mu A_v - \partial_v A_\mu$, $Z(\phi) := 1-|\phi|^2$ and $n_{\mu\nu} := \eta_{\mu\nu}(\partial\hat{n})^2 - \partial_\mu\hat{n}\cdot\partial_\nu\hat{n}$. $\nabla_\mu\phi := (\partial_\mu - \mathrm{i}gA_\mu)\phi$ is the $U(1)$ covariant derivative induced by the gauge rotation $U(\alpha\hat{n})$. We note that for $A_\mu = 0$ the theory becomes

$$\mathcal{L}^M = -\frac{(Z(\phi))^2}{4g^2}H_{\mu\nu}^2 - \frac{1}{4g^2}\left\{(n_{\mu\nu}-\mathrm{i}H_{\mu\nu})(\partial^\mu\phi)^\dagger\partial^\nu\phi + \mathrm{h.c}\right\}, \quad (3)$$

in which the media-like factor $Z(\phi)$ resembles the dia-electric factor in the DCS model[8, 9] and the gauge-invariant kernel in the effective model[7], both of which account for the QCD vacuum effects: $Z(\phi\to 0) = 1$ in the perturbative (normal) vacuum (say, inside hadrons) and $Z(\phi\to\phi_0)\neq 1$ in the nonperturbative (NP) vacuum (say, outside hadrons).

The topological variable $\hat{n}(x)$, which defines the homotopy $\pi_2(V)$ of the relevant region V, plays the role of the singular transformation from the global basis $\{\tau^{1-3}\}$ to the local basis $\{\hat{n},\partial_\mu\hat{n},\partial_\mu\hat{n}\times\hat{n}\}$. This suggests that the QCD vacuum can be topologically different with the perturbative vacuum due to the nontrivial homotopic class of the map $\hat{n}$. The validity of the local basis in the region V depends upon the regularity of $\partial_\mu\hat{n}$ in V which is violated at isolated singularities z_i. Note that in the slowly-varying limit of $\hat{n}$ (that is, the norm $||\partial_\mu\hat{n}||$ is negligible on the average), the decomposition (1) ceases to make sense due to the degeneracy of $\{\hat{n},\partial_\mu\hat{n},\partial_\mu\hat{n}\times\hat{n}\}$, and in this case one can instead use the commonly-used expression $A^a\tau^a$.

In the DCS model[8, 9] for hadrons, the theory admits two vacua: one is the perturbative vacuum with the scalar $\sigma = 0$ inside the soliton and the other is the NP vacuum $\sigma = \sigma_0$ outside the soliton. The soliton is the field-theoretical counterpart of the bag in the bag model[8, 9]. Comparing this with the idea of two vacua, it is very suggestive to investigate the small-g limit of the dynamics (2) by assuming $\langle||\partial\hat{n}||\rangle\sim O(g)$ and $\partial\phi\sim o(g)$ as $g\to 0$. This yields $\langle||H_{\mu\nu}||\rangle/g\to 0$ $\langle||n_{\mu\nu}-\mathrm{i}H_{\mu\nu}||\rangle/g^2\to\mathrm{const}$. The theory then becomes an Abelian electrodynamics

$$\mathcal{L}^{\mathrm{E}} = -\frac{1}{4}F_{\mu\nu}^2\ . \quad (4)$$

Let us consider a bag-like picture of a glueball or a hadron with the two vacua separated by the bag boundary region. We assume the existence of a fixed point of the Gell-Man-Low beta function and $g\to g_{\mathrm{s}}$ monotonously while changing the position x from the bag center $\boldsymbol{x}=0$ to $|\boldsymbol{x}| = +\infty$ (see Ref. [13]). The two limits $g\to 0$ (the infrared limit) and $g\to g_{\mathrm{s}}\sim 1$ (the ultraviolet limit) correspond to the perturbative vacuum inside the bag (or soliton) and the NP vacuum outside, respectively. The dual structure of QCD in these two limits implies that asymptotically one can view the model (4) as the chromo-electric dynamics for the inside of the bag while (3) as the chromo-magnetic dynamics for the outside.

