Brazilian Journal of Probability and Statistics (2002), 16, pp. 205–217.

AN ESTIMATION ALGORITHM FOR THE FALSE ALARM PROBABILITY IN SYSTEMS WITH UNCERTAIN OBSERVATIONS

R. Caballero-Águila[1], **A. Hermoso-Carazo**[2] and **J. Linares-Pérez**[2]

[1] *Departamento de Estadística e I. O. Universidad de Jaén. Paraje Las Lagunillas, s/n, 23071 Jaén, Spain. E-mail: raguila@ujaen.es*

[2] *Departamento de Estadística e I. O. Universidad de Granada. Campus Fuentenueva, s/n, 18071 Granada, Spain. E-mails: ahermoso@ugr.es and jlinares@ugr.es*

Summary

A recursive algorithm to estimate the false alarm probability in discrete linear systems is proposed. This algorithm is obtained from a bayesian viewpoint, considering successive approximations of mixture distributions to obtain the posterior densities of the unknown probability given the observations. Numerical simulations of the observations of a scalar system have been obtained for different values of the false alarm probability; the performance of the algorithm with these simulated values shows that it is feasible.

Key words: Estimation algorithm; false alarm probability; uncertain observations.

1 Introduction

Usually, the estimation theory in linear stochastic systems assumes that, at any time, the state to be estimated is present in the observations. However, in many practical situations, such as in communication problems or images processing, there exists a positive probability (*false alarm probability*) that the observed data do not contain the signal to be estimated. These situations are described by systems whose observation equation include a multiplicative noise component, modeled by a sequence of Bernoulli random variables. These systems are known as *systems with uncertain observations.*

The state estimation problem in these systems, when the uncertainty in the observations is described by independent random variables, has been treated by several authors. For example, Nahi (1969), Hermoso and Linares (1994, 1995) treated such problem under a state-space model approach, and Nakamori (1997) addressed the problem assuming that only covariance information on the signal and noises in the observation equation is available. In all the cases it is assumed that the false alarm probability at any time is known. When it is not the case, estimates of it can be considered for adapting the corresponding estimation algorithms. So, the problem of estimating the false alarm probability in a system with uncertain observations can be interesting, not only itself (as, for example, in signal detection problems), but also in estimation of signals.

In this paper, by assuming a state-space model, we treat the false alarm probability estimation problem from the observations of the system. Concretely, we consider a discrete-time linear system in which the additive noises of the state and observation equation are correlated, and the uncertainty is modeled by a sequence of independent Bernoulli random variables; it is assumed that the probability that the observations contain the state process is unknown, but fixed throughout the time. In order to estimate this probability, we use a bayesian approach; specifically, our purpose is to obtain the Bayes estimators of that, assuming a quadratic loss function.

Due to an ever-increasing computational complexity as a result of the uncertainty in the observations, the Bayes estimator of the probability is unfeasible in practice. For this reason, it becomes necessary to find approximations which are viable from a computational point of view.

First, by using successive approximations of gaussian mixtures, we propose a method for the computation of the approximations for the posterior densities of the unknown probability, given the observations. These approximations have also a mixture form and, consequently, their computation involves an additional complexity, which, obviously, depends on the selected prior density. We treat the problem by considering a Beta as the prior distribution and, by means of the new approximations of mixture distributions, we obtain a recursive algorithm, which allows us to obtain estimations of the unknown probability with a great computational simplicity. For reference about of the approximations of mixture distributions and applications (in particular, applications to linear dynamic systems) see Titterington et al. (1985).

2 System model

Let us consider the following kind of discrete-time linear systems with uncertain observations

$$\begin{aligned} x_{k+1} &= A_k x_k + w_k, \quad k \geq 0 \\ z_k &= u_k C_k x_k + v_k, \quad k \geq 0 \end{aligned}$$

where x_k is the $n \times 1$ state vector and z_k is the $m \times 1$ observation vector at time k. The additive disturbances $\{w_k;\ k \geq 0\}$, $\{v_k;\ k \geq 0\}$ are n-dimensional and m-dimensional stochastic processes, respectively, and the multiplicative noise $\{u_k;\ k \geq 0\}$ is a scalar stochastic process which describes the presence or not of the state at the observations. Finally, A_k, C_k are known deterministic matrices of appropriate dimensions.

