4-14 THREE-PHASE CIRCUITS (三相电路)

1. Introduction

Thomas Alva Edison

(1847-1931)

A balanced three-phase system is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°.

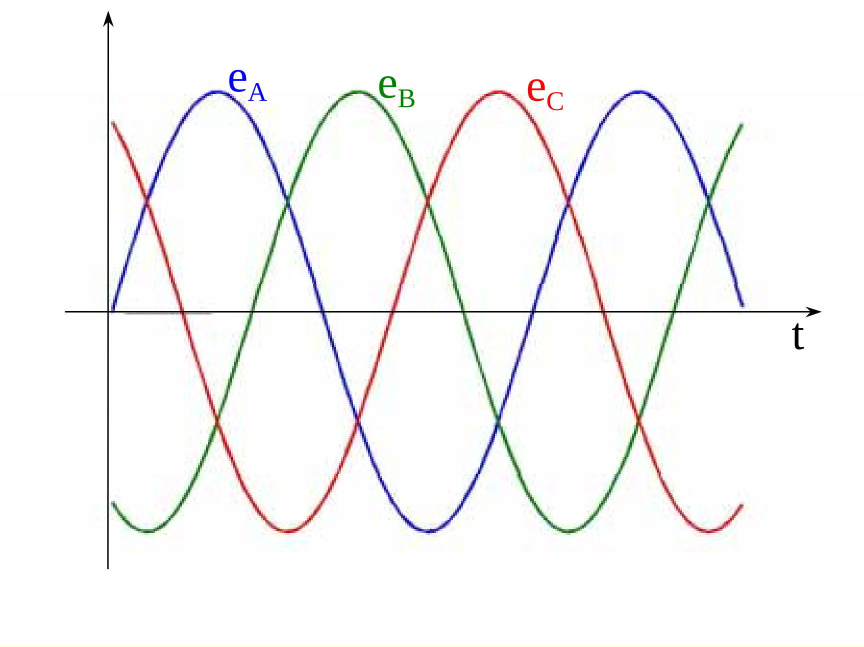
Three-phase systems are important for at least three reasons：

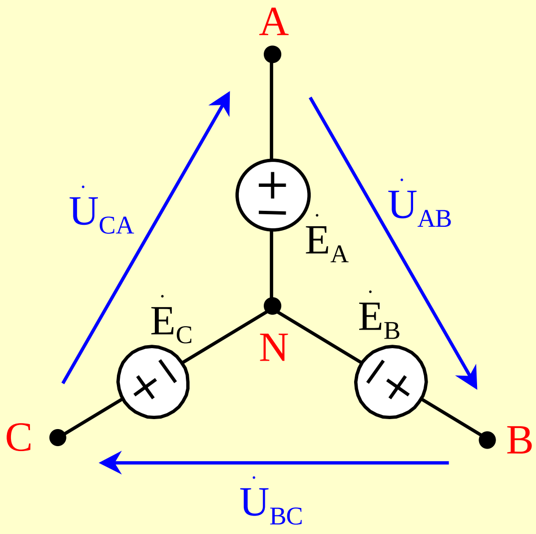
- Nearly all electric power is generated and distributed in three-phase.
- The instantaneous power in a three-phase system can be constant.
- For the same amount of power, the three-phase system is more economical than the single-phase.

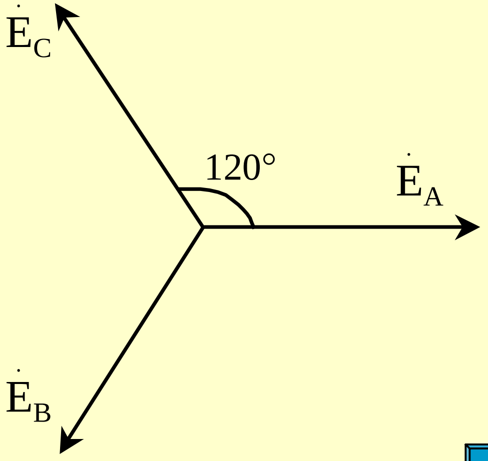
Balanced Three-Phase Voltages

Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.

$\dot{E}_A , \dot{E}_B , \dot{E}_C$: phase voltages.

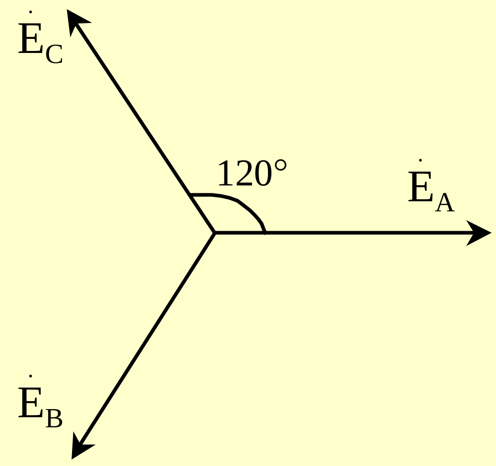






The phase sequence is the time order in which the voltages pass through their respective maximum values. (相序)

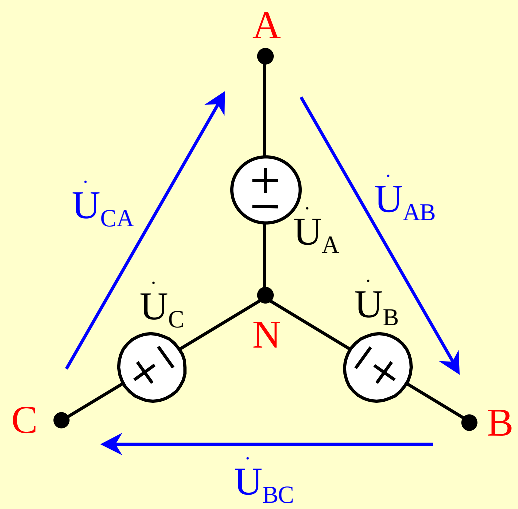
The abc sequence or positive sequence.
The acb sequence or negative sequence.

