

基于输入输出线性化方法的一类非线性 系统观测器设计*

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摘要 研究一类非线性系统的观测器设计问题. 基于输入输出线性化方法提出了一类非线性系统的观测器设计. 并且此非线性系统具有多输入多输出的特点, 证明了在适当条件下, 提出的观测器保证了观测误差渐近趋于零. 仿真例表明了所得结果的有效性.

关键词 非线性系统, 线性化, 状态观测器, Lyapunov 函数, 坐标变换.

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1 引言

近年来, 非线性系统观测器设计是非常活跃的研究领域. 一些观测器存在性的充分条件已经建立起来, 观测器设计的一些构造方法也已经提出. 文献 [1] 首先提出了在李代数条件下带可线性化误差动态的非线性观测器设计. 文献 [2] 引入了一个非线性观测器的标准形式, 在这个标准形式下, 系统的非线性部分仅仅与原系统输入输出有关. 文献 [3] 推广这类非线性系统, 提出了一种观测器设计, 这种设计基于变换到推广的观测器标准形式, 这种形式依赖于输入变量的 n 阶导数. 文献 [4] 对非线性观测器的存在性提出了充分的类 Lyapunov 条件, 并且证明了这个条件等价于线性情况的可检测性条件. 但是一般而言, Lyapunov 函数的构造相当困难. 文献 [5] 对在某确定函数的全局 Hölder 条件下的全相对阶的非线性系统, 推广了类 Luengerger 观测器. 文献 [6] 提出了在 Lipschitz 连续条件下, 当可观性映射是半微分同胚时的观测器设计方法. 文献 [7] 推广了文献 [6] 的结果, 提出了非 Lipschitz 可观性型的观测器设计方法. 文献 [8] 讨论了依赖于状态变换和外数值微分法的观测器设计. 文献 [9] 提出了一类带误差动态可线性化的多输入多输出系统的非线性标准型观测器设计方法. 文献 [10] 讨论了用动态观测器误差线性化方法来设计观测器.

本文提出了一类非线性系统观测器设计方法, 这个方法是基于输入输出线性化方法, 利用坐标变换将系统状态变换为标准形式来研究, 文中证明了提出的观测器在适当条件下保证了观测误差渐近趋于零.

在下面的讨论中一些符号说明如下: A 是 Hurwitz 矩阵是指其所有特征值 λ 都有 $\text{Re}(\lambda) < 0$; 对任何一个整数 r, I_r 表示 $r \times r$ 阶单位阵; $\|\cdot\|$ 代表在欧几里得空间中向

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量的欧几里得范数; $D_1f(x_1, x_2, x_3), D_2f(x_1, x_2, x_3), D_3f(x_1, x_2, x_3)$ 分别表示连续可微函数 $f(x_1, x_2, x_3)$ 关于第 1 个, 第 2 个, 第 3 个自变量的 Jacobian 矩阵.

2 问题描述

考虑如下非线性系统

$$\begin{cases} \dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2, \\ y_1 = h_1(x), y_2 = h_2(x), \end{cases} \quad (1)$$

其中 $x \in R^n$ 为系统状态, $u_i \in R, y_i \in R (i = 1, 2)$ 分别为系统的输入输出, $f(x)$ 和 $g_i(x) (i = 1, 2)$ 为 R^n 中的光滑向量场, $h_i(x) (i = 1, 2)$ 为光滑函数. 设 $f(x^0) = 0, h_i(x^0) = 0$, 即 x^0 是非强迫系统的平衡点.

首先给出下列假设.

假设 1 系统 (1) 可以通过坐标变换 $[Z_1^T, Z_2^T, \eta^T]^T = \Phi(x)$ 化成如下标准型.

$$\begin{aligned} \dot{z}_j^i &= z_{j+1}^i, \quad j = 1, 2, \dots, r_i - 1, \\ \dot{z}_{r_i}^i &= a_i(Z_1, Z_2, \eta) + b_{i1}(Z_1, Z_2, \eta)u_1 + b_{i2}(Z_1, Z_2, \eta)u_2, \\ \dot{\eta} &= q(Z_1, Z_2, \eta), \\ y_i &= z_1^i, \quad i = 1, 2, \end{aligned} \quad (2)$$

其中 $Z^i = [z_1^i, z_2^i, \dots, z_{r_i}^i]^T, \eta = [\eta_1, \eta_2, \dots, \eta_{m-r}]^T, r = r_1 + r_2, i = 1, 2$.