In the following, we suppose that all the random vectors which appear in the system are defined on the same probability space $(\Omega, \mathcal{A}, P)$.

We assume that the initial state, x_0, and the additive and multiplicative noises, $\{w_k;\ k \geq 0\}$, $\{v_k;\ k \geq 0\}$ and $\{u_k;\ k \geq 0\}$, satisfy the hypotheses which are detailed below.

H1. x_0 is a gaussian vector with zero mean and covariance matrix Σ_0.

H2. The noise process $\left\{ \left(w_k^T, v_k^T\right)^T;\ k \geq 0 \right\}$ is a gaussian white sequence with zero means and covariance matrices $\begin{pmatrix} Q_k & S_k \\ S_k^T & R_k \end{pmatrix}$.

H3. $\{u_k;\ k \geq 0\}$ is a sequence of independent Bernoulli random variables with $P\left[u_k = 1\right] = p$, for all $k \geq 0$.

H4. $\{u_k;\ k \geq 0\}$ is independent of $(x_0, \{w_k;\ k \geq 0\}, \{v_k;\ k \geq 0\})$.

H5. For all k, (w_k, v_k) is independent of $(x_0, w_0, \ldots, w_{k-1}, v_0, \ldots, v_{k-1})$.

3 Problem statement

In the above system, we suppose that the false alarm probability $1-p$, fixed throughout the time (H3), is unknown. So, the proposed problem is to provide a recursive algorithm for the estimation of the unknown parameter p. For this purpose, we assume a bayesian approach and, then, we treat the problem of finding the Bayes estimator of p, given a specific prior density and assuming a quadratic loss function; that is, our aim is to obtain $\overline{p}_k = E\left\{p/Z^k\right\}$, where $Z^k = \{z_0, \ldots, z_k\}$.

Clearly, to solve this problem we need to obtain the posterior density given the observations, for the selected prior density.

If we denote by $f\left(p/Z^{-1}\right)$ the prior density for p, and by $f\left(p/Z^k\right)$, $k \geq 0$, the posterior density given $\{z_0, \ldots, z_k\}$, from the Bayes theorem it follows that

$$f\left(p/Z^k\right) = \frac{f\left(z_k/p, Z^{k-1}\right) f\left(p/Z^{k-1}\right)}{\int f\left(z_k/p, Z^{k-1}\right) f\left(p/Z^{k-1}\right) dp}, \quad k \geq 0$$

where $f\left(z_k/p, Z^{k-1}\right) = pf_1\left(z_k/Z^{k-1}\right) + (1-p)f_0\left(z_k/Z^{k-1}\right)$ with $f_i\left(z_k/Z^{k-1}\right) = f\left(z_k/u_k = i, Z^{k-1}\right)$, $f_i\left(z_0/Z^{-1}\right) = f\left(z_0/u_0 = i\right)$, $i = 0, 1$.

From the expression of $f\left(z_k/p, Z^{k-1}\right)$, it is easy to prove that the denominator of the posterior density, density for z_k conditional on $\{z_0, \ldots, z_{k-1}\}$, is given by

$$f\left(z_k/Z^{k-1}\right) = \overline{p}_{k-1} f_1\left(z_k/Z^{k-1}\right) + (1 - \overline{p}_{k-1}) f_0\left(z_k/Z^{k-1}\right), \quad k \geq 0$$

and then,

$$f\left(p/Z^k\right) = \frac{\left[pf_1\left(z_k/Z^{k-1}\right) + (1-p)f_0\left(z_k/Z^{k-1}\right)\right] f\left(p/Z^{k-1}\right)}{\overline{p}_{k-1} f_1\left(z_k/Z^{k-1}\right) + (1 - \overline{p}_{k-1}) f_0\left(z_k/Z^{k-1}\right)}, \quad k \geq 0.$$

Hence, the recursive computation of the posterior density $f\left(p/Z^k\right)$ requires obtaining $f_i\left(z_k/Z^{k-1}\right)$, for $i = 0, 1$.