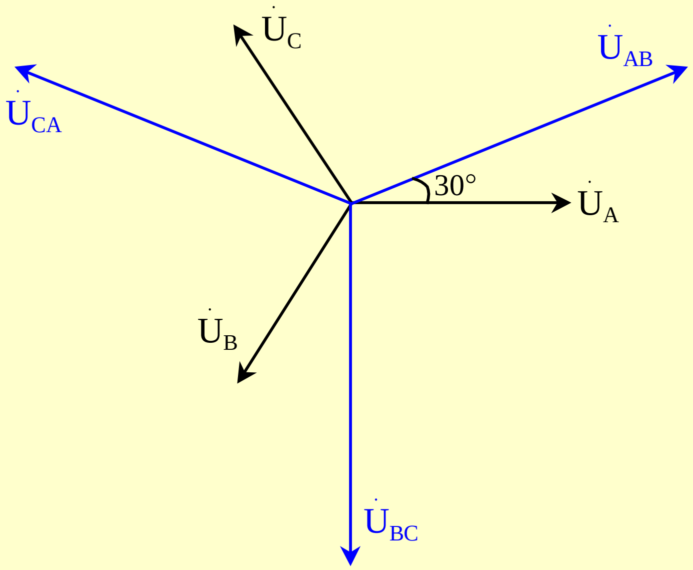


$e_A=E_m sin\omega t$， $e_B=E_m sin(\omega t-120°)$， $e_C=E_m sin(\omega t+120°)$

$\dot{E}_A = E\angle 0^\circ$， $\dot{E}_B = E\angle -120^\circ$， $\dot{E}_C = E\angle 120^\circ$

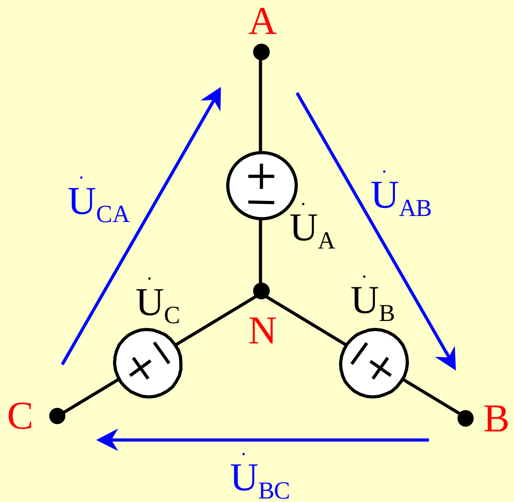
Y-connected three-phase voltage source

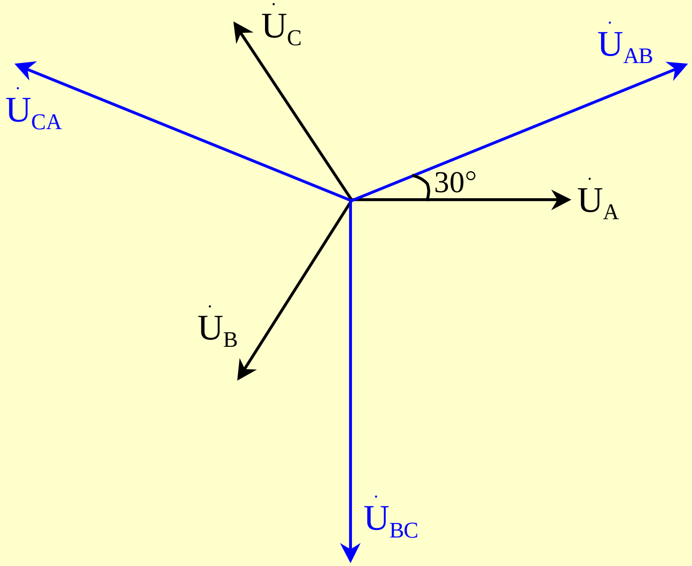




N：the neutral line. (中点)

$\dot{U}_{AB}, \dot{U}_{BC}, \dot{U}_{CA}$：the line-to-line voltages or simply line voltages. (线电压)





$$\dot{U}_A = U\angle 0^\circ$$

$$\dot{U}_{AB} = \dot{U}_A - \dot{U}_B = \sqrt{3}\dot{U}_A\angle 30^\circ$$

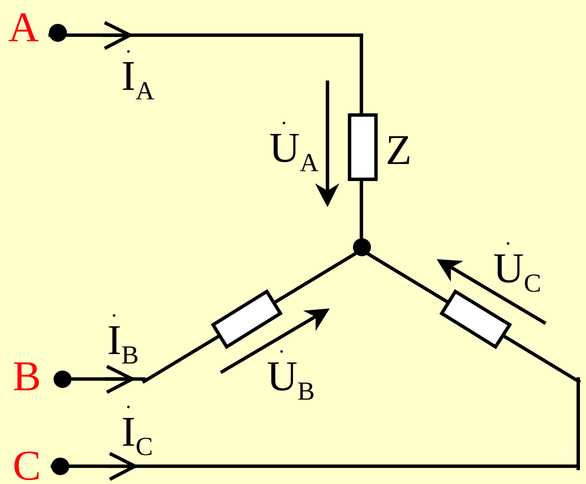
$$\dot{U}_{BC} = \dot{U}_{AB}\angle -120^\circ = \sqrt{3}\dot{U}_B\angle 30^\circ$$

$$\dot{U}_{CA} = \dot{U}_{AB}\angle 120^\circ = \sqrt{3}\dot{U}_C\angle 30^\circ$$

$$U_l = \sqrt{3}U_{ph}$$

Balanced Y-Y connection

A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.