To reconcile the bag picture with the dual superconductor mechanism of the confinement[14] one needs to set the average norm $\langle||\partial\hat{n}||\rangle = \langle(\partial\hat{n})^2\rangle^{1/2}\to 0$ as $g\to 0$ and the magnetic field fluctuatations $\langle(H_{\mu\nu})^2\rangle^{1/2}\propto\langle(\partial\hat{n})^2\rangle\to H$ (a constant), increasingly as $|\boldsymbol{x}|\to+\infty$ since $||\partial\hat{n}||$ measures the density of monopoles which should tend to vanish inside the bag ($g\approx 0$). This implies the following: as $|\boldsymbol{x}|$ goes from 0 to $+\infty$, the monopoles density will increases, say, from $\rho=0$ to ρ_0, since the region where singularities in the magnetic field $\boldsymbol{C}_{\mu\nu} = -g^{-1}H_{\mu\nu}\hat{n}$ ocuur increases in number as $\hat{n}(x)$ is varying appreciably. This agrees qualitatively with the Abelian projection[10] where the QCD vacuum is in the condensed monopoles system, with the normal vacuum filled with the chromo-electric flux $F_{\mu\nu}$.

3 Multi-monopoles in the magnetic vacuum

We consider here the qualitative behavior of the monopole density $\rho_{\mathrm{m}}(\boldsymbol{x})$. As is known, the magnetic charge density is given by[3]

$$\rho_{\mathrm{ch}}(\boldsymbol{x}) = \frac{1}{4\pi}\epsilon^{ijk}\epsilon_{abc}\partial_i n^a\partial_j n^b\partial_i n^c = \sum_i\frac{w(\boldsymbol{z}_i)}{g}\delta^3(\boldsymbol{x}-\boldsymbol{z}_i) \quad (5)$$

where $w(\boldsymbol{z}_i)$ stands for the winding number of the map $\hat{n}(x)$ at the singularity (monopole) $\boldsymbol{z}_i$. The total magnetic charge $G_{\rm m} = \int_{V_{\rm out}} \rho_{\rm ch}(\boldsymbol{x}){\rm d}\boldsymbol{x}$ in $V_{\rm out}$ is then given by

$$G_{\rm m} = \sum_{\boldsymbol{z}_i \in V_{\rm out}} \frac{w(\boldsymbol{z}_i)}{g} \ . \tag{6}$$

We note here that $G_{\rm m}$ is a topological invariant under the map deformation of $\hat{n}(x)$.

Let ε be the scale of the core radius of monopoles, over which ∂n varies. It follows from (5) that $\rho_{\rm ch}(\boldsymbol{x}) \approx (1/g)w(\boldsymbol{z}_i)/\varepsilon^3$. Let $w(z_i) = w$ be equal for all monopoles; the monopole density is then

$$\rho_{\rm m}(\boldsymbol{x}) = \frac{\rho_{\rm ch}(\boldsymbol{x})}{(2\pi/g)} \approx \frac{w(\boldsymbol{x})}{2\pi\varepsilon^3} \ . \tag{7}$$

Since the vacuum outside is colorless one must have $G_{\rm m} = 0$, which implies that monopoles do occur only as monopole-anti-monopole pairs. The length scale $\Lambda_{\rm QCD}^{-1}$ of QCD can be introduced by the QCD cutoff $\Lambda_{\rm QCD}$. In the case $\Lambda_{\rm QCD}$=0.5 GeV, this scale is about 0.4 fm. If $\varepsilon \approx 0.4$ fm is chosen, the monopole density depends then mainly on $w(\boldsymbol{x})$, the winding numbers of $\hat{n}(\boldsymbol{x})$ at the local sites $\boldsymbol{x}$ of monopoles.