By reason of the hypotheses on the system, it is clear that $f_0\left(z_k/Z^{k-1}\right)$ is the density of the gaussian distribution $\mathcal{N}(0,\ R_k)$. However, the computation of $f_1\left(z_k/Z^{k-1}\right)$ grows in complexity as k increases, as a result of the uncertainty in the observations $\{z_0, \ldots, z_{k-1}\}$.

To solve this problem, we propose to seek approximations, $\widetilde{f}_1\left(z_k/Z^{k-1}\right)$, which avoid this ever-increasing computational complexity. These approximations will provide, in their turn, approximations for the posterior density, $\widetilde{f}\left(p/Z^k\right)$, and for the Bayes estimators of p, $\widetilde{p}_k = \int p\widetilde{f}\left(p/Z^k\right) dp$. This task will be carried out in the next section.

4 Estimators of the false alarm probability

The problem stated in the above section will be approached by approximating mixtures of gaussian distributions by gaussian distributions with the corresponding parameters. Next, we outline the procedure.

Estimator of p given Z^0

For $k=0$, it is clear that $f_1\left(z_0/Z^{-1}\right)$ is the density of the gaussian distribution $\mathcal{N}\left(0,\ C_0\Sigma_0C_0^T+R_0\right)$; this provides the density $f\left(p/Z^0\right)$ directly and, hence, $\overline{p}_0$ can be exactly calculated when the prior distribution is specified.

Estimator of p given Z^1

From the system equations, it is also clear that $f_1\left(z_1/Z^0\right)$ is determined by $f\left(x_0,w_0,z_0/Z^{-1}\right)$ and

$$f\left(x_0,w_0,z_0/Z^{-1}\right)=\overline{p}_{-1}f_1\left(x_0,w_0,z_0/Z^{-1}\right)+(1-\overline{p}_{-1})f_0\left(x_0,w_0,z_0/Z^{-1}\right)$$

where $f_i\left(x_0,w_0,z_0/Z^{-1}\right)=f\left(x_0,w_0,z_0/u_0=i\right)$, for $i=0,1,$ is the density of the gaussian distribution

$$\mathcal{N}\left(\left(\begin{array}{c}0\\0\\0\end{array}\right),\left(\begin{array}{ccc}\Sigma_0 & 0 & i\Sigma_0C_0^T\\ 0 & Q_0 & S_0\\ iC_0\Sigma_0 & S_0^T & iC_0\Sigma_0C_0^T+R_0\end{array}\right)\right).$$

Then, we approximate $f\left(x_0,w_0,z_0/Z^{-1}\right)$ (mixture of gaussian distributions) by a single gaussian distribution whose mean and covariance matrix are equal to those of the mixture, that is,

$$\mathcal{N}\left(\left(\begin{array}{c}0\\0\\0\end{array}\right),\left(\begin{array}{ccc}\Sigma_0 & 0 & \overline{p}_{-1}\Sigma_0C_0^T\\ 0 & Q_0 & S_0\\ \overline{p}_{-1}C_0\Sigma_0 & S_0^T & \Pi_0\end{array}\right)\right)$$

with $\Pi_0=\overline{p}_{-1}C_0\Sigma_0C_0^T+R_0$.

This approximated distribution provides the distribution of (x_0,w_0) given Z^0,

$$\mathcal{N}\left(\left(\begin{array}{c}K_0z_0\\ S_0\Pi_0^{-1}z_0\end{array}\right),\ \left(\begin{array}{cc}\Sigma_{0/0} & -K_0S_0^T\\ -S_0K_0^T & Q_0-S_0\Pi_0^{-1}S_0^T\end{array}\right)\right)$$

with $K_0=\overline{p}_{-1}\Sigma_0C_0^T\Pi_0^{-1}$ and $\Sigma_{0/0}=\Sigma_0-K_0\Pi_0K_0^T$.