注 1 式 (2) 可写成如下的矩阵形式

$$\begin{aligned} \dot{Z}_i &= A_{r_i}Z_i + B_{r_i}[a_i(Z_1, Z_2, \eta) + b_{i1}(Z_1, Z_2, \eta)u_1 + b_{i2}(Z_1, Z_2, \eta)u_2], \\ \dot{\eta} &= q(Z_1, Z_2, \eta), \\ y_i &= C_iZ_i, \quad i = 1, 2, \end{aligned} \quad (3)$$

其中 $A_{r_i} \in R^{r_i \times r_i}, B_{r_i} \in R^{r_i \times 1}, C_i \in R^{1 \times r_i}$,

$$A_{r_i} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B_{r_i} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \quad C_i = [1 \quad 0 \quad \cdots \quad 0], \quad i = 1, 2.$$

假设 2 函数 $a_i(Z_1, Z_2, \eta), b_{ik}(Z_1, Z_2, \eta), i, k = 1, 2$, 是全局 Lipschitz 的.

注 2 如果假设 1 成立, 且 u_1, u_2 是有界的, 则存在正常数 $l_1, l_2, l_3, l_4, l_5, l_6$ 使得

$$\begin{aligned} & \left\| a_1(\bar{Z}_1, \bar{Z}_2, \bar{\eta}) + \sum_{k=1}^2 b_{1k}(\bar{Z}_1, \bar{Z}_2, \bar{\eta})u_k - a_1(Z_1, Z_2, \eta) - \sum_{k=1}^2 b_{1k}(Z_1, Z_2, \eta)u_k \right\| \\ & \leq l_1 \|\bar{Z}_1 - Z_1\| + l_2 \|\bar{Z}_2 - Z_2\| + l_3 \|\bar{\eta} - \eta\|, \\ & \left\| a_2(\bar{Z}_1, \bar{Z}_2, \bar{\eta}) + \sum_{k=1}^2 b_{2k}(\bar{Z}_1, \bar{Z}_2, \bar{\eta})u_k - a_2(Z_1, Z_2, \eta) - \sum_{k=1}^2 b_{2k}(Z_1, Z_2, \eta)u_k \right\| \\ & \leq l_4 \|\bar{Z}_1 - Z_1\| + l_5 \|\bar{Z}_2 - Z_2\| + l_6 \|\bar{\eta} - \eta\|, \quad \forall \bar{Z}_1, Z_1 \in R^{r_1}, \bar{Z}_2, Z_2 \in R^{r_2}, \eta \in R^{n-r}. \end{aligned} \quad (4)$$

假设 3 $q(Z_1, Z_2, \eta)$ 关于 Z_1, Z_2 是全局 Lipschitz 的, 即存在正常数 l_7, l_8 使得

$$\begin{aligned} & \|q(\bar{Z}_1, Z_2, \eta) - q(Z_1, Z_2, \eta)\| \\ & \leq l_7 \|\bar{Z}_1 - Z_1\|, \\ & |q(\bar{Z}_1, \bar{Z}_2, \eta) - q(\bar{Z}_1, Z_2, \eta)| \\ & \leq l_8 \|\bar{Z}_2 - Z_2\|, \quad \forall \bar{Z}_1, Z_1 \in R^{r_1}, \bar{Z}_2, Z_2 \in R^{r_2}, \eta \in R^{n-r}. \end{aligned} \quad (5)$$

假设 4 存在一个正定矩阵 $P_3 \in R^{(n-r) \times (n-r)}$ 和一个正常数 k_0 使得

$$v^T P_3 \{D_3 q(Z_1, Z_2, \eta)\} v \leq -\frac{1}{2} k_0 \|v\|^2, \quad \forall Z_1 \in R^{r_1}, Z_2 \in R^{r_2}, \eta \in R^{n-r}, v \in R^{n-r}. \quad (6)$$

本文的目的是对于系统 (1) 设计观测器, 使观测误差渐近趋于零.