We now examine these multi-monopoles using the Skyrme-Faddeev(SF) model[5] as a magnetic dynamics. The SF model reads

$$\mathcal{L}^{\rm SF} = \frac{\mu_{\rm F}^2}{2}(\partial_\mu \hat{n})^2 - \frac{\alpha}{4}(\partial_\mu \hat{n} \times \partial_\nu \hat{n})^2, \tag{8}$$

The static energy is

$$E^{\rm SF} = \int {\rm d}\boldsymbol{x} \left[\frac{\mu_{\rm F}^2}{2}(\nabla \hat{n})^2 + \frac{\alpha}{4}(\partial_i \hat{n} \times \partial_j \hat{n})^2 \right]. \tag{9}$$

For simplicity, the $\hat{n}$-configuration is chosen to be $(n^1, n^2, n^3) = (\cos w\varphi \sin w\theta,\ \sin w\varphi \sin w\theta,\ \cos w\theta)$, which has an integer winding number w. Direct calculation shows that

$$(\nabla \hat{n})^2 \propto w^2 \ . \tag{10}$$

Owing to the topological reasons this proportionality holds also for an alternative w-winding map $\hat{n}'$, a map continuously deformed from the above $\hat{n}$.

Using the virial theorem and (10), one finds that the classical energy (9) is given by

$$E^{\rm SF} = \int {\rm d}\boldsymbol{x} \mu_{\rm F}^2 (\nabla \hat{n})^2 \propto w^2 \ . \tag{11}$$

We see that the local energy (11) of a monopole with w-winding (i.e., magnetic charge $2\pi w/g$) is larger than that of w monopoles with unit winding ($w = 1$). Therefore, if the NP vacuum of QCD is highly nontrivial in the sense that $\hat{n}(x)$ accommodates singularities with nonzero winding densely distributed in it, such a vacuum can be a stable system of monopoles with unit winding ($w = 1$), in contrast to a system with monopolies of higher winding ($|w| > 1$).

For a bag with soft boundary, its boundary can be taken to be a transition region $V_{\rm ao}$ between two vacua. Due to its complexity, we try to give a rather qualitative picture for $V_{\rm ao}$ from the point of view of dual superconductors. Let us suppose that the variation of the monopole density $\rho_{\rm m}(\boldsymbol{x}) \propto w(\boldsymbol{x})$ according (7) happens mainly over $V_{\rm ao}$. As $|\boldsymbol{x}|$ decreases, the number of topological singularities decreases to zero over this region, which agrees with the analysis in section 2 (as $|\boldsymbol{x}|$ goes from inside of the bag to the outside, the monopole density increases from 0 to a nonzero value ρ_0). This is comparable with the core structure of the Abrikosov vortex in type-II superconductors where the density of Cooper pairs rises from zero to a uniform value as one goes from the core center to the outside of the vortex. In the region outside of the bag, the dominant variable is given by $\hat{n}$ and the related energy is given by the classical energy (9).

4 Abelian-Higgs phase and its model

To obtain a calculational procedure within the dual superconductor mechanism, we need an effective model for the transition region $V_{\rm ao}$. As discussed in section 2, $\phi(x)$ in (2) can play the role of a soliton field interpolating in between the two vacua: $\phi(x) = 0$ and $\phi(x) = v (\neq 0)$. It is then very useful to take the monopole density $\rho_{\rm m}(\boldsymbol{x})$ to be proportional to the square of the norm of $\phi(x)$ in the (2): $\rho_{\rm m}(\boldsymbol{x}) \propto |\phi(x)|^2$. In the language of field theory, this implies to choose $\phi(x)$ as the monopole field, similar to the wavefunction of Cooper pairs in a superconductor. Writing $\phi(x) = \Phi(x) + \delta\phi$, where $\Phi(x)$ is the monopole condensate and $\delta\phi$ its fluctuation, one has

$$\langle \phi(x) \phi^\dagger(y) \rangle \approx \Phi(x)\Phi^*(y), \quad \text{for } x^0 > y^0 \ . \tag{12}$$