From this distribution and using the system equations, we obtain that $\widetilde{f}_1\left(z_1/Z^0\right)$ is the density of the distribution $\mathcal{N}\left(C_1\widehat{x}_{1/0},\ C_1\Sigma_{1/0}C_1^T+R_1\right)$, where

$$\begin{aligned}\widehat{x}_{1/0}&=\left(A_0K_0+S_0\Pi_0^{-1}\right)z_0\\ \Sigma_{1/0}&=A_0\Sigma_{0/0}A_0^T+Q_0-S_0\Pi_0^{-1}S_0^T-A_0K_0S_0^T-S_0K_0^TA_0^T.\end{aligned}$$

Finally, the replacement of $\widetilde{f}_1\left(z_1/Z^0\right)$ in the expression of $f\left(p/Z^1\right)$, together with $f\left(p/Z^0\right)$, provides an approximation, $\widetilde{f}\left(p/Z^1\right)$, and, from it, we obtain $\widetilde{p}_1$.

In the following step, we start from the distribution for (x_1, w_1, z_1) conditional on Z^0, which, by reason of the above approximation is also approximated by

$$\widetilde{f}\left(x_1, w_1, z_1/Z^0\right) = \overline{p}_0 f_1\left(x_1, w_1, z_1/Z^0\right) + (1-\overline{p}_0) f_0\left(x_1, w_1, z_1/Z^0\right)$$

where $f_i\left(x_1, w_1, z_1/Z^0\right) = f\left(x_1, w_1, z_1/u_1 = i, Z^0\right)$, for $i = 0, 1$, is the density of the gaussian distribution

$$\mathcal{N}\left(\left(\begin{array}{c} \widehat{x}_{1/0} \\ 0 \\ iC_1\widehat{x}_{1/0} \end{array}\right), \left(\begin{array}{ccc} \Sigma_{1/0} & 0 & i\Sigma_{1/0}C_1^T \\ 0 & Q_1 & S_1 \\ iC_1\Sigma_{1/0} & S_1^T & iC_1\Sigma_{1/0}C_1^T + R_1 \end{array}\right)\right).$$

Hence, in this step, we also start from a distribution which is mixture of two gaussian distributions. This fact remains true in the posterior steps.

Estimator of p given Z^{k+1}

The proposed procedure provides a recursive method for obtaining the densities $\widetilde{f}_1\left(z_{k+1}/Z^k\right)$. In fact, let us assume that, for an arbitrary $k \geq 1$, $f\left(x_k, w_k, z_k/Z^{k-1}\right)$ is approximated by

$$\begin{aligned} \widetilde{f}\left(x_k, w_k, z_k/Z^{k-1}\right) &= \widetilde{p}_{k-1}\widetilde{f}_1\left(x_k, w_k, z_k/Z^{k-1}\right) \\ &\quad +(1-\widetilde{p}_{k-1})\widetilde{f}_0\left(x_k, w_k, z_k/Z^{k-1}\right) \end{aligned}$$

where $\widetilde{f}_i\left(x_k, w_k, z_k/Z^{k-1}\right)$, approximation of $f\left(x_k, w_k, z_k/u_k = i, Z^{k-1}\right)$ for $i = 0, 1$, is the density of the gaussian distribution

$$\mathcal{N}\left(\left(\begin{array}{c} \widehat{x}_{k/k-1} \\ 0 \\ iC_k\widehat{x}_{k/k-1} \end{array}\right), \left(\begin{array}{ccc} \Sigma_{k/k-1} & 0 & i\Sigma_{k/k-1}C_k^T \\ 0 & Q_k & S_k \\ iC_k\Sigma_{k/k-1} & S_k^T & iC_k\Sigma_{k/k-1}C_k^T + R_k \end{array}\right)\right).$$

