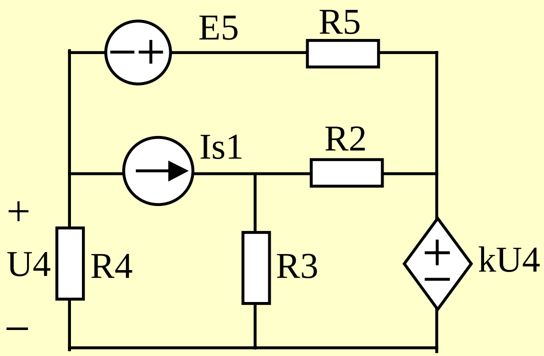
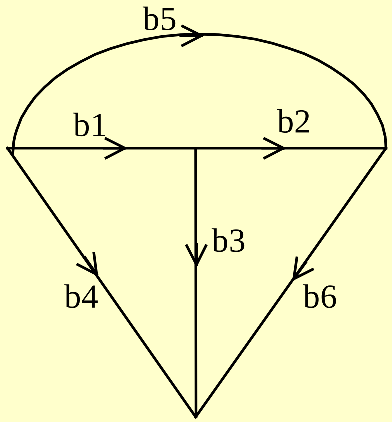
CHAPTER 2 METHODS OF ANALYSIS

2-1 NETWORK TOPOLOGY





连通图

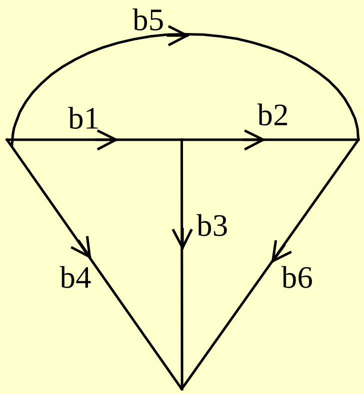
有向拓扑图

A given circuit and its linear graph (or simply a graph)

Node：A point at which two or more elements have a common connection.

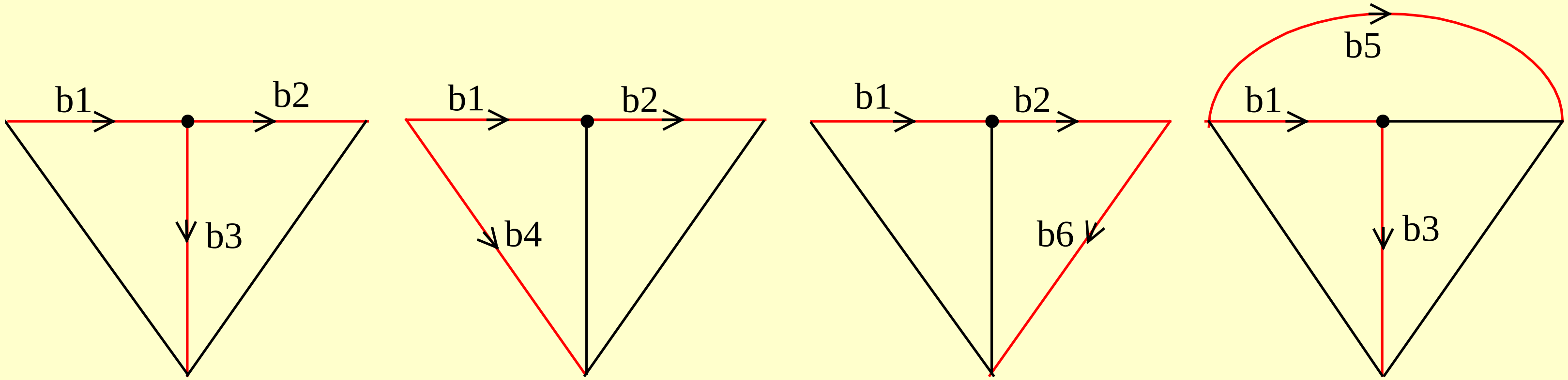
Branch：A single path, containing one simple element, which connects one node to any other node.

Tree（树）：A set of branches which does not contain any loops and connects every node to every other nodes.



Cotree（余树）：After a tree has been specified, those branches that re not part of the tree form the cotree.

Link（连支）：Any branch belonging to the cotree.