$I_l = I_{ph}$

Phase current：相电流

Line current：线电流

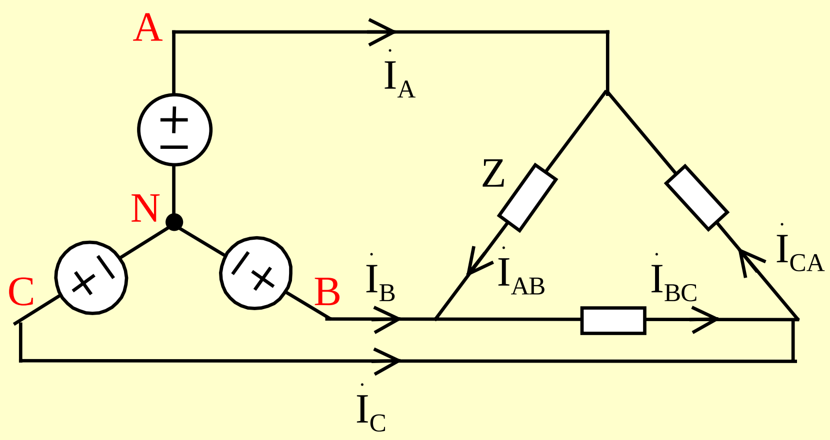
$$\dot{I}_A = \frac{\dot{U}_A}{Z}$$

$$\dot{I}_B = \frac{\dot{U}_B}{Z} = \dot{I}_A \angle -120^\circ$$

$$\dot{I}_C = \frac{\dot{U}_C}{Z} = \dot{I}_A \angle 120^\circ$$

A balanced Y-Δ system consists of a balanced Y-connected source feeding a balanced Δ-connected load.

Line voltages：



$$\dot{U}_{AB} = U_{AB}\angle 0^\circ$$

$$\dot{U}_{BC} = \dot{U}_{AB}\angle -120^\circ$$

$$\dot{U}_{CA} = \dot{U}_{AB}\angle 120^\circ$$

Phase currents：

$$\dot{I}_{AB} = \frac{\dot{U}_{AB}}{Z}$$

$$\dot{I}_{BC} = \frac{\dot{U}_{BC}}{Z} = \dot{I}_{AB}\angle -120^\circ, \quad \dot{I}_{CA} = \frac{\dot{U}_{CA}}{Z} = \dot{I}_{AB}\angle 120^\circ$$

Line currents：

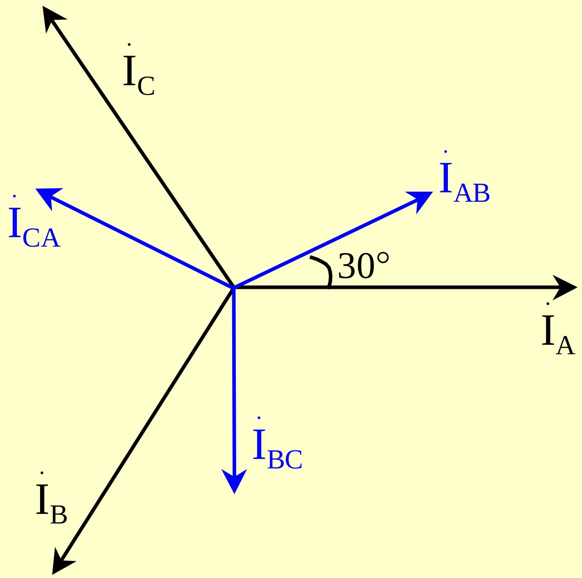
$$\because\ \dot{I}_A = \dot{I}_{AB} - \dot{I}_{CA}$$

$$\dot{I}_A = \sqrt{3}\,\dot{I}_{AB}\angle -30^\circ$$

$$\dot{I}_B = \sqrt{3}\,\dot{I}_{BC}\angle -30^\circ = \dot{I}_A\angle -120^\circ$$

$$\dot{I}_C = \sqrt{3}\,\dot{I}_{CA}\angle -30^\circ = \dot{I}_A\angle 120^\circ$$