As in the first step, we approximate this mixture by the density of the gaussian distribution

$$\mathcal{N}\left(\left(\begin{array}{c} \widehat{x}_{k/k-1} \\ 0 \\ \widetilde{p}_{k-1}C_k\widehat{x}_{k/k-1} \end{array}\right), \left(\begin{array}{ccc} \Sigma_{k/k-1} & 0 & \widetilde{p}_{k-1}\Sigma_{k/k-1}C_k^T \\ 0 & Q_k & S_k \\ \widetilde{p}_{k-1}C_k\Sigma_{k/k-1} & S_k^T & \Pi_k \end{array}\right)\right)$$

where $\Pi_k = \widetilde{p}_{k-1} C_k \Sigma_{k/k-1} C_k^T + R_k + \widetilde{p}_{k-1}(1-\widetilde{p}_{k-1}) C_k \widehat{x}_{k/k-1} \widehat{x}_{k/k-1}^T C_k^T$.

So, the distribution of (x_k, w_k) given Z^k is approximated by a single gaussian distribution

$$\mathcal{N}\left(\left(\begin{array}{c} \widehat{x}_{k/k-1} + K_k\left[z_k - \widetilde{p}_{k-1} C_k \widehat{x}_{k/k-1}\right] \\ S_k \Pi_k^{-1}\left[z_k - \widetilde{p}_{k-1} C_k \widehat{x}_{k/k-1}\right] \end{array}\right), \left(\begin{array}{cc} \Sigma_{k/k} & -K_k S_k^T \\ -S_k K_k^T & Q_k - S_k \Pi_k^{-1} S_k^T \end{array}\right)\right)$$

with $K_k = \widetilde{p}_{k-1} \Sigma_{k/k-1} C_k^T \Pi_k^{-1}$ and $\Sigma_{k/k} = \Sigma_{k/k-1} - K_k \Pi_k K_k^T$.

As a consequence, taking into account the system equations, the approximation $\widetilde{f}_1\left(z_{k+1}/Z^k\right)$ is the density of the gaussian distribution

$$\mathcal{N}(C_{k+1} \widehat{x}_{k+1/k}, C_{k+1} \Sigma_{k+1/k} C_{k+1}^T + R_{k+1}),$$

where

$$\begin{array}{rcl} \widehat{x}_{k+1/k} & = & A_k \widehat{x}_{k/k-1} + \left(A_k K_k + S_k \Pi_k^{-1}\right)\left[z_k - \widetilde{p}_{k-1} C_k \widehat{x}_{k/k-1}\right] \\ \Sigma_{k+1/k} & = & A_k \Sigma_{k/k} A_k^T + Q_k - S_k \Pi_k^{-1} S_k^T - A_k K_k S_k^T - S_k K_k^T A_k^T. \end{array}$$

The approximation $\widetilde{f}_1\left(z_{k+1}/Z^k\right)$ provides $\widetilde{f}\left(p/Z^{k+1}\right)$ and, from this, we obtain $\widetilde{p}_{k+1}$.

This procedure provides a method for the computation of $\widetilde{f}\left(p/Z^k\right)$ and the estimator $\widetilde{p}_k$. However, its application presents an additional difficulty, due to the fact that this posterior density also has a mixture form,

$$\widetilde{f}\left(p/Z^k\right) = \delta_k \frac{p\widetilde{f}\left(p/Z^{k-1}\right)}{\widetilde{p}_{k-1}} + (1-\delta_k)\frac{(1-p)\widetilde{f}\left(p/Z^{k-1}\right)}{1-\widetilde{p}_{k-1}}$$

with

$$\delta_k = \frac{\widetilde{p}_{k-1}\widetilde{f}_1\left(z_k/Z^{k-1}\right)}{\widetilde{p}_{k-1}\widetilde{f}_1\left(z_k/Z^{k-1}\right) + (1-\widetilde{p}_{k-1})\, f_0\left(z_k/Z^{k-1}\right)}$$

Obviously, the difficulty at the computation depends on the selected prior distribution. In the following section, by considering a *Beta* as the prior distribution, we propose a new approximation for the Bayes estimators of p.