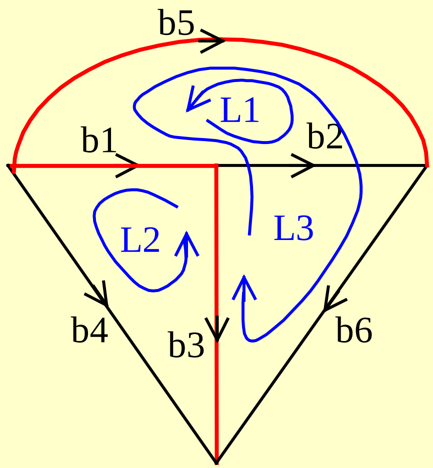


Check Your Understanding：

一个有n个节点、b条支路的连通图，连支数l为？

$l=b-(n-1)$

基本回路（单连支回路）：该回路中只包含一条连支，其它都是树支。规定基本回路的方向与其中连支的方向一致。



L1：Ub1+Ub2－Ub5=0

L2：－Ub1－Ub3+Ub4=0

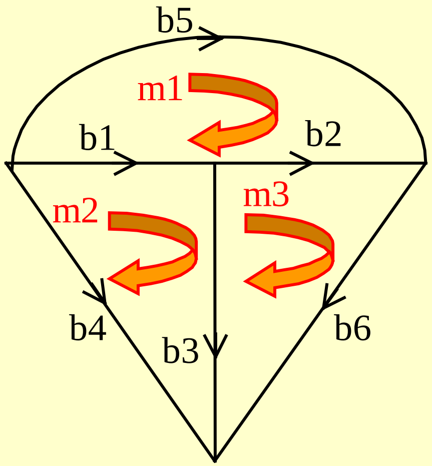
L3：－Ub1－Ub3+Ub5+Ub6=0

Mesh(网孔)：A loop which does not contain any other loops within it.

m1：$-Ub1-Ub2+Ub5=0$

m2：$Ub1+Ub3-Ub4=0$

m3：$Ub2+Ub6-Ub3=0$



L1：$Ub1+Ub2-Ub5=0$

L2：$-Ub1-Ub3+Ub4=0$

L3：$-Ub1-Ub3+Ub5+Ub6=0$

2-2 支路电流法 BRANCH ANALYSIS

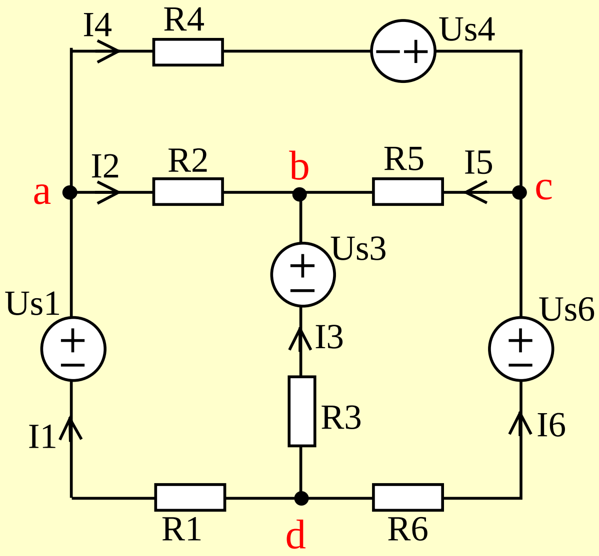
以支路电流为未知量，直接应用KCL和KVL建立电路方程，从方程中解出各支路电流。

a． $-I_1+I_2+I_4=0$

b． $-I_2-I_3-I_5=0$

c． $-I_4+I_5-I_6=0$

d． $I_1+I_3+I_6=0$



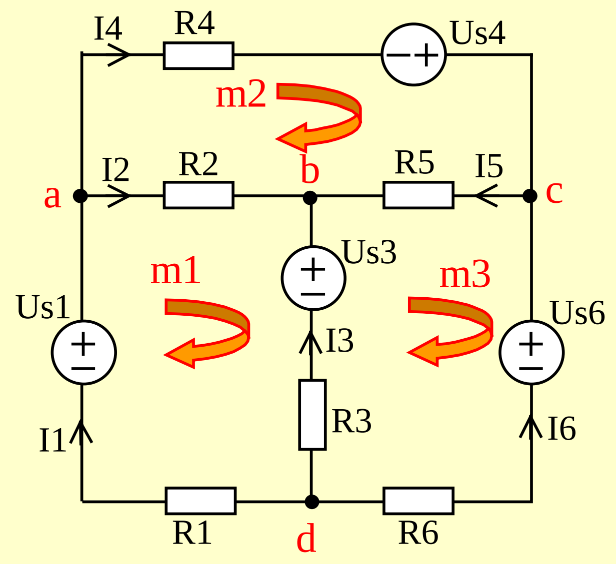
独立节点数：全部节点数－1

An independent node is a node whose voltage cannot be derived from the voltage of another node.

m1： $U_{S1}-U_{S3}=R_2I_2-R_3I_3+R_1I_1$ ；

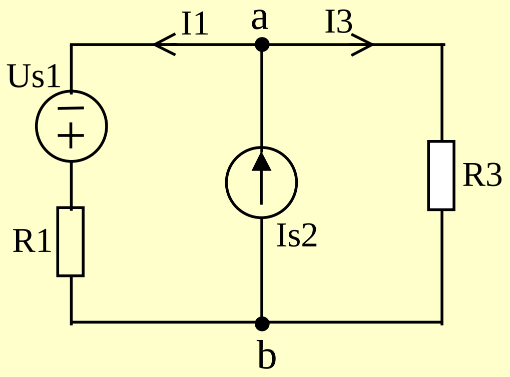
m2： $U_{S4}=R_5I_5-R_2I_2+R_4I_4$ ；

m3： $U_{S3}-U_{S6}= -R_5I_5-R_6I_6+R_3I_3$



例：$R_1=1\Omega$，$R_3=2\Omega$，$U_{S1}=2V$，$I_{S2}=0.5A$，求I_3，P_U，P_I。

解：



a：$I_1+I_3=I_{S2}$ (1)