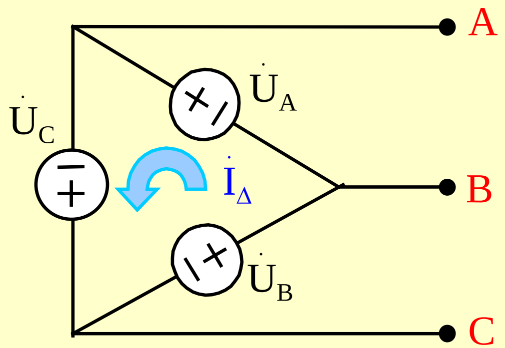
$$I_l = \sqrt{3}\,I_{ph}$$



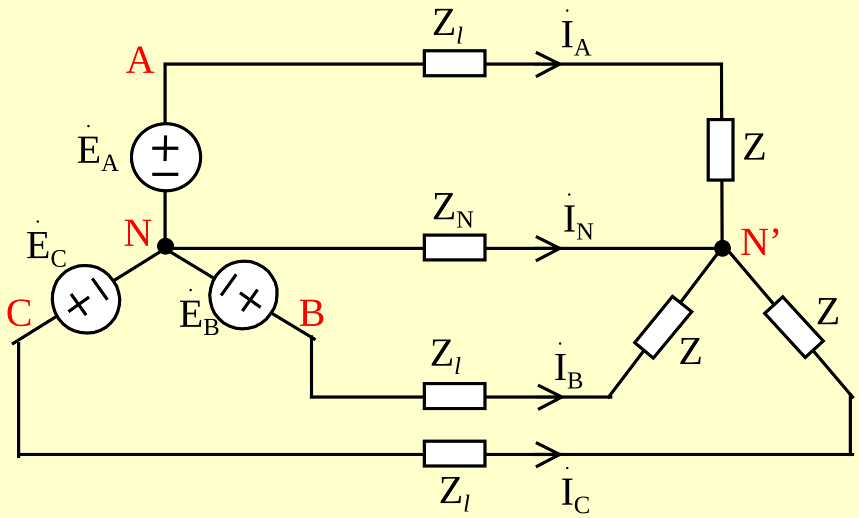
Δ-connected sources

$$\dot{U}_A + \dot{U}_B + \dot{U}_C = 0$$

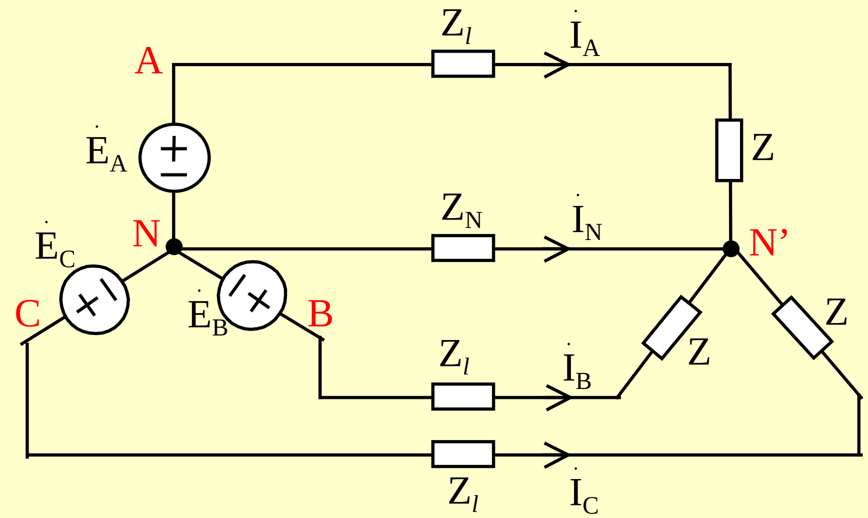
Any slight imbalance in the phase voltages will result in unwanted circulating currents.



2. Balanced Three-Phase Connection/Per-Phase Equivalent Circuits



$$\dot{U}_{N'N} = \frac{\dfrac{\dot{E}_A}{Z_l+Z} + \dfrac{\dot{E}_B}{Z_l+Z} + \dfrac{\dot{E}_C}{Z_l+Z}}{\dfrac{1}{Z_l+Z} + \dfrac{1}{Z_l+Z} + \dfrac{1}{Z_l+Z} + \dfrac{1}{Z_N}} = 0$$



$$\dot{I}_A = \frac{\dot{E}_A}{Z_l+Z}, \quad \dot{I}_B = \dot{I}_A \angle -120^\circ,$$

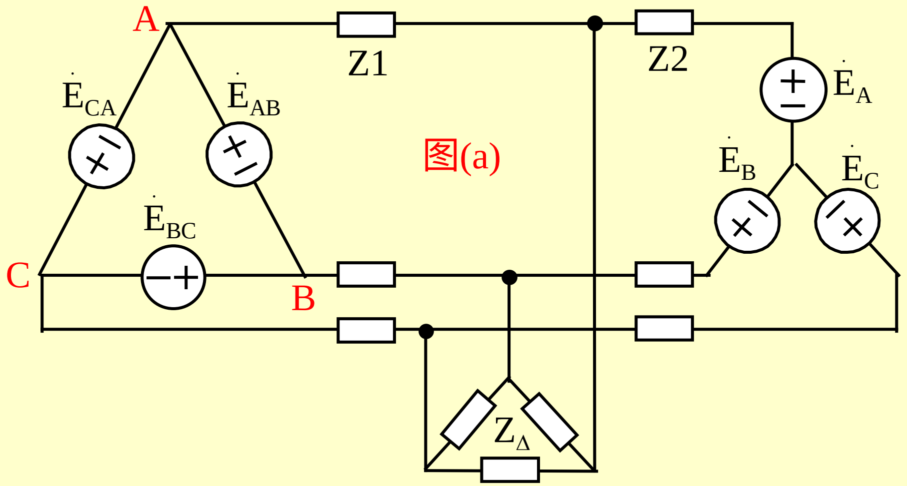
$$\dot{I}_C = \dot{I}_A \angle 120^\circ, \quad \dot{I}_N = 0$$

Phase voltages：

$$\dot{U}_A = \dot{I}_A Z, \quad \dot{U}_B = \dot{U}_A \angle -120^\circ, \quad \dot{U}_C = \dot{U}_A \angle 120^\circ$$

- The neutral line can be removed without affecting the system.
- The balanced three-phase circuit may be replaced with its single-phase equivalent circuit.