In the bag picture of hadrons with a soft boundary region $V_{\rm ao}$, there are three regions with different scales : $V_{\rm B} := \{\boldsymbol{x} | \boldsymbol{x}$ is inside the bag, but not in $V_{\rm ao}\}$, $V_{\rm ao}$ and $V_{\rm out} := \{\boldsymbol{x} | \boldsymbol{x}$ is outside the bag and $V_{\rm ao}\}$, in the increasing order of length scale.

As discussed in section 2, $V_{\rm B}$ and $V_{\rm out}$ can be taken to be in the phase of the perturbative QCD phase and the NP condensate phase, respectively. The relevant variables are the ultraviolet gluon fields A_μ^a ($a = 1,2,3$) for the former and the infrared variable $\hat{n}$ for the latter. Here, we take (A_μ, ϕ) as the relevant variables for the region $V_{\rm ao}$, and derive the corresponding effective model from (2) by treating $\hat{n}$ as a background field. As will be seen in the following, the effective model for this region is the AH model, and we call the phase for describing $V_{\rm ao}$ the Abelian-Higgs phase.

Let us write

$$\partial_\mu \hat{n}(x) = M {\rm e}_\mu(x) \tag{13}$$

with $M=||\partial_\mu \hat{n}(\boldsymbol{x})||$. Clearly, $M\to 0$ for $\boldsymbol{x}\to 0$ and $M\to M_0$ for $\boldsymbol{x}\to\infty$. For simplicity, we assume $M\approx \text{const}<M_0$ in V_{ao}. Then one can find $(\partial_\mu\hat{n})^2=M^2\{(\text{e}_0)^2-\sum_i(\text{e}_i)^2\}=-2M^2$, $H_{\mu\nu}=M^2h_{\mu\nu}$ where $h_{\mu\nu}=\hat{n}\cdot(\text{e}_\mu\times\text{e}_\nu)=\sin\theta_{\mu\nu}$, with $\theta_{\mu\nu}$ the angle between e_μ and e_ν in the iso-space. We have also

$$\begin{aligned}\partial_\mu\hat{n}\cdot\partial_\nu\hat{n} &= M^2\cos\theta_{\mu\nu}\ ,\\ n_{\mu\nu} &= \eta_{\mu\nu}(\partial\hat{n})^2-\partial_\mu\hat{n}\cdot\partial_\nu\hat{n}=\\ &\quad -M^2(2\eta_{\mu\nu}-\cos\theta_{\mu\nu})\ .\\ (H_{\mu\nu})^2 &= M^4h^2_{\mu\nu}=\frac{M^4}{2}\sum_{\mu\nu}(1-\cos2\theta_{\mu\nu}).\end{aligned}$$

For the magnetic field fluctuatation $H := \langle (H_{\mu\nu})^2\rangle^{1/2}$ one has

$$H^2=\frac{M^4}{2}\left\{\sum_{\mu\nu}\langle 1-\cos2\theta_{\mu\nu}\rangle\right\}\approx 6M^2$$

where $\sum_{\mu\nu}1=12$. Here, we have used the random phase approximation (RPA)

$$\sum_{\mu\nu}\langle\cos2\theta_{\mu\nu}\rangle\approx 0\ .$$

Then, one has $M^2=H/\sqrt{6}$ and the reformulated YM Lagrangian (2) becomes

$$\begin{aligned}\mathcal{L}^{\text{YM}} &= -\frac{1}{4}F^2_{\mu\nu}+\frac{M^2Z(\phi)}{4g}F^{\mu\nu}h_{\mu\nu}-\\ &\quad \frac{M^4Z(\phi)^2}{4g^2}h^2_{\mu\nu}+\frac{M^2}{2g^2}\{[2\eta_{\mu\nu}+\cos\theta_{\mu\nu}+\\ &\quad \text{i}\sin\theta_{\mu\nu}](\nabla^\mu\phi)^\dagger\nabla^\nu\phi+\text{h.c}\}.\end{aligned}$$