5 Estimators approximation

Since p is the parameter of the Bernoulli random variables $\{u_k;\ k \geq 0\}$, let us specify a Beta as prior distribution of p. This is justified if we take into account that the Beta family is conjugate for the sampling of the Bernoulli distribution; so, if the variables u_k were observable and the estimation of p were made from them, with a $\beta(\alpha_0, \beta_0)$ as prior distribution, the Bayes estimator of p given $\{u_0, \ldots, u_k\}$, would be the mean value of the distribution $\beta\left(\alpha_0 + \sum_{i=0}^{k} u_i,\ \beta_0 + \sum_{i=0}^{k} (1 - u_i)\right)$.

In our case, if $f(p/Z^{-1}) \equiv \beta(\alpha_0, \beta_0)$, the prior estimator is $\overline{p}_{-1} = \alpha_0(\alpha_0 + \beta_0)^{-1}$, and the posterior density is given by the following mixture

$$f\left(p/Z^0\right) = \delta_0 \beta(\alpha_0 + 1, \beta_0) + (1 - \delta_0)\, \beta(\alpha_0, \beta_0 + 1)$$

where

$$\delta_0 = \frac{\overline{p}_{-1} f_1\left(z_0/Z^{-1}\right)}{\overline{p}_{-1} f_1\left(z_0/Z^{-1}\right) + (1 - \overline{p}_{-1}) f_0\left(z_0/Z^{-1}\right)}.$$

So, as a result of the mixture form of $f\left(p/Z^0\right)$, and since each posterior density is mixture of two distributions, the density $\widetilde{f}\left(p/Z^k\right)$ will be a mixture of 2^{k+1} distributions.

In order to avoid the computational complexity, we propose to approximate $f\left(p/Z^0\right)$ by $\widehat{f}\left(p/Z^0\right)$, the density of the distribution $\beta(\alpha_0 + \delta_0, \beta_0 + 1 - \delta_0)$. This approximation is justified by the argument explained at the beginning of this section, taking into account that $\delta_0 = E\{u_0/Z^0\}$ and, also, from the fact that the mean values of the approximation and the true distribution (which provides the Bayes estimator) are the same.

For the subsequent steps we proceed in an analogous way; so, if

$$\widehat{f}\left(p/Z^{k-1}\right) = \beta\left(\alpha_0 + \sum_{i=0}^{k-1} \widehat{\delta}_i,\ \beta_0 + \sum_{i=0}^{k-1} \left(1 - \widehat{\delta}_i\right)\right)$$

where

$$\widehat{\delta}_i = \frac{\widehat{p}_{i-1} \widehat{f}_1\left(z_i/Z^{i-1}\right)}{\widehat{p}_{i-1} \widehat{f}_1\left(z_i/Z^{i-1}\right) + (1 - \widehat{p}_{i-1})\, f_0\left(z_i/Z^{i-1}\right)}; \quad i = 1, \ldots, k-1$$

the posterior distribution of p given Z^k will be mixture of two Beta distributions, with mixture parameter $\widehat{\delta}_k$, and will be approximated by the distribution

$$\beta\left(\alpha_0 + \sum_{i=0}^{k} \widehat{\delta}_i,\ \beta_0 + \sum_{i=0}^{k} \left(1 - \widehat{\delta}_i\right)\right).$$

So, the estimator of p given the observations $\{z_0, \ldots, z_k\}$ will be

$$\widehat{p}_k = \frac{\alpha_0 + \sum_{i=0}^{k} \widehat{\delta}_i}{\alpha_0 + \beta_0 + k + 1}.$$

The main advantage of these approximations is that the estimators can be obtained by the following recursive relation

$$\begin{aligned} \widehat{p}_k &= \widehat{p}_{k-1} - \frac{1}{\alpha_0+\beta_0+k+1}\left[\widehat{p}_{k-1} - \widehat{\delta}_k\right], \quad k \geq 0 \\ \widehat{p}_{-1} &= \frac{\alpha_0}{\alpha_0+\beta_0} \end{aligned}$$

which provides an easy method to obtain estimations of p.