例：电源为三相对称，$\dot{E}_{AB}=380\angle 0^{\circ}V$，$\dot{E}_{A1}=220\angle 0^{\circ}V$，$Z_1=j4\Omega$，$Z_2=j3\Omega$，$Z_{\Delta}=(90+j60)\Omega$，求负载 Z_{Δ}的相电压和相电流。

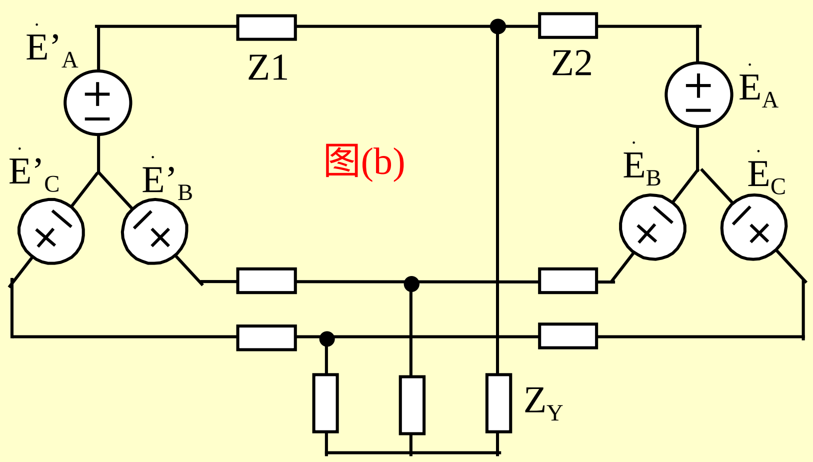


解：

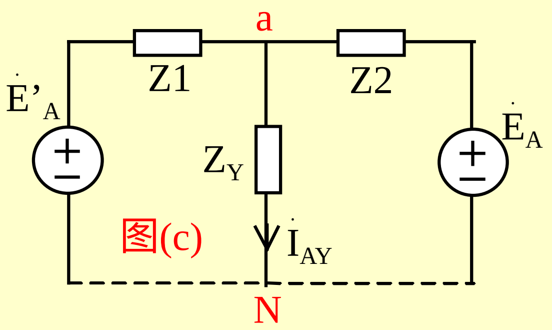
图(b)：

$$\dot{E}'_A=\frac{\dot{E}_{AB}}{\sqrt{3}}\angle -30^{\circ}=220\angle -30^{\circ}V$$

$$Z_Y=\frac{1}{3}Z_{\Delta}=30+j20\,\Omega$$



$$\dot{U}_{aN}=\frac{\dot{E}'_{A}/Z_1+\dot{E}_{A1}/Z_2}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_Y}}=207\angle-15^\circ V$$



$$\dot{I}_{AY}=\frac{\dot{U}_{aN}}{Z_Y}=5.75\angle-48.7^\circ A$$

$$\dot{I}_{AB}=\frac{\dot{I}_{AY}}{\sqrt{3}}\angle 30^\circ=3.32\angle-18.7^\circ A$$

$$\dot{U}_{AB}=Z_\Delta \dot{I}_{AB}=359\angle 15^\circ V$$

$$\dot{U}_{AB}=359\angle 15^\circ V,\ \dot{U}_{BC}=359\angle-105^\circ V,\ \dot{U}_{CA}=359\angle-225^\circ V$$

$$\dot{I}_{AB}=3.32\angle-18.7^\circ A,\ \dot{I}_{BC}=3.32\angle-138.7^\circ A,\ \dot{I}_{CA}=3.32\angle-258.7^\circ A$$

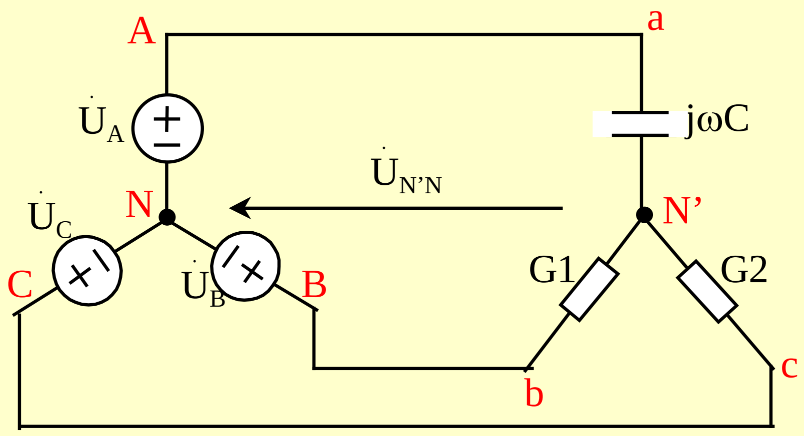
4-15 Unbalanced Three-Phase Systems

An unbalanced system is due to unbalanced voltage sources or an unbalanced load.

$$\dot{U}_A = U_{ph}\angle 0^\circ,$$

$$\dot{U}_B = U_{ph}\angle -120^\circ,$$

$$\dot{U}_C = U_{ph}\angle 120^\circ$$



$$\dot{U}_{N'N} = \frac{j\omega C\dot{U}_A + G_1\dot{U}_B + G_2\dot{U}_C}{G_1 + G_2 + j\omega C}$$