In the RPA, one has $\langle h_{\mu\nu}\rangle\approx 0$, $\langle h^2_{\mu\nu}\rangle\approx 6$, $\langle \text{e}^{\text{i}\theta_{\mu\nu}}(\nabla^\mu\phi)^\dagger\nabla^\nu\phi\rangle\approx 0$, which leads to the following averaged Lagrangian

$$\mathcal{L}^{\text{AH}}=-\frac{1}{4}F^2_{\mu\nu}+\frac{2H}{\sqrt{6}g^2}(\nabla^\mu\phi)^\dagger\nabla^\nu\phi-\frac{H^2}{4g^2}\langle Z(\phi)^2\rangle \quad (14)$$

where the equation $\langle(\nabla^\mu\phi)^\dagger\nabla_\mu\phi\rangle=(\nabla_\mu\Phi(x))^*\nabla^\mu\Phi(x)$, which follows from Eq. (12), has been used.

Using the Wick's theorem and the Bose symmetry of the scalar field, one finds

$$\begin{aligned}\left\langle(\phi^\dagger\phi)^2\right\rangle &= \left\langle\phi^\dagger\phi\right\rangle\left\langle\phi^\dagger\phi\right\rangle+\left\langle\phi^\dagger\phi^\dagger\right\rangle\left\langle\phi\phi\right\rangle+\\ &\quad \left\langle\phi^\dagger\phi\right\rangle\left\langle\phi^\dagger\phi\right\rangle=2\left\langle\phi^\dagger\phi\right\rangle^2\end{aligned}$$

$$\begin{aligned}\langle(Z(\phi))^2\rangle &= \left\langle 1+(\phi^\dagger\phi)^2-2\phi^\dagger\phi\right\rangle\approx\\ &\quad 1+2|\Phi^*\Phi|^2-2\Phi^*\Phi=\\ &\quad 2(|\Phi|^2-1/2)^2+1/4\ .\end{aligned}$$

Using the above relations and rescaling Φ so that it gets the dimension of mass

$$\sqrt{\frac{3}{2}}\frac{m}{g}\Phi(x)\to\Phi(x)\ , \quad (15)$$

we arrive at the following dual AH model with an added constant

$$\mathcal{L}^{\text{eff}}=-\frac{1}{4}F^2_{\mu\nu}+|(\partial_\mu-\text{i}gA_\mu)\Phi|^2-V(\Phi)-\frac{H^2}{8g^2}\ . \quad (16)$$

where the replacement (15) was used. The potential $V(\Phi)$ is given by

$$V(\Phi)=\frac{\lambda^2}{4}(|\Phi|^2-v^2)^2\ , \quad (17)$$

where

$$\begin{aligned}\lambda &= \sqrt{3}g\ ,\\ v &= \frac{\sqrt{H}}{\sqrt[4]{6}g}\ .\end{aligned} \quad (18)$$

It has the Mexico-hat form, implying two vacua $\Phi=0$ and $\Phi=v$. As mentioned before, Φ is assumed to be the monopole condensate, up to a constant. So, the two vacua correspond to the perturbative vacuum and NP vacuum, as expected in section 3. The static energy associated with the dual AH model (16) is then given by

$$E^{\text{AH}}=\int_{V_{\text{ao}}}\text{d}\boldsymbol{x}\left\{\frac{1}{2}\boldsymbol{B}^2+|D_i\Phi|^2+V(\Phi)+\frac{H^2}{8g^2}\right\}. \quad (19)$$

5 Glueball energy

The model (16) is nothing but the dual AH model suggested as the effective model of the dual superconductor picture[15] for the confining phase of QCD. It is known that this model admits the Nielsen-Olesen vortex solution[16], and the dual Meissner effect is measured by two scales: the coherent length $\xi=1/m_\Phi$ and the penetrating length $\lambda_L=1/m_A$. For the studies of the AH model as a long-distance gluodynamics in the lattice framework, see Ref. [17].