3 观测器设计

考虑如下非线性观测器

$$\dot{\bar{x}} = f(\bar{x}) + \sum_{k=1}^2 g_k(\bar{x}) u_k + \left[\frac{\partial \Phi(\bar{x})}{\partial \bar{x}} \right]^{-1} \begin{bmatrix} K_1(\varepsilon) & 0_{r_1} \\ 0_{r_2} & K_2(\varepsilon) \\ 0_{n-r} & 0_{n-r} \end{bmatrix} \begin{bmatrix} y_1 - h_1(\bar{x}) \\ y_2 - h_2(\bar{x}) \end{bmatrix}, \quad (7)$$

其中

$$K_i(\varepsilon) = \begin{bmatrix} \frac{k_{1i}}{\varepsilon} & \frac{k_{2i}}{\varepsilon^2} & \cdots & \frac{k_{r_i i}}{\varepsilon^{r_i}} \end{bmatrix}^T, \quad 0_{r_i} = [0 \ 0 \ \cdots \ 0]_{1 \times r_i}^T, \quad i = 1, 2.$$

$k_{1i}, k_{2i}, \dots, k_{r_i i}$ 使得 $s^{r_i} + k_{1i}s^{r_i-1} + \cdots + k_{r_i i} = 0$, $i = 1, 2$, 是 Hurwitz 的. 令 $P_i, i = 1, 2$ 是下列矩阵方程

$$A_i^T P_i + P_i A_i = -I_{r_i} \quad (8)$$

的解, 其中

$$A_i = \begin{bmatrix} -k_{1i} & 1 & 0 & \cdots & 0 \\ -k_{2i} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_{r_i i} & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

令 x_0 和 \bar{x}_0 分别是 (1) 和 (7) 的初始状态, 且 $e_0 = \bar{x}_0 - x_0$. 下面陈述主要结果.

定理 1 考虑非线性系统 (1) 和观测器 (7), 如果假设 1-4 都满足, 并且输入 $u_j, j = 1, 2$ 是有界的, 则对任意初始条件 e_0, x_0 , 当 $0 < \varepsilon < \varepsilon^*$ 时, 有 $\lim_{t \rightarrow \infty} \|\bar{x}(t) - x(t)\| = 0$, 其中

$$\varepsilon^* \leq \min \left(\frac{1}{4l_1 \|P_1\|}, \frac{1}{4l_5 \|P_2\|}, \frac{k_0}{32(l_3 \|P_1\| + l_7 \|P_3\|)^2}, \frac{3k_0}{64(l_6 \|P_2\| + l_8 \|P_3\|)^2}, \frac{\sqrt{3}}{8(l_2 \|P_1\| + l_4 \|P_2\|)} \right).$$

证 在整个证明的过程中都假设 $0 < \varepsilon < 1$. 在新的坐标系 $[Z_1^T, Z_2^T, \eta^T]^T = \Phi(x)$, 观测器 (7) 可以变换为

$$\begin{aligned} \dot{\bar{Z}}_i &= A_{r_i} \bar{Z}_i + B_{r_i} \left[a_i(\bar{Z}_1, \bar{Z}_2, \bar{\eta}) + \sum_{k=1}^2 b_{ik}(\bar{Z}_1, \bar{Z}_2, \bar{\eta}) u_k \right] + K_i(\varepsilon)(y_i - C_i \bar{Z}_i), \\ \dot{\bar{\eta}} &= q(\bar{Z}, \bar{\eta}), \quad i = 1, 2. \end{aligned} \quad (9)$$

定义观测值误差为 $e_1 = \bar{Z}_1 - Z_1, e_2 = \bar{Z}_2 - Z_2, e_3 = \bar{\eta} - \eta$. 则由 (3) 和 (9) 可以得到