$U_{ab}=R_1I_1-U_{S1}$，$U_{ab}=R_3I_3$

$\therefore R_1I_1-U_{S1}=R_3I_3$

回路电压方程：$U_{S1}=R_1I_1-R_3I_3$ (2)

$\Rightarrow I_1=1A$，$I_3=-0.5A$

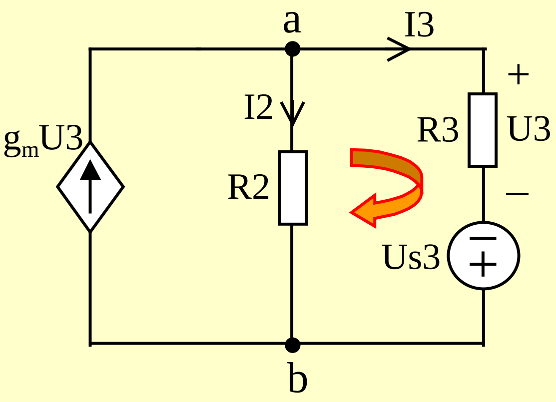
$\Rightarrow U_{ab}=R_3I_3=-1V$

$P_U=U_{S1}I_1=2W$（发出功率），

$P_I=U_{ab}I_{S2}=-0.5W$（吸收）

例：$R_2=2\Omega$，$R_3=3\Omega$，$U_{S3}=3V$，$g_m=1S$，求I_2，I_3。

解：



a：$g_mU_3=I_2+I_3$，

$U_3=R_3I_3$，$\Rightarrow I_2 -2I_3=0$

网孔电压方程：$U_{S3}= - R_2I_2+ R_3I_3$，

$\Rightarrow -2I_2+3I_3=3$

$\Rightarrow I_2= -6A$，$I_3= -3A$

2-3 回路电流法 LOOP ANALYSIS

以基本回路电流为独立变量，在基本回路中建立KVL方程，解方程组得回路电流，随后确定支路电流。

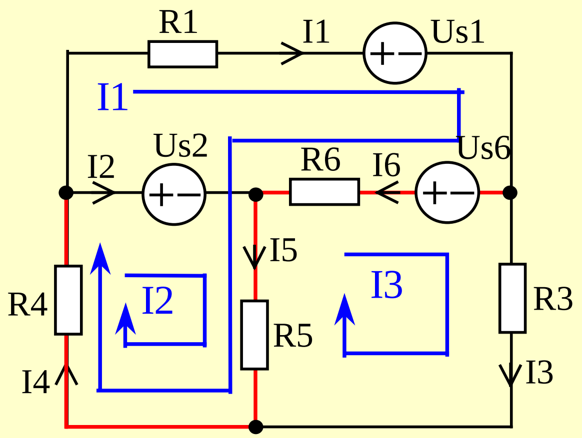
基本回路电流：在基本回路（单连支回路）中，由连支电流形成的环流，简称回路电流。

Steps：

constructing a suitable tree；

assigning a current reference to each link；

KVL equations must be written around each of loops.



I1：$-U_{S1}+U_{S6}=R_1I_1+R_6I_6+R_5I_5+R_4I_4$

∵ $I_4=I_1+I_2$,

$I_5=I_4-I_3=I_1+I_2-I_3$,

$I_6=I_1-I_3$

∴ I1：$(R_1+R_4+R_5+R_6)I_1+(R_4+R_5)I_2-(R_5+R_6)I_3=-U_{S1}+U_{S6}$

I2：$(R_4+R_5)I_1+(R_4+R_5)I_2-R_5I_3=-U_{S2}$

I3：$-(R_5+R_6)I_1-R_5I_2+(R_3+R_5+R_6)I_3=-U_{S6}$

即：$R_{11}I_1+R_{12}I_2+R_{13}I_3=\sum_{1} U_S$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \sum_1 U_S \\ \sum_2 U_S \\ \sum_3 U_S \end{bmatrix}$$

R_{ij} (i=j)：自电阻, Sum of the resistances in loop i

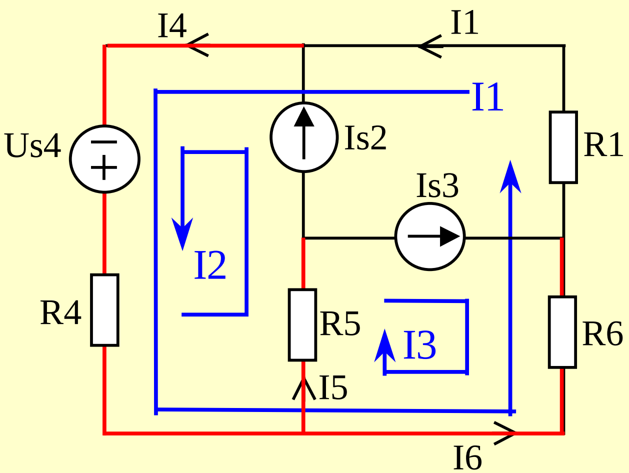
R_{ij} (i≠j)：互电阻, Sum of the resistances in common with loops i and j.