6 Numerical example

We consider the following scalar system

$$\begin{aligned} x_{k+1} &= 0.5x_k + w_k, \quad k \geq 0 \\ z_k &= u_k x_k + v_k, \quad k \geq 0 \end{aligned}$$

where the initial state, x_0, is a random variable with distribution $\mathcal{N}(0,1)$, and $\left\{(w_k, v_k)^T ;\ k \geq 0\right\}$ is a gaussian white sequence with zero means and covariance matrices

$$\begin{pmatrix} E\left\{w_k^2\right\} & E\left\{w_k v_k\right\} \\ E\left\{w_k v_k\right\} & E\left\{v_k^2\right\} \end{pmatrix} = \begin{pmatrix} 19/3 & -19/9 \\ -19/9 & 19/3 \end{pmatrix}, \ k \geq 0.$$

The multiplicative noise $\{u_k;\ k \geq 0\}$ is a sequence of independent Bernoulli variables with unknown parameter p.

In order to test the effectiveness of the proposed estimators, we have obtained numerical simulations for the observations of this system, considering different values of the parameter p; specifically, $p = 1/4$, $p = 1/2$ and $p = 3/4$.

In each case, we have performed one hundred iterations of the proposed algorithm by assuming that the prior distribution of the parameter p is a Beta distribution $\beta\left(\sqrt{1+\frac{19}{3}}, \sqrt{\frac{19}{3}}\right)$; these parameters specify the standard deviation of the first observation when the false alarm probability is zero and one, respectively, and their quotient is the signal plus noise-to-noise ratio at the first observation. So, we have chosen the parameters taking into account that $\widehat{p}_{-1} = \dfrac{\alpha_0}{\alpha_0+\beta_0}$ is an increasing function of $\dfrac{\alpha_0}{\beta_0}$.

The successive estimations of p, obtained by using the observations simulated with each value of the false alarm probability, are displayed in the below table and figures. A slow but clear decreasing and increasing tendency of the estimations can be noticed in the extreme cases, $p = 1/4$ and $p = 3/4$, respectively. In the case $p = 1/2$, we observe that the estimations are stabilized about the true parameter value. This is due to the fact that the prior estimation is very close to this value.

Table 1

Estimations of p with prior distribution $\beta\left(\sqrt{1+\frac{19}{3}},\sqrt{\frac{19}{3}}\right)$

$p = 1/4$	$p = 1/2$	$p = 3/4$
0.51831723324638	0.51831723324638	0.51831723324638
0.51537459841362	0.51849459535199	0.51582408954547
0.50697186992435	0.53232497295275	0.56614192451664
0.53281837909697	0.54677559593503	0.55750046166646
0.52162208776814	0.53640230540166	0.55627415406129
0.51137081848336	0.52918576968116	0.54618263087425
0.50444868431272	0.53038698985860	0.53977703131954
0.49589963533815	0.52631709855551	0.53251052778447
0.48969963701413	0.52529539431987	0.56591004795231
0.48241461397994	0.51813996350773	0.55850530838332
0.47650225536945	0.51126243890472	0.55781536025395
0.47060131946910	0.52560969952050	0.55215781576029
0.47261096162675	0.51951568027991	0.54965501359023
0.46707097406715	0.51447553089033	0.54847736443736
0.46858231671134	0.51043408111051	0.54300749627540
0.46477173705491	0.50526614838821	0.54353750695478
0.46188604562814	0.51574778478497	0.54132684275379
0.45768066890013	0.52114595059111	0.54086162768936
0.46373728255425	0.51677276228482	0.53987010004842
0.46368497744269	0.51427306402818	0.53886514789470
0.46005798351903	0.51347787014840	0.54126207262092
0.45723742102163	0.51231645636639	0.54217252676857
0.45419977525343	0.51331602702717	0.53975637818390
0.45264711984599	0.51006210059039	0.53604880055837
0.45113060932491	0.50947591275222	0.53351328969618
0.45060013698069	0.50764917678509	0.53182275846705
0.45106816617602	0.50552212987869	0.52914488502297
0.45629627408432	0.51717417379169	0.52766099271959
0.45323368663999	0.51878900144625	0.53533390382609
0.45136563244343	0.51573184561503	0.53259902687039
0.45076538643869	0.51281811227351	0.53105678130386

