Delay-dependent L_2-L_∞ Filter for Singular Time-delay Systems

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Abstract The problem of L_2-L_∞ filtering is discussed for singular time-delay systems. Attention is focused on the design of full-order filter that guarantees the delay-dependent exponential admissibility and a prescribed noise attenuation level in L_2-L_∞ sense for the filtering error dynamics. The desired filter can be constructed by solving certain linear matrix inequality (LMI). Numerical examples are given to show that the methods have less conservatism.

Key words Singular systems, delay-dependent, L_2-L_∞ filters, linear matrix inequality (LMI)

Over the past decades, the filtering problem has been extensively investigated