$\sum_1 U_S$：Sum of all independent voltage sources in loop 1， with voltage rise treated as positive.

例：$R_1=1\Omega$，$R_4=4\Omega$，$R_5=5\Omega$，$R_6=6\Omega$，$I_{S2}=2A$，$I_{S3}=3A$，$U_{S4}=4V$，求各支路电流。

解：

选456为树。



$I_2=I_{S2}=2A$，$I_3=I_{S3}=3A$

I1回路：$(R_1+R_4+R_6)I_1+R_4I_2-R_6I_3=U_{S4}$

$\Rightarrow I_1=14/11=1.27A$，

$I_4=I_1+I_2=36/11A$，

$I_5=I_2+I_3=5A$，

$I_6=I_1-I_3= -19/11A$。

例：$R_1=1\Omega$，$R_2=2\Omega$，$R_3=3\Omega$，$R_4=4\Omega$，$I_{S5}=5A$，要使$I_3=0$，求g_m。

解：

选树如图。

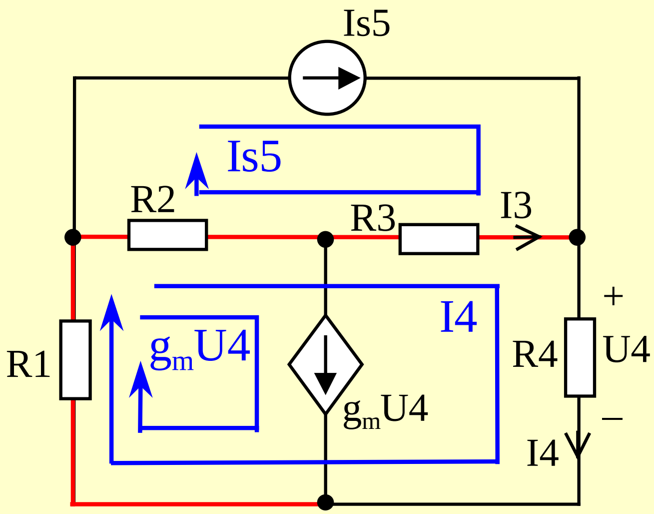
I4回路：$(R_1+R_2+R_3+R_4)I_4+(R_1+R_2)\, g_m U_4 -(R_2+R_3)I_{S5}=0$

$U_4=R_4I_4$

$$I_4=\frac{25}{10+12g_m}$$

$$I_3=I_4-I_{S5}=\frac{25}{10+12g_m}-5=0$$

$$g_m=-\frac{5}{12}S$$



2-4 网孔电流法 MESH ANALYSIS

A loop is any closed path, and a mesh is a loop which does not contain any other loops within it.

A mesh current is a current that flows only around the perimeter of a mesh, may often be identified as a branch current.

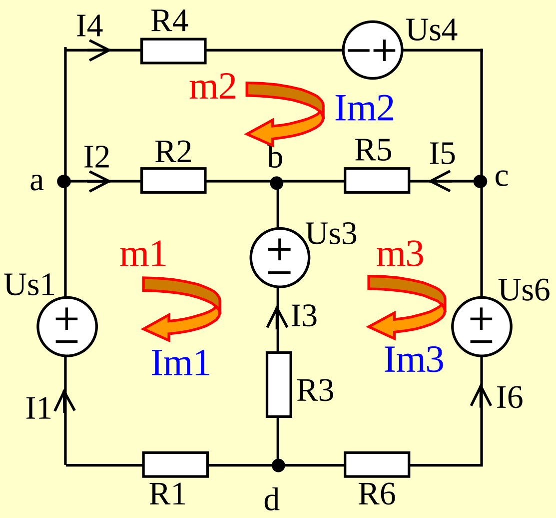
I_{m1} : $(R_1+R_2+R_3)I_{m1}-R_2I_{m2}-R_3I_{m3}=U_{S1}-U_{S3}$

I_{m2} ： $-R_2I_{m1}+(R_2+R_4+R_5)I_{m2}-R_5I_{m3}=U_{S4}$

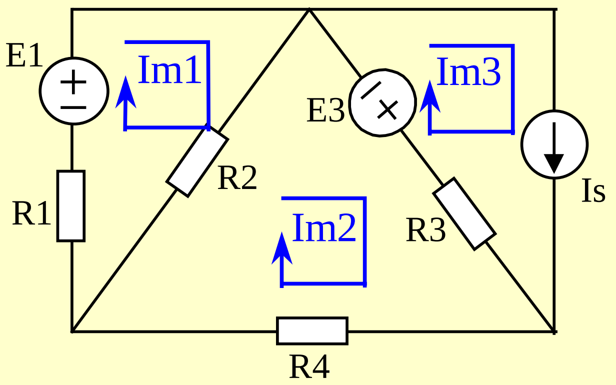
I_{m3} ： $-R_3I_{m1}-R_5I_{m2}+(R_3+R_5+R_6)I_{m3}=U_{S3}-U_{S6}$