相序指示器

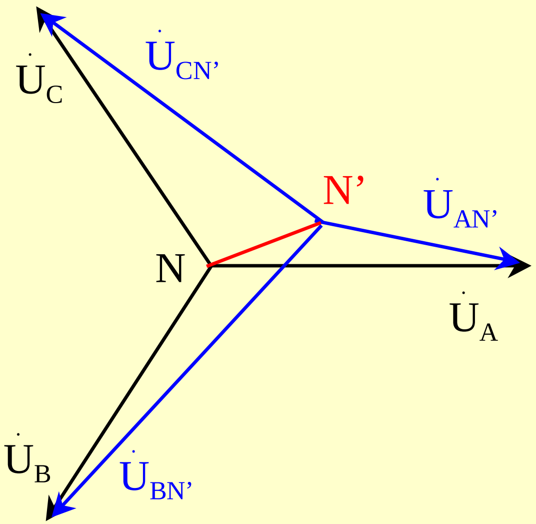
$$= \frac{\omega C\dot{U}(j1 + 1\angle -120^\circ + 1\angle 120^\circ)}{\omega C(2 + j1)} = 0.632U_{ph}\angle 108.4^\circ \quad (\omega C = G_1 = G_2)$$

$$\dot{U}_{BN'} = \dot{U}_B - \dot{U}_{N'N} = U_{ph}\angle -120^\circ - 0.632U_{ph}\angle 108.4^\circ = 1.5U_{ph}\angle -101.6^\circ$$

$$\dot{U}_{CN'} = \dot{U}_C - \dot{U}_{N'N} = U_{ph}\angle 120^\circ - 0.632U_{ph}\angle 108.4^\circ = 0.4U_{ph}\angle 138.4^\circ$$

$$\dot{U}_{N'N} \neq 0$$

The neutral of the load is no longer at the same voltage as the neutral of the source.



Phase A：jωC

Phase B：G_1

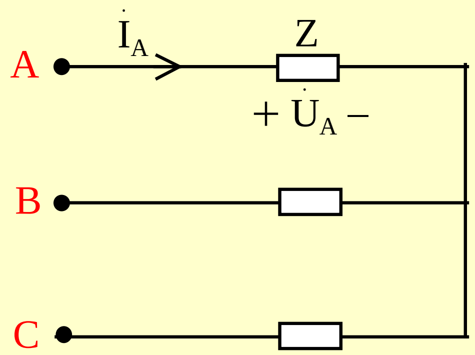
Phase C：G_2

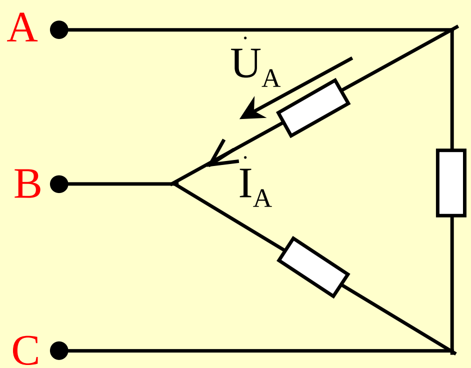
Check Your Understanding

When there exists a neutral wire in the unbalanced Y connection, is the neutral of the load at the same voltage as the neutral of the source?

4-16 POWER IN A BALANCED SYSTEM

1. Complex Power In A Balanced System





The per-phase complex power：

$$\dot{U}_A = U_{ph}\angle\phi_u\,,\quad \dot{I}_A = I_{ph}\angle\phi_i\,,\quad \phi_z = \phi_u - \phi_i$$

$$\tilde{S}_A = \dot{U}_A \overset{*}{I}_A = U_{ph}\angle\phi_u I_{ph}\angle -\phi_i = U_{ph} I_{ph}\angle\phi_u - \phi_i$$

$$= U_{ph} I_{ph}\cos\phi_z + jU_{ph} I_{ph}\sin\phi_z = P_A + jQ_A$$

The per-phase apparent power： $S_A = U_{ph} I_{ph}$

$$pf_A = \frac{|P_A|}{S_A} = \cos\phi_z$$

The total complex power：

$$\tilde{S} = \tilde{S}_A + \tilde{S}_B + \tilde{S}_C = 3U_{ph} I_{ph} \cos\phi_z + j3U_{ph} I_{ph} \sin\phi_z = 3P_A + j3Q_A = P + jQ$$

The total apparent power： $S=3S_A=3U_{ph}I_{ph}$

$$pf = \frac{|P|}{S} = \frac{3|P_A|}{3S_A} = pf_A$$

Balanced Y-connected load： $I_{ph}=I_l\,,\ U_{ph}=U_l/\sqrt{3}$

Balanced △-connected load： $U_{ph}=U_l\,,\ I_{ph}=I_l/\sqrt{3}$

The total complex power：

$$\tilde{S}=\tilde{S}_A+\tilde{S}_B+\tilde{S}_C=3U_{ph}I_{ph}\cos\phi_z+j3U_{ph}I_{ph}\sin\phi_z=3P_A+j3Q_A=P+jQ$$

$$\tilde{S}=\sqrt{3}U_lI_l\cos\phi_z+j\sqrt{3}U_lI_l\sin\phi_z=P+jQ$$

The total apparent power： $S=\sqrt{3}U_lI_l$

2. The Instantaneous Power

The phase voltages for a balanced load：

$$u_A = \sqrt{2} U_{ph} \sin(\omega t + \phi_u)$$

$$u_B = \sqrt{2} U_{ph} \sin(\omega t + \phi_u - 120^\circ), \quad u_C = \sqrt{2} U_{ph} \sin(\omega t + \phi_u + 120^\circ)$$