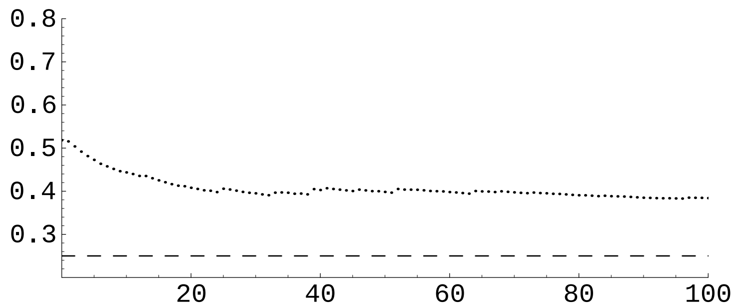

Figure 1
Estimations of $p = 0.25$

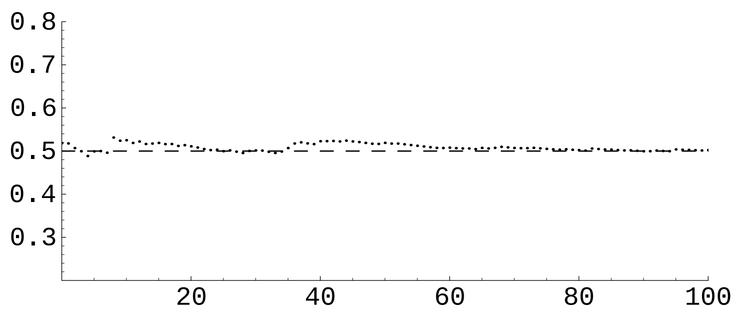

Figure 2
Estimations of $p = 0.5$

7 Conclusions

In this paper, we consider a system with uncertain observations in which the additive noises are correlated and the false alarm probability is unknown. We propose a recursive estimation algorithm for that probability, based on the successive observations of the system. For our purpose, we use a bayesian approach, by specifying a Beta prior distribution for the unknown parameter and considering a quadratic loss function. By means of successive approximations of mixture distributions, we obtain a recursive algorithm which provides approximations for the Bayes estimators of the

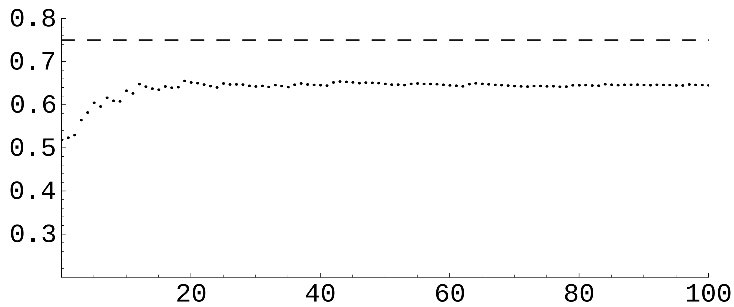

Figure 3
Estimations of $p = 0.75$

parameter. These estimators can be used for adapting the state estimation algorithms.

Acknowledgments

This work has been partially supported by the "Ministerio de Ciencia y Tecnología" under contract BFM2002-00932.

(Received May, 2002. Revised December, 2002.)

References

Hermoso, A. and Linares, J. (1994). Linear estimation for discrete-time systems in the presence of time-correlated disturbances and uncertain observations. *IEEE Transactions on Automatic Control*, **AC-39**, 1636–1638.

Hermoso, A. and Linares, J. (1995). Linear smoothing for discrete-time systems in the presence of correlated disturbances and uncertain observations. *IEEE Transactions on Automatic Control*, **AC-40**, 1486–1488.

Nahi, N. E. (1969). Optimal recursive estimation with uncertain observation. *IEEE Transactions on Information Theory*, **IT-15**, 457–462.

Nakamori, S. (1997). Estimation technique using covariance information with uncertain observations in linear discrete-time systems. *Signal Processing*, **58**, 309–317.

Titterington, D.M., Smith, A.F.M. and Makov, U.M. (1985), *Statistical Analysis of Finite Mixture Distributions*. New York: John Wiley and Sons.