$$\begin{aligned} \dot{e}_1 &= A_1(\varepsilon)e_1 + B_{r_1} \left[a_1(\bar{Z}_1, \bar{Z}_2, \bar{\eta}) + \sum_{k=1}^2 b_{1k}(\bar{Z}_1, \bar{Z}_2, \bar{\eta})u_k - a_1(Z_1, Z_2, \eta) \right. \\ &\quad \left. - \sum_{k=1}^2 b_{1k}(Z_1, Z_2, \eta)u_k \right], \\ \dot{e}_2 &= A_2(\varepsilon)e_2 + B_{r_2} \left[a_2(\bar{Z}_1, \bar{Z}_2, \bar{\eta}) + \sum_{k=1}^2 b_{2k}(\bar{Z}_1, \bar{Z}_2, \bar{\eta})u_k - a_2(Z_1, Z_2, \eta) \right. \\ &\quad \left. - \sum_{k=1}^2 b_{2k}(Z_1, Z_2, \eta)u_k \right], \\ \dot{e}_3 &= q(\bar{Z}_1, \bar{Z}_2, \bar{\eta}) - q(\bar{Z}_1, \bar{Z}_2, \eta) + q(\bar{Z}_1, \bar{Z}_2, \eta) - q(\bar{Z}_1, Z_2, \eta) \\ &\quad + q(\bar{Z}_1, Z_2, \eta) - q(Z_1, Z_2, \eta), \end{aligned} \quad (10)$$

其中

$$A_i(\varepsilon) = A_{r_i} - K_i(\varepsilon)C_i = \begin{bmatrix} -\frac{k_{1i}}{\varepsilon} & 1 & 0 & \cdots & 0 \\ -\frac{k_{2i}}{\varepsilon^2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{k_{r_i i}}{\varepsilon^{r_i}} & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad i = 1, 2.$$

定义

$$H_i(\varepsilon) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \varepsilon & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \varepsilon^{r_i-1} \end{bmatrix}, \quad i = 1, 2,$$

显然有 $A_i(\varepsilon) = \varepsilon^{-1}H_i^{-1}(\varepsilon)A_iH_i(\varepsilon)$, $i = 1, 2$. 从 (8) 式可以得出

$$P_i(\varepsilon) = H_i^T(\varepsilon)P_iH_i(\varepsilon), \quad i = 1, 2, \quad (11)$$

满足

$$A_i^T(\varepsilon)P_i(\varepsilon) + P_i(\varepsilon)A_i(\varepsilon) = -\varepsilon^{-1}H_i^T(\varepsilon)H_i(\varepsilon), \quad i = 1, 2. \quad (12)$$

选取 Lyapunov 函数为 $V(e_1, e_2, e_3) = \sum_{i=1}^2 \varepsilon^{(2-2r_i)} e_i^T P_i(\varepsilon) e_i + e_3^T P_3 e_3$, 由 (10) 式、(11) 式和 (12) 式可以得到

$$\begin{aligned} \dot{V} &= - \sum_{i=1}^2 \varepsilon^{(1-2r_i)} e_i^T H_i^T(\varepsilon) H_i(\varepsilon) e_i + 2 \sum_{i=1}^2 \varepsilon^{(2-2r_i)} B_{r_i}^T H_i^T(\varepsilon) P_i H_i(\varepsilon) e_i \\ &\quad \cdot \left[a_i(\bar{Z}_1, \bar{Z}_2, \bar{\eta}) + \sum_{j=1}^2 b_{ij}(\bar{Z}_1, \bar{Z}_2, \bar{\eta})u_j - a_i(Z_1, Z_2, \eta) - \sum_{k=1}^2 b_{ik}(Z_1, Z_2, \eta)u_k \right] \\ &\quad + \dot{e}_3^T P_3 e_3 + e_3^T P_3 \dot{e}_3. \end{aligned}$$

由 (6) 式和 (10) 式可得

$$\begin{aligned} & \dot{e}_3^T P_3 e_3 + e_3^T P_3 \dot{e}_3 \\ & \leq -k_0 \|e_3\|^2 + 2[q(\bar{Z}_1, \bar{Z}_2, \eta) - q(\bar{Z}_1, Z_2, \eta) + q(\bar{Z}_1, Z_2, \eta) - q(Z_1, Z_2, \eta)]^T P_3 e_3, \end{aligned} \quad (13)$$