Phase currents：

$$i_A = \sqrt{2} I_{ph} \sin(\omega t + \phi_i)$$

$$i_B = \sqrt{2} I_{ph} \sin(\omega t + \phi_i - 120^\circ), \quad i_C = \sqrt{2} I_{ph} \sin(\omega t + \phi_i + 120^\circ)$$

The per-phase instantaneous power：

$$p_A = u_A i_A = 2U_{ph} I_{ph} \sin(\omega t + \phi_u) \sin(\omega t + \phi_i)$$

$$= U_{ph} I_{ph} \cos(\phi_u - \phi_i) - U_{ph} I_{ph} \cos(2\omega t + \phi_u + \phi_i)$$

$$p_B = U_{ph} I_{ph} \cos(\phi_u - \phi_i) - U_{ph} I_{ph} \cos(2\omega t + \phi_u + \phi_i - 240^\circ)$$

$$p_C = U_{ph} I_{ph} \cos(\phi_u - \phi_i) - U_{ph} I_{ph} \cos(2\omega t + \phi_u + \phi_i + 240^\circ)$$

$$\because\ \cos(2\omega t+\phi_u+\phi_i)+\cos(2\omega t+\phi_u+\phi_i-240^\circ)+\cos(2\omega t+\phi_u+\phi_i-240^\circ)=0$$

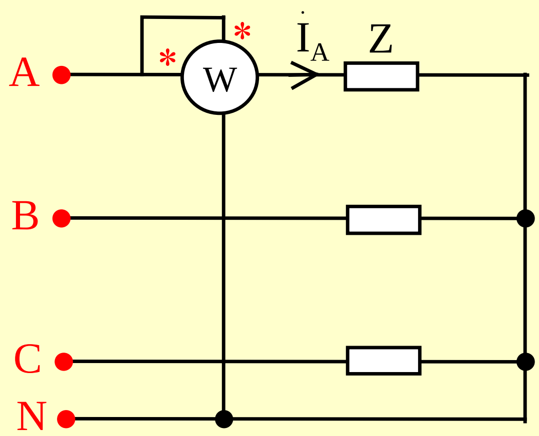
The total instantaneous power：

$$p_A+p_B+p_C=3U_{ph}I_{ph}\cos(\phi_u-\phi_i)=P$$

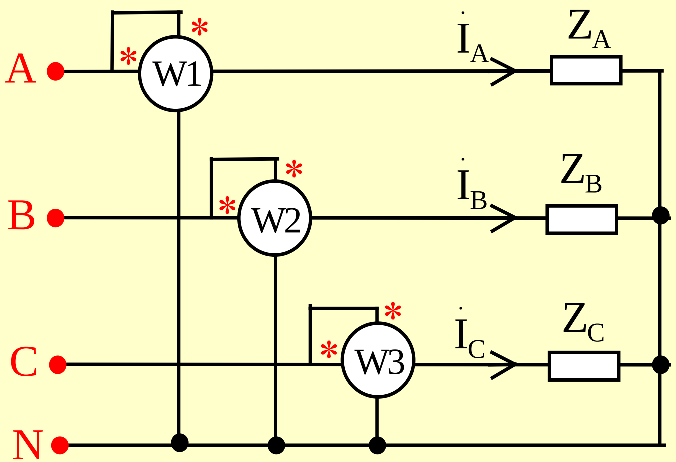
- The total instantaneous power in a balanced three-phase system is constant. This result is true whether the load is Y- or Δ-connected.
- Less material is needed to deliver the same power with a three-phase system than is required for a single-phase system.

3. Power Measurement In Three-Phase Systems

Three-Phase Four-Wire System



Balanced Circuit



Unbalanced Circuit

$W = U_A I_A \cos(\phi_{uA} - \phi_{iA}) = P_A$

The total average power：$P = 3P_A$

$W_1 = U_A I_A \cos(\phi_{uA} - \phi_{iA}) = P_A$

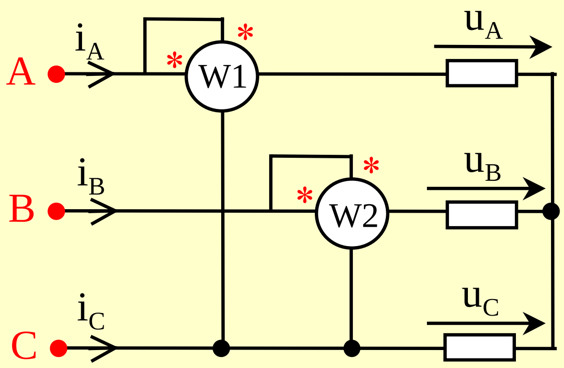
$W_2 = U_B I_B \cos(\phi_{uB} - \phi_{iB}) = P_B$

$W_3 = U_C I_C \cos(\phi_{uC} - \phi_{iC}) = P_C$

$P = P_A + P_B + P_C$

Three-Phase Three-Wire System

The two-wattmeter method



The algebraic sum of the two wattmeter reading is the total power drawn by the load, regardless of load unbalance or source unbalance.

$$p_1+p_2=u_{AC}\, i_A+u_{BC}\, i_B=(u_A-u_C)i_A+(u_B-u_C)i_B=u_A i_A+u_B i_B+u_C i_C=p_A+p_B+p_C\ ,$$

$$i_C=-i_A-i_B$$

$$W_1+W_2=\frac{1}{T}\int_0^T (p_1+p_2)dt=\frac{1}{T}\int_0^T (p_A+p_B+p_C)dt=P_A+P_B+P_C$$