$I_1=I_{m1}$ ， $I_2=I_{m1}-I_{m2}$， $I_3=-I_{m1}+I_{m3}$，

$I_4=I_{m2}$， $I_5=I_{m2}-I_{m3}$， $I_6=-I_{m3}$



例：E_1=1V，E_3=6V，I_S=6A，R_1=3Ω，R_2=2Ω，R_3=1Ω，R_4=4Ω，求网孔电流。



解：

$I_{m3}=I_S=6A$

Im1网孔：

$(R_1+R_2)I_{m1}-R_2I_{m2}=E_1$

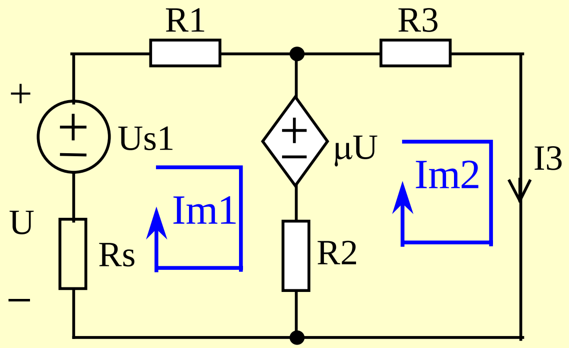
Im2网孔：

$-R_2I_{m1}+(R_2+R_3+R_4)I_{m2}-R_3I_{m3}=E_3$

即：$5I_{m1}-2I_{m2}=1$，$-2I_{m1}+7I_{m2}-1\times6=6$

$\Rightarrow I_{m1}=1A$，$I_{m2}=2A$。

例：$R_S=R_1=1\Omega$，$R_2=2\Omega$，$R_3=3\Omega$，$\mu=3$，要使$I_3=3A$，确定Us1。



解：

Im1网孔：

$(R_1+R_2+R_S)I_{m1}-R_2I_{m2}=U_{S1}-\mu U$

Im2网孔：

$-R_2I_{m1}+(R_2+R_3)I_{m2}=\mu U$

$U=U_{S1}-R_SI_{m1}$，

代入上两式，得：

$[(1-\mu)R_S+R_1+R_2]I_{m1}-R_2I_{m2}=(1-\mu)U_{S1}$；

$(\mu R_S-R_2)I_{m1}+(R_2+R_3)I_{m2}=\mu U_{S1}$

$\Rightarrow 7I_{m2}=5U_{S1}$

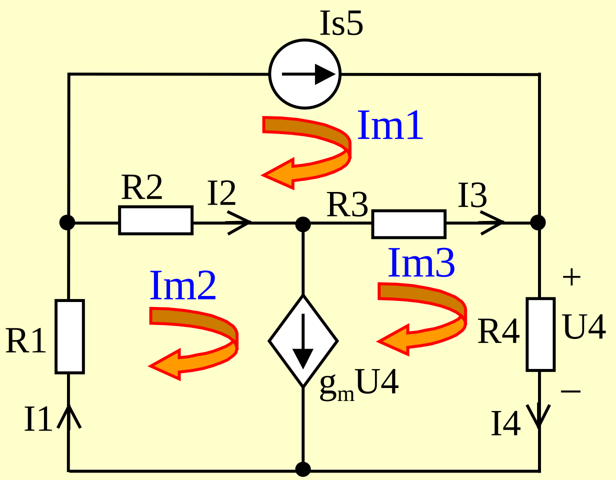
令：$I_3=I_{m2}=3A$，$\Rightarrow U_{S1}=4.2V$

Check Your Understanding：Calculate the mesh currents.

Im1：$I_{m1}=I_{S5}$

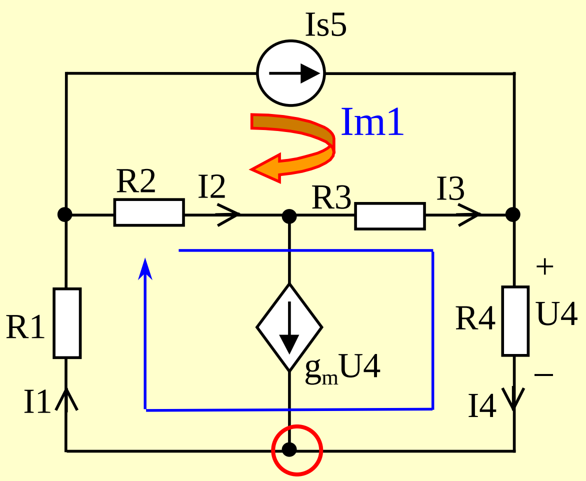
Im2：$-R_2I_{m1}+(R_1+R_2)\ I_{m2}=0$

?



$R_1I_1+R_2I_2+U_{gU4}=0 \qquad U_{gU4}=R_3I_3+R_4I_4$

A supermesh results when two meshes have a dependent or independent current source in common.