令 $\xi_i = H_i(\varepsilon)e_i$, $i = 1, 2$, 因为 $\|B_{r_i}^T H_i(\varepsilon)\| = \varepsilon^{r_i-1}$, $i = 1, 2$, 由 (4) 式, (5) 式和 (13) 式可得

$$\begin{aligned} \dot{V} & \leq - \sum_{i=1}^2 \varepsilon^{(1-r_i)} \|\xi_i\|^2 + 2\varepsilon^{(1-r_1)} l_1 \|P_1\| \|\xi_1\| \|e_1\| + 2\varepsilon^{(1-r_1)} l_2 \|P_1\| \|\xi_1\| \|e_2\| \\ & \quad + 2\varepsilon^{(1-r_1)} l_3 \|P_1\| \|\xi_1\| \|e_3\| + 2\varepsilon^{(1-r_2)} l_4 \|P_2\| \|\xi_2\| \|e_1\| + 2\varepsilon^{(1-r_2)} l_5 \|P_2\| \|\xi_2\| \|e_2\| \\ & \quad + 2\varepsilon^{(1-r_2)} l_6 \|P_2\| \|\xi_2\| \|e_3\| - k_0 \|e_3\|^2 + 2l_7 \|e_1\| \|P_3\| \|e_3\| + 2l_8 \|e_2\| \|P_3\| \|e_3\|, \end{aligned}$$

又因为 $\|H_i^{-1}(\varepsilon)\| = \varepsilon^{(1-r_i)}$, $i = 1, 2$, 所以 $\|e_i\| \leq \varepsilon^{(1-r_i)} \|\xi_i\|$, $i = 1, 2$, 则有

$$\begin{aligned} \dot{V} & \leq - \sum_{i=1}^2 \varepsilon^{(1-2r_i)} \|\xi_i\|^2 + 2\varepsilon^{(2-2r_1)} l_1 \|P_1\| \|\xi_1\|^2 + 2\varepsilon^{(2-r_1-r_2)} l_2 \|P_1\| \|\xi_1\| \|\xi_2\| - k_0 \|e_3\|^2 \\ & \quad + 2\varepsilon^{(1-r_1)} l_3 \|P_1\| \|\xi_1\| \|e_3\| + 2\varepsilon^{(2-2r_2)} l_5 \|P_2\| \|\xi_2\|^2 + 2\varepsilon^{(2-r_1-r_2)} l_4 \|P_2\| \|\xi_1\| \|\xi_2\| \\ & \quad + 2\varepsilon^{(1-r_2)} l_6 \|P_2\| \|\xi_2\| \|e_3\| + 2\varepsilon^{(1-r_1)} l_7 \|P_3\| \|\xi_1\| \|e_3\| + 2\varepsilon^{(1-r_2)} l_8 \|P_3\| \|\xi_2\| \|e_3\|. \end{aligned}$$

因为 $\varepsilon \leq \min(\frac{1}{4l_1 \|P_1\|}, \frac{1}{4l_5 \|P_2\|})$, 所以

$$\begin{aligned} \dot{V} & \leq -k_0 \left[\frac{1}{2} \|e_3\| - \frac{2}{k_0} \varepsilon^{(1-r_2)} (l_3 \|P_1\| + l_7 \|P_3\|) \|\xi_1\| \right]^2 + \frac{4}{k_0} \varepsilon^{(2-2r_1)} (l_3 \|P_1\| + l_7 \|P_3\|)^2 \|\xi_1\|^2 \\ & \quad - \frac{1}{2} k_0 \|e_3\|^2 - \frac{1}{2} \varepsilon^{(1-2r_1)} \|\xi_1\|^2 - k_0 \left[\frac{1}{2} \|e_3\| - \frac{2}{k_0} \varepsilon^{(1-r_2)} (l_6 \|P_2\| + l_8 \|P_3\|) \|\xi_2\| \right]^2 \\ & \quad - \frac{1}{2} \varepsilon^{(1-2r_2)} \|\xi_2\|^2 + \frac{4}{k_0} \varepsilon^{(2-2r_2)} (l_6 \|P_2\| + l_8 \|P_3\|)^2 \|\xi_2\|^2 \\ & \quad + 2\varepsilon^{(2-r_1-r_2)} (l_2 \|P_1\| + l_4 \|P_2\|) \|\xi_1\| \|\xi_2\| \\ & \leq -\frac{1}{2} k_0 \|e_3\|^2 - \frac{1}{4} \varepsilon^{(1-2r_1)} \|\xi_1\|^2 - \varepsilon^{(1-2r_1)} \left[\frac{1}{2} \|\xi_1\| - 2\varepsilon^{(1-r_2+r_1)} (l_2 \|P_1\| + l_4 \|P_2\|) \|\xi_2\| \right]^2 \\ & \quad + 4\varepsilon^{(3-2r_2)} (l_2 \|P_1\| + l_4 \|P_2\|)^2 \|\xi_2\|^2 - \frac{1}{2} \varepsilon^{(1-2r_2)} \|\xi_2\|^2 \\ & \quad + \frac{4}{k_0} \varepsilon^{(2-2r_1)} (l_3 \|P_1\| + l_7 \|P_3\|)^2 \|\xi_1\|^2 + \frac{4}{k_0} \varepsilon^{(2-2r_2)} (l_6 \|P_2\| + l_8 \|P_3\|)^2 \|\xi_2\|^2 \\ & \leq -\frac{1}{2} k_0 \|e_3\|^2 - \left[\frac{1}{4} \varepsilon^{(1-2r_1)} - \frac{4}{k_0} \varepsilon^{(2-2r_1)} (l_3 \|P_1\| + l_7 \|P_3\|)^2 \right] \|\xi_1\|^2 \\ & \quad - \left[\frac{1}{2} \varepsilon^{(1-2r_2)} - \frac{4}{k_0} \varepsilon^{(1-2r_2)} (l_6 \|P_2\| + l_8 \|P_3\|)^2 - 4\varepsilon^{(3-2r_2)} (l_2 \|P_1\| + l_4 \|P_2\|)^2 \right] \|\xi_2\|^2. \end{aligned}$$