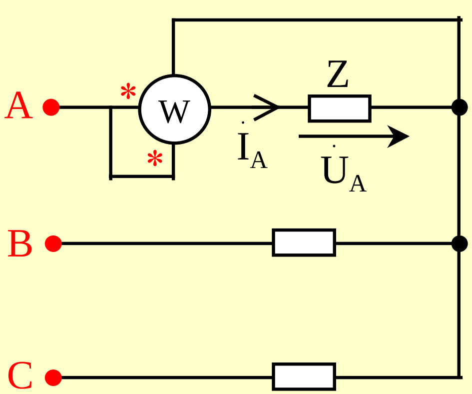
Check Your Understanding

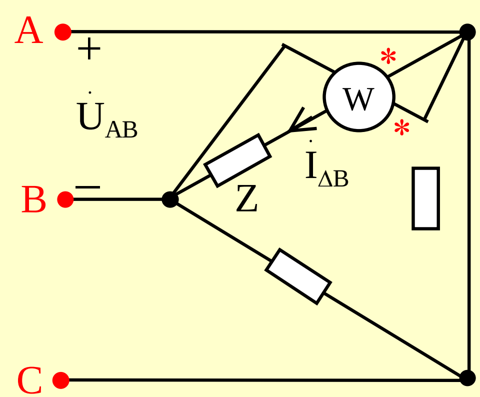
Find the other methods of connection to measure the total power by two-wattmeter method.

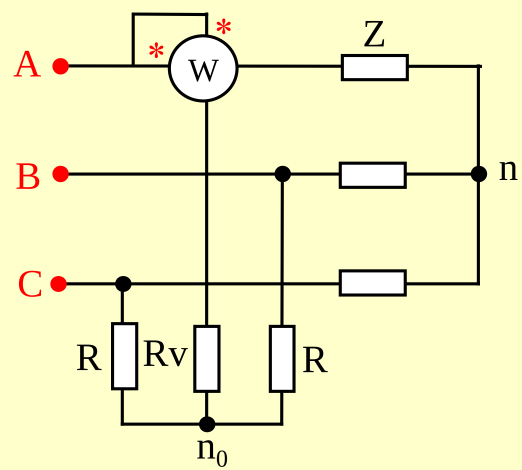
u_{BA}， i_B and u_{CA}， i_C ；u_{CB}， i_C and u_{AB}， i_A 。

The one-wattmeter method

In the balanced three-phase three-wire circuit, the wattmeter is connected in such a way that reads the power taken by one phase of three-phase load.

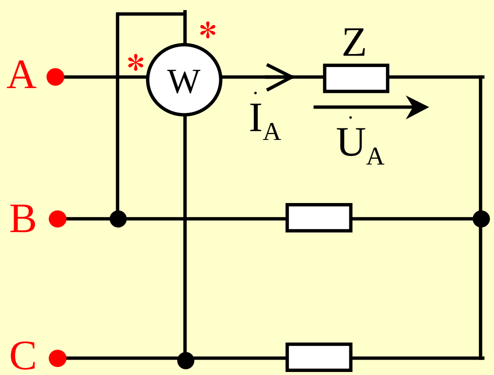


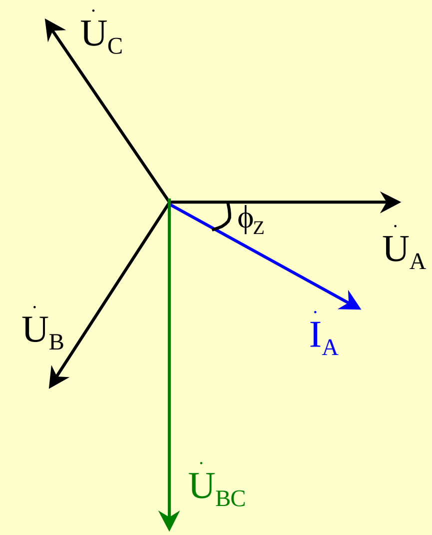




n_0：人造中点

The method of connecting one wattmeter to measure the total reactive power taken by a three-phase load.





$$\dot{U}_A = U_{ph}\angle 0^\circ \qquad \dot{I}_A = \frac{\dot{U}}{Z} = \frac{U_{ph}\angle 0^\circ}{|Z|\angle\phi_z} = I_l\angle -\phi_z$$

$$\dot{U}_{BC} = \sqrt{3}\dot{U}_B\angle 30^\circ = \sqrt{3}(U_{ph}\angle -120^\circ)\angle 30^\circ = U_l\angle -90^\circ$$

$$W = U_{BC} I_A \cos(\phi_{uBC} - \phi_{iA}) = U_l I_l \cos(-90^\circ + \phi_z) = U_l I_l \sin\phi_z$$

$$\because\ \tilde{S} = \sqrt{3}U_l I_l \cos\phi_z + j\sqrt{3}U_l I_l \sin\phi_z = P + jQ$$

$$\therefore\ \sqrt{3}W = \sqrt{3}U_l I_l \sin\phi_z = Q$$

Inductive load：$\phi_z > 0$， W>0， Q>0，
Capacitive load：$\phi_z < 0$， W<0， Q<0。

Three methods of connection：

$$\dot{U}_{BC},\ \dot{I}_A\ ;\ \dot{U}_{CA},\ \dot{I}_B\ ;\ \dot{U}_{AB},\ \dot{I}_C\ 。$$