$R_1I_1+R_2I_2+R_3I_3+R_4I_4=0$， $I_2=I_1-I_{S5}$， $I_3=I_4-I_{S5}$

$I_1=g_mU_4+I_4$

$U_4=R_4I_4$

$I_1=Im_2$， $I_4=Im_3$

2-5 节点电压法
NODE-VOLTAGE ANALYSIS / NODAL ANALYSIS

一、Steps：

Select a node as the reference node.

Apply KCL to each of the n-1 nonreference nodes. Use Ohm’s law to express the branch currents in terms of node voltages.

Solve the resulting simultaneous equations to obtain the unknown node voltages.

Node 1：

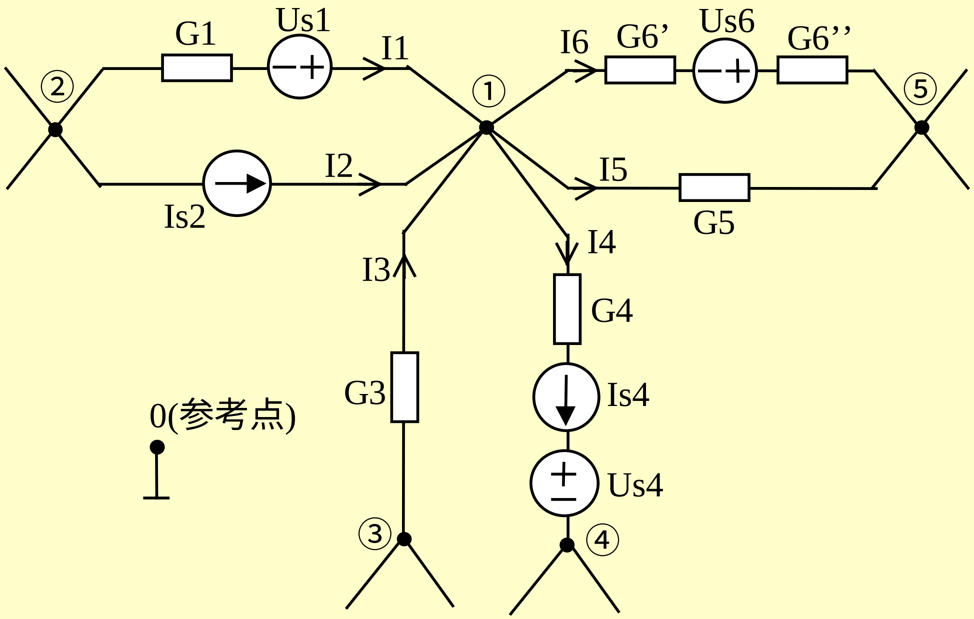
$-I_1-I_{S2}-I_3+I_{S4}+I_5+I_6=0$

$I_1=G_1(U_{②}-U_{①}+U_{S1})$，

$I_3=G_3(U_{③}-U_{①})$，

$I_5=G_5(U_{①}-U_{⑤})$，

$I_6=G_6(U_{①}-U_{⑤}+U_{S6})$



$$G_6=\frac{1}{R_6'+R_6''}=\frac{G_6'G_6''}{G_6'+G_6''}$$

$$(G_1+G_3+G_5+G_6)U_{①}-G_1U_{②}-G_3U_{③}-(G_5+G_6)U_{⑤}=G_1U_{S1}-G_6U_{S6}+I_{S2}-I_{S4}$$

$$G_{11}U_{①}+G_{12}U_{②}+G_{13}U_{③}+G_{15}U_{⑤}=\sum_{(1)}GU_S+\sum_{(1)}I_S$$

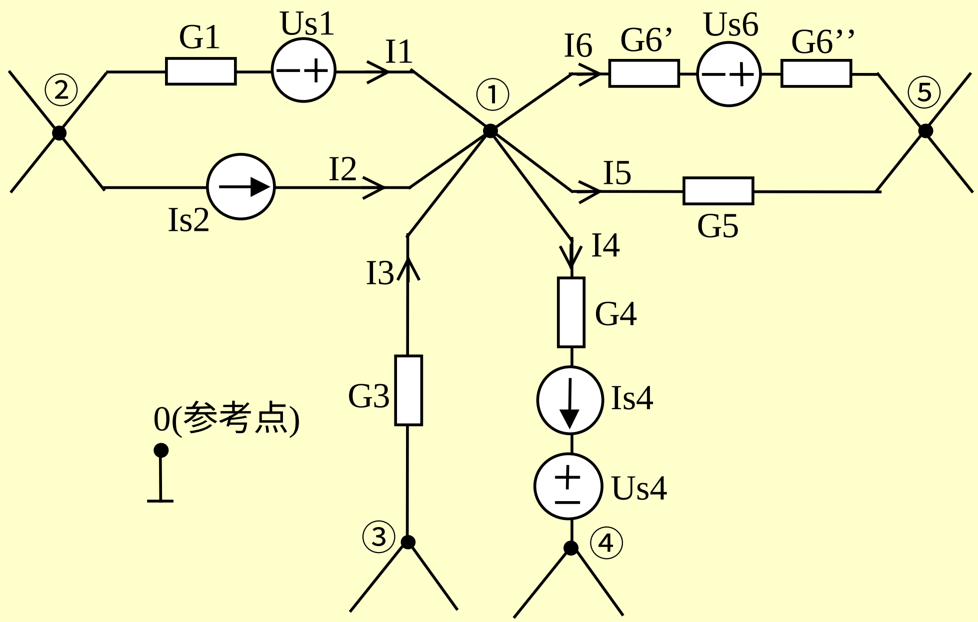
$$G_{11}U_{①}+G_{12}U_{②}+G_{13}U_{③}+G_{15}U_{⑤}=\sum_{(1)} GU_S+\sum_{(1)} I_S$$