又因为 $\varepsilon \leq \min(\frac{k_0}{32(l_3 \|P_1\| + l_7 \|P_3\|)^2}, \frac{3k_0}{64(l_6 \|P_2\| + l_8 \|P_3\|)^2}, \frac{\sqrt{3}}{8(l_2 \|P_1\| + l_4 \|P_2\|)})$, 所以

$$\dot{V} \leq -\frac{1}{2} k_0 \|e_3\|^2 - \frac{1}{8} \varepsilon^{(1-2r_1)} \|\xi_1\|^2 - \frac{1}{8} \varepsilon^{(1-2r_2)} \|\xi_2\|^2.$$

证毕.

进一步做出如下假设.

假设 5 假设存在正常数 $l_9, l_{10}, l_{11} (< \frac{1}{2\|P_4\|})$, 使得 $R(Z_1, Z_2, \eta) = q(Z_1, Z_2, \eta) - A_3\eta$ 满足 $\|R(\bar{Z}_1, \bar{Z}_2, \bar{\eta}) - R(Z_1, Z_2, \eta)\| \leq l_9\|\bar{Z}_1 - Z_1\| + l_{10}\|\bar{Z}_2 - Z_2\| + l_{11}\|\bar{\eta} - \eta\|$, 其中 A_3 是任意一个 Hurwitz 矩阵, P_4 是 $A_3^T P_4 + P_4 A_3 = -I$ 的解.

定理 2 考虑系统 (1) 和观测器 (7), 如果假设 1, 假设 2, 假设 5 成立, 并且输入 u_1, u_2 是有界的, 那么存在正常数 k'_0 , 使得对任意初始状态 e_0, x_0 , 当 $0 < \varepsilon < \varepsilon^+$ 时, 有 $\lim_{t \rightarrow \infty} \|\bar{x}(t) - x(t)\| = 0$, 其中

$$\varepsilon^+ \leq \min \left(\frac{1}{4l_1\|P_1\|}, \frac{1}{4l_5\|P_2\|}, \frac{k'_0}{32(l_3\|P_1\| + l_9\|P_4\|)^2}, \frac{3k'_0}{64(l_6\|P_2\| + l_{10}\|P_4\|)^2}, \frac{\sqrt{3}}{8(l_2\|P_1\| + l_4\|P_2\|)} \right).$$

证 定义 Lyapunov 函数 $V(e_1, e_2, e_3) = \sum_{i=1}^2 \varepsilon^{(2-2r_i)} e_i^T P_i(\varepsilon) e_i + e_3^T P_4 e_3$. 可得

$$\begin{aligned} \dot{e}_3^T P_4 e_3 + e_3^T P_4 \dot{e}_3 &= 2e_3^T P_4 (R(\bar{Z}_1, \bar{Z}_2, \bar{\eta}) - R(Z_1, Z_2, \eta)) + e_3^T (A_3^T P_4 + P_4 A_3) e_3 \\ &\leq (2l_{11}\|P_4\| - 1)\|e_3\|^2 + 2l_9\|e_3\|\|P_4\|\|e_1\| + 2l_{10}\|e_3\|\|P_4\|\|e_2\|. \end{aligned}$$

因为 $l_{11} < \frac{1}{2\|P_4\|}$, 所以 $2l_{11}\|P_4\| - 1 < 0$, 取 $k'_0 = -(2l_{11}\|P_4\| - 1)$, 类似于定理 1 的证明, 当 $0 < \varepsilon < \varepsilon^+$ 时, 有

$$\dot{V} \leq -\frac{1}{2}k'_0\|e_3\|^2 - \frac{1}{8}\varepsilon^{(1-2r_1)}\|\xi_1\|^2 - \frac{1}{8}\varepsilon^{(1-2r_2)}\|\xi_2\|^2.$$

证毕.

4 举 例

本节通过一个具体的仿真例子来说明本文所提出的设计方法的正确性和可行性. 考虑如下非线性系统

$$\begin{cases} \dot{x} = \begin{bmatrix} 0.01 \sin x_1 + x_2 + 0.5x_3 \\ 0.02 \sin 2x_2 + x_4 \\ -1.5x_1 - 0.01 \sin 2x_3 \\ -\sin x_4 - 2x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u_2, \\ y_1 = h_1(x) = x_1 + x_2, \quad y_2 = h_2(x) = x_2 + 2x_3, \end{cases} \quad (14)$$

极易验证系统 (14) 的相对阶 $r = \{r_1, r_2\} = \{1, 1\}$, 分布 $G = \text{span}\{g_1, g_2\}$ 是对合的. 选择坐标变换 $[z_1 \ z_2 \ \eta_1 \ \eta_2]^T = \Phi(x) = [x_1 + x_2 \ x_2 + 2x_3 \ x_3 \ x_4]^T$, 系统 (14) 可以变换为如下标准型

$$\begin{aligned} \dot{z}_1 &= 0.01 \sin(z_1 - z_2 + 2\eta_1) + z_2 - 1.5\eta_1 + 0.02 \sin(z_2 - 2\eta_1) + \eta_2 + u_1, \\ \dot{z}_2 &= 0.02 \sin 2(z_2 - 2\eta_1) + \eta_2 - 3z_1 + 3z_2 - 6\eta_1 - 0.02 \sin 2\eta_1 + u_1 + u_2, \\ \dot{\eta}_1 &= -1.5z_1 + 1.5z_2 - 3\eta_1 - 0.01 \sin 2\eta_1, \\ \dot{\eta}_2 &= -\sin \eta_2 - 2\eta_2. \end{aligned}$$

可以验证系统 (14) 满足假设 1-4, 应用 (7), 状态观测器为

$$\dot{\bar{x}} = \begin{bmatrix} 0.01 \sin \bar{x}_1 + \bar{x}_2 + 0.5\bar{x}_3 \\ 0.02 \sin 2\bar{x}_2 + \bar{x}_4 \\ -1.5\bar{x}_1 - 0.01 \sin 2\bar{x}_3 \\ -\sin \bar{x}_4 - 2\bar{x}_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u_2 + \begin{bmatrix} \frac{k_{11}}{\varepsilon} & \frac{k_{12}}{\varepsilon} \\ \varepsilon & \varepsilon \\ 0 & \frac{k_{12}}{\varepsilon} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 - \bar{x}_1 - \bar{x}_2 \\ y_2 - \bar{x}_2 - 2\bar{x}_3 \end{bmatrix}.$$

取 $l_1 = 2, l_2 = 5, l_3 = 8, l_4 = 4, l_5 = 6, l_6 = 15, l_7 = 2, l_8 = 2, k_0 = 2, k_{11} = 1, k_{12} = 2, \varepsilon = 0.001, P_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, 所以 $P_1 = 0.5, P_2 = 0.25, \|P_1\| = 0.5, \|P_2\| = 0.25, \|P_3\| = \sqrt{2}$. 取输入 $u_1 = 10, u_2 = -100$, 初始值 $x_0 = [10 \ 60 \ 80 \ 20.5]^T, \bar{x}_0 = [10 \ 70 \ 89 \ 20]^T, e_0 = [0 \ 10 \ 9 \ 0.5]^T$. 仿真结果如图 1-4 所示. 从图可见, 估计误差渐近趋于零 ($t \rightarrow \infty$).

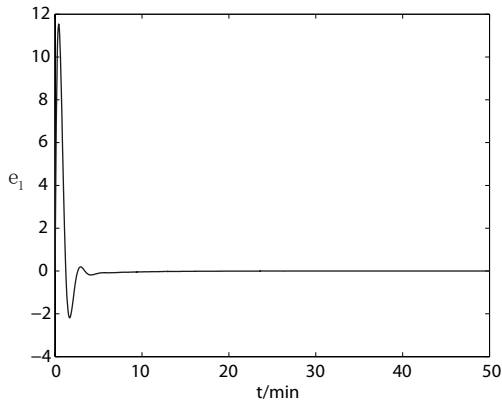


图 1 误差 $e_1 = \bar{x}_1 - x_1$ 的轨线

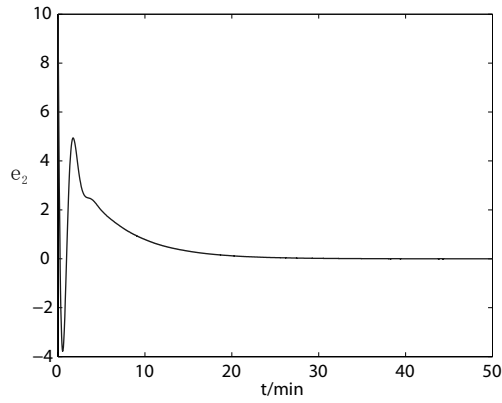


图 2 误差 $e_2 = \bar{x}_2 - x_2$ 的轨线

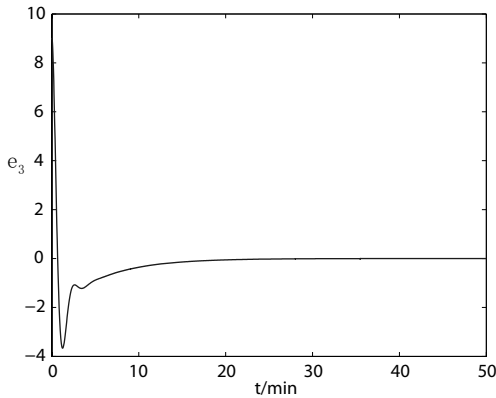


图 3 误差 $e_3 = \bar{x}_3 - x_3$ 的轨线

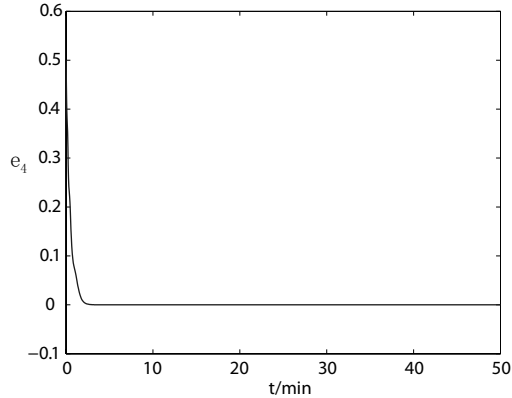


图 4 误差 $e_4 = \bar{x}_4 - x_4$ 的轨线

5 结 论

本文研究了一类非线性系统观测器设计问题, 结合输入输出线性化提出了一类非线性系统的观测器设计方法. 将文献 [5] 所讨论的单输入单输出非线性系统推广到一类多输入多输出非线性系统, 并给出了保证状态估计误差渐近趋于零的充分条件. 最后, 仿真实例验证了所得结果的有效性.

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OBSERVER DESIGN FOR A CLASS OF NONLINEAR SYSTEM BASED ON INPUT OUTPUT LINEARIZATION APPROACH

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Abstract The problem of observer design for a class of nonlinear system is considered. Based on input-output linearization approach, the observer design for the nonlinear system is developed. The nonlinear system presented in this paper is of characteristic of multi-input multi-output. Under appropriate conditions, it is proved that the proposed observer assured that the observation error converges to zero asymptotically. A simulation result shows the validity of the results.

Key words Nonlinear system, linearization, state observer, Lyapunov function, coordinate transformation.