The masses m_Φ for the Higgs field Φ and m_A for the chromo-electric field A_μ can be determined by the potential parameter λ and v in (18). They are

$$\begin{aligned}m_\Phi &= \lambda v=\frac{\sqrt{3H}}{\sqrt[4]{6}}\ ,\\ m_A &= \sqrt{2}gv=\frac{\sqrt{2H}}{\sqrt[4]{6}}\ .\end{aligned} \quad (20)$$

With (20), one finds that the Ginzburg-Landau parameter for the NP vacuum medium as

$$\kappa=\frac{m_\Phi}{m_A}=\frac{\sqrt{3}}{\sqrt{2}},\ (\text{type}-\text{II}). \quad (21)$$

The result (21) predicts a type-II superconductor vacuum. The Nielsen-Olesen vortex solution indicates that Φ increases from zero near the vortex core and approaches a nonzero constant v far away from the vortex core.

Given that the stable gluon flux is confined in the bag, one expects that the energy of the gluon field within the bag stabilizes the normal vacuum $\Phi=0$ by compensating a chromo-electric energy term whose density is

$$\frac{H^2}{8g^2}=V(0)-V(v)\ . \tag{22}$$

Here, the bag is taken to be the wall limit of confined multi-vortices[1].

In the cylindrically symmetric case the field strength in $V_{\rm ao}$ is written as $B=\nabla\times A(r)$, where $A(r)$ denotes the nonvanishing component of $\boldsymbol{A}$ along the longitudinal direction $\hat{\theta}$, and the gluon field in V_B as (B,B,B) for simplicity. The gluon energy in V_B is given by $E_{\rm A}=(3B^2/2)V_B$. Collecting the energies in all regions one has

$$E=\frac{3B^2}{2}V_B+E^{\rm AH}+E^{\rm SF}\ . \tag{23}$$

Here, $E^{\rm SF}$ in (23) is taken to be the energy in $V_{\rm out}$. Owing to the requirement of continuity and the approximated uniformity of the condensate in $V_{\rm out}$, the energy density u_0 in the SF model equals approximately the dual AH energy density at the boundary of $V_{\rm ao}$ and $V_{\rm out}$: $u_0\approx H^2/(8g^2)$. One gets then

$$E\ =\ \frac{3B^2}{2}V_B+\frac{B^2}{2}V_{\rm ao}+\int_{V_{\rm ao}+V_{\rm out}}{\rm d}\boldsymbol{x}\frac{H^2}{8g^2}+ \int_{V_{\rm ao}}{\rm d}\boldsymbol{x}\{|D_i\Phi|^2+V(\Phi)\}. \tag{24}$$

Let R be the size of the bag, C the bag equator with the section $A(C)$. Being a vortex formed in normal vacuum ($\Phi=0$), the chromo-electric flux passing through $A(C)$ is quantized by the monopole condensate field $\Phi=\rho\exp({\rm i}N\theta)$ with N-multiply quantized vortices $\Phi_A(C)=2N\pi/g$. N is the quantum number of the vortex within the bag. Notice that $\Phi_A(C)\approx B\pi R^2$ and thereby $B=2N/(gR^2)$. Adding the energy $[V(0)-v(\Phi_0)]V_{B+{\rm ao}}$, which is due to the vacuum energy density difference (22), and discarding the infinite constant contribution from the integration over $V_{\rm ao}+V_{\rm out}$, we obtain the glueball energy

$$\begin{aligned}E\ =\ &\frac{2N^2}{g^2R^4}[2V_B+V_{B+{\rm ao}}]+\frac{H^2}{8g^2}V_{B+{\rm ao}}+\\ &\int_{V_{\rm ao}}{\rm d}\boldsymbol{x}\{|D_i\Phi|^2+V(\Phi)\}=\\ &\frac{8\pi N^2}{3g^2R}\left[1+2\left(1-\frac{\lambda_{\rm L}}{R}\right)^3\right]+\frac{\pi H^2}{6g^2}R^3+\\ &\int_{V_{\rm ao}}{\rm d}\boldsymbol{x}\{|D_i\Phi|^2+V(\Phi)\},\end{aligned} \tag{25}$$

where we have chosen $V_{B+{\rm ao}}:=V_B+V_{\rm ao}=\dfrac{4}{3}\pi R^3$, $V_B=4\pi(R-\lambda_{\rm L})^3/3$ and $V_{\rm ao}=4\pi[R^3-(R-\lambda_{\rm L})^3]/3$. Here, the bag boundary thickness was chosen to be $\lambda_{\rm L}$, which equals approximately $1/m_A=\sqrt[4]{6}/\sqrt{2H}$. The bag wall tension can be taken as $T_{\rm W}:=(1/V_{\rm ao})\int_{V_{\rm ao}}{\rm d}\boldsymbol{x}\{|D_i\Phi|^2+V(\Phi)\}$. We see that the first two terms have the form of the MIT-bag energy in the thin-wall limit $\lambda_{\rm L}/R\rightarrow 0$. Minimization of the energy (25) fixes R as a function of (N,g,H). Recalling that $m_\Phi^2\propto H\propto\langle(\nabla\hat{n})^2\rangle$ (see the Eq. (20)), one sees that the dual AH model (16) and (25) provides a calculational procedure for the glueball energy, with two parameters H and N. Here, g can be chosen as $g_{\rm s}=(4\pi\alpha_{\rm s})^{1/2}$.

We note here that our framework for computing the glueball energy is comparable to that of the holographic dual model[18, 19] of QCD based on the AdS/QCD correspondence[20]. This can be seen from the following arguments. (1) In modeling the glueballs both employ the "string/field" correspondence or "duality" . In our model it is the "electric-magnetic" duality, which has a gravitational analogy to a black hole in color space[2]. The holographic model is based on the supergravity duality of QCD[21]. (2) Both models introduce a finite cutoff to truncate the regime where conformal field modes (the massless gluon field modes for the former and the string modes for the latter) can propagate. (3) In the "hard-wall" or "thin-wall" limit both models provide an analogue to the MIT bag model. The bag is described by a step function, given by the scalar condensate Φ in our framework, and by a metric factor in the holographic model. In spite of these similarities, one can see that our model differs from the holographic model (e.g., the AdS slice dual model[18]) in that the field modes, confined inside the bag in our model, are the flux tubes of the gluon field in the form of multi-vortices, while the counterparts in the holographic model are the lightest string modes in a higher dimensional string theory[19]. Therefore, the duality in our model is actually that between field and vortices which end on the bag boundary, and can be viewed as the prototype of the string/field duality in string theory within the framework of field theory.

Explicit calculation of the glueball mass depends on the solution of the dual AH model (16) which is to be used to calculate the last integration concerning the bag wall tension $T_{\rm W}$ in (25). The magnetic condensation H can be given by the one-loop effective potential calculation[22] $\sqrt{H}=\Lambda\exp\left(-\dfrac{6\pi^2}{11g_{\rm s}^2}+\dfrac{1}{2}\right)$, where Λ is the QCD cutoff ($\approx 0.3\sim 0.5$ GeV). The further calculations and the comparison with the lattice prediction $M_{0^{++}}=1.61\pm 0.15$ GeV[23] as well as the holographic prediction 1.3 GeV (for $\Lambda=0.26$ GeV) for the mass of the glueball 0^{++} will be presented in a forthcoming paper.

6 Summary

The dual structure of the $SU(2)$ YM theory is revisited associated with the bag picture of hadrons and using the reparametrization, called connection decomposition. It is shown that the theory admits an Abelian-Higgs phase, which is effectively described by a dual Abelian-Higgs model, with a Higgs vacuum constant added. This phase corresponds to the soft boundary region of the bag which is the transition region between the normal vacuum and NP vacuum of QCD. Applying a bag picture for the glueball, we presented a calculational procedure for the glueball energy, based on the idea of wall-vortices.

The author is grateful to C-R Ji for his hospitality and valuable discussions during the visit to NC State University. Thanks goes also to C. Liu, P. Wang for many helpful discussions.

References

1 Bologonesi S. Nucl. Phys. B, 2005, **730**(1-2): 127—149; Nucl. Phys. B, 2006, **741**(1-2): 1—16
2 JIA Duo-Jie, AI De-Zhen. HEP & NP, 2007, **31**(5): 64—67 (in Chinese)
3 DUAN Yi-Shi, GE Mo-Lin. Sci. Sin., 1979, **11**: 1072 (in Chinese)
4 Cho Y M. Phys. Rev. D, 1980, **21**(4): 1080—1088
5 Faddeev L D, Niemi A J. Phys. Rev. Lett., 1999, **82**(8): 1624—1627
6 Langmann E, Niemi A J. Phys. Lett. B, 1999, **463**(2-4): 252—256
7 't Hooft G. Nucl. Phys. A, 2003, **721**(30): C3—C19; Nucl. Phys. B (Proc.Suppl.), 2003, **121**: 333—340
8 Lee T D. Particle Physics and Introduction to Field Theory. Amsterdam: Harwood Academic, 1983. 23—30
9 Wilets L. Nontopological Soliton, World Scientific Lecture Notes in Physics, Vol. 24. Singapore: World Scientific, 1989
10 't Hooft G. Nucl. Phys. B, 1981, [FS3]**190**(3): 455—458
11 JIA D, DUAN Y S. Mod. Phys. Lett. A, 2001, **16**(29): 1863—1869; DUAN Y S et al. J. Math. Phys., 2000, **41**: 4379
12 JIA Duo-Jie, LEE Xi-Guo. HEP & NP, 2003, **27**(4): 293—298 (in Chinese)
13 Gribov V N. Orsay Lectures on Confinement (II), in The Gribov Theory of Guark Confinement. Ed. Nyiri J. Singapore: World Scientific, 2001
14 Numbu Y. Phys. Rev. D, 1974, **10**: 4262—4268; 't Hooft G. High Energy Physics. In: Zichichi A ed. EPS International Conference, Palermo Bologna: Editrice Compositori, 1975; Mandelstam S. Phys. Rep. C, 1976, **23**(3): 245—249
15 Suzuki T. Prog. Theor. Phys., 1988, **80**(6): 929—934
16 Nielsen H B. Olesen P. Nucl. Phys. B, 1973, **61**(24): 45—61
17 Kato S et al. Nucl. Phys. B, 1998, **520**(1): 323—344; Schilling K, Bali G S et al. Nucl. Phys., 1998 (Proc. Suppl.), **63**:A-C: 519—521; Gubarev F V et al. Phys. Lett. B, 1999, **468**(1): 134—137
18 Boschi-Filho H, Braga N R F. Eur. Phys. J. C, 192004, **32**: 529—533; J. High Energy Phys., 2003, **05**: 009—015
19 Teramond Guy F de, Brodsky S J. Phys. Rev. Lett., 2005, **94**: 201601
20 Polchinski J, Strassler M J. Phys. Rev. Lett., 2002, **88**: 031601
21 Witten E. Adv. Theor. Math. Phys., 1998, **2**: 253—291; Gross D J, Ooguri H. Phys. Rev. D, 1998, **58**(10): 106002
22 Cho Y M, Pak D G. Phys. Rev. D, 2002, **65**(7): 074027
23 Teper M J. hep-lat/9711011