G_{ij}（i=j）：自电导，Sum of the conductances connected to node i .

G_{ij}（i≠j）：互电导，Negative of the sum of the conductances directly connecting nodes i and j .

（Turn OFF all sources.）

$\sum_{(1)} GU_S+\sum_{(1)} I_S$ ：Sum of all sources directly connected to node 1, with currents entering the node treated as positive.



$$G_6 = \frac{1}{R_6' + R_6''} = \frac{G_6' G_6''}{G_6' + G_6''}$$

$(G_1+G_3+G_4+G_5+G_6)U_{①}-G_1U_{②}-G_3U_{③}-G_4U-(G_5+G_6)U_{⑤}$

$= G_1U_{S1}- G_6U_{S6}+I_{S2}-I_{S4}+G_4U_{S4}$

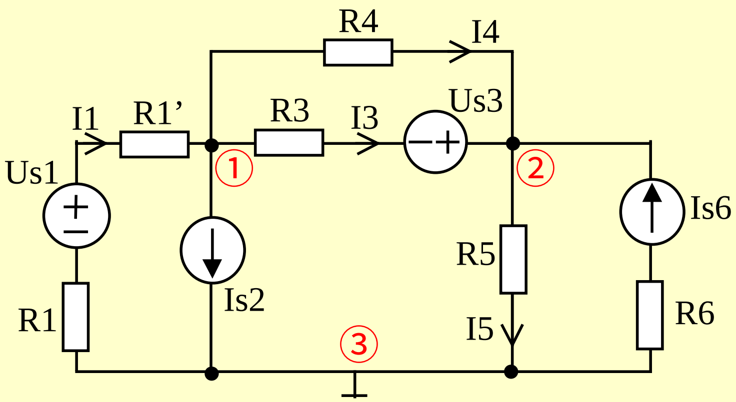
?

$(G_1+G_3+G_5+G_6)U_{①}-G_1U_{②}- G_3U_{③}-(G_5+G_6)U_{⑤}= G_1U_{S1}- G_6U_{S6}+I_{S2}-I_{S4}$

$(G_1+G_3+ \boldsymbol{G_4}+G_5+G_6)U_{①}-G_1U_{②}- G_3U_{③}- \boldsymbol{G_4U_{④}}-(G_5+G_6)U_{⑤}= G_1U_{S1}- G_6U_{S6}+I_{S2}-I_{S4}+\boldsymbol{G_4U_{S4}}$

Turn OFF all sources !!! 电源置零

例：$R_1=R_1'=0.5\Omega$，
$R_3=R_4=R_5=R_6=1\Omega$，$U_{S1}=1V$，
$U_{S3}=3V$，$I_{S2}=2A$，$I_{S6}=6A$，
求各支路电流。



解：取节点③为参考节点：

节点①：

$$\left(\frac{1}{R_1+R_1'}+\frac{1}{R_3}+\frac{1}{R_4}\right)U_{(1)}-\left(\frac{1}{R_3}+\frac{1}{R_4}\right)U_{(2)}=\frac{U_{S1}}{R_1+R_1'}-\frac{U_{S3}}{R_3}-I_{S2}$$

节点②：

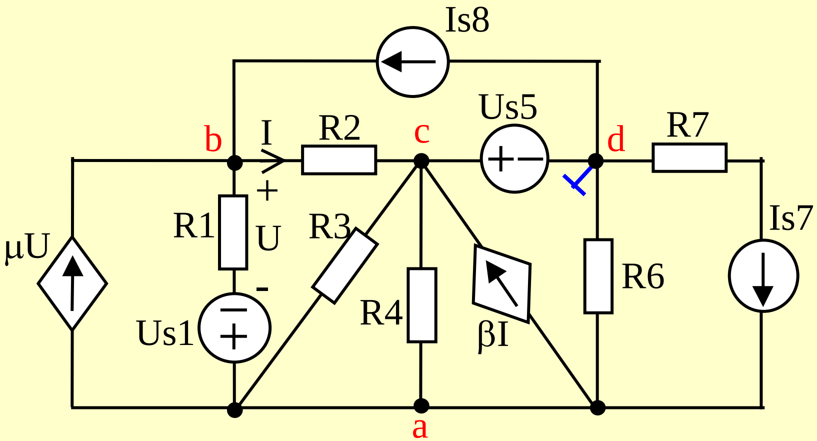
$$-\left(\frac{1}{R_3}+\frac{1}{R_4}\right)U_{(1)}+\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)U_{(2)}=\frac{U_{S3}}{R_3}+I_{S6}$$

$\Rightarrow U_{①}=1.2V$，　$U_{②}=3.8V$。

$I_1=(U_{S1}-U_{①})/(R_1+R_1')=-0.2A$，$I_3=(U_{①}-U_{②}+U_{S3})/R_3=0.4A$

$I_4=(U_{①}-U_{②})/R_4=-2.6A$，$I_5=U_{②}/R_5=3.8A$

例：$R_1=R_2=R_3=R_4=R_6=2\Omega$，$R_7=6\Omega$，$U_{S1}=4V$，$U_{S5}=10V$，$I_{S7}=1A$，$I_{S8}=4A$，$\mu=2$，$\beta=2$，求各节点电压。



解：选d为参考节点，$U_C=10V$

a：$(G_1+G_3+G_4+G_6)Ua-G_1Ub-(G_3+G_4)Uc=G_1U_{S1}-\mu U-\beta I+I_{S7}$

b：$-G_1Ua+(G_1+G_2)Ub-G_2Uc=\mu U - G_1U_{S1}+I_{S8}$

$$(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2})U_a-\frac{1}{2}U_b-(\frac{1}{2}+\frac{1}{2})U_c=\frac{1}{2}\times 4-2U-2I+1$$

$$-\frac{1}{2}U_a+(\frac{1}{2}+\frac{1}{2})U_b-\frac{1}{2}U_c=-\frac{1}{2}\times 4+2U+4$$

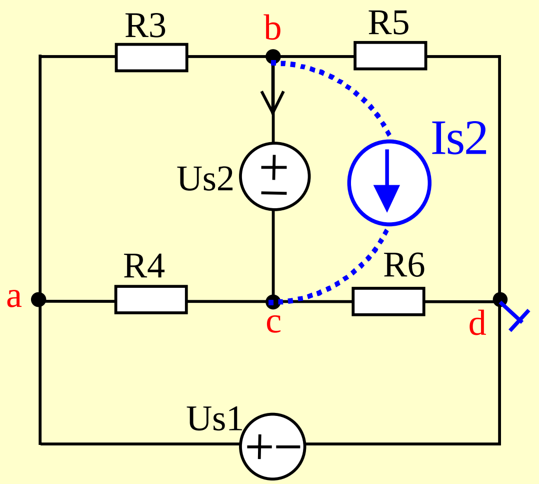
U=Ub-Ua+4； $\Rightarrow$Ub=10V

I=0.5×(Ub-Uc)=0.5×(Ub-10) $\Rightarrow$Ua=14V

二、改进节点法 MODIFIED NODAL ANALYSIS

U_{S2}：无伴电压源

Assign an unknown current to the branch which contains the voltage source.



a：$Ua=U_{S1}$

b：$-G_3Ua+(G_3+G_5)Ub= -I_{S2}$

c：$-G_4Ua+(G_4+G_6)U_C= I_{S2}$

$Ub -Uc= U_{S2}$

The other method is to treat node b, node c and the voltage source together as a sort of supernode and apply KCL to both nodes at the same time.

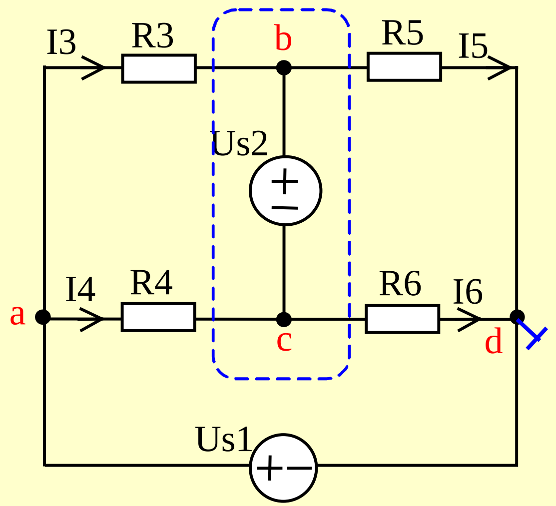
A supernode may be regarded as a closed surface enclosing the voltage source and its two nodes.

a：$U_a = U_{S1}$ (1)

b&c：$I_3 + I_4 = I_5 + I_6$

$$\frac{U_a - U_b}{R_3} + \frac{U_a - U_c}{R_4} = \frac{U_b}{R_5} + \frac{U_c}{R_6} \qquad (2)$$

$U_b - U_c = U_{S2}$ (3)



SUMMARY

Which method is the best or most efficient? The choice is dictated by two factors.

The first factor is the nature of the particular network. The key is to select the method that results in the smallest number of equations.

The second factor is the information required.

By the way, nodal analysis is more amenable to solution by computer, as it is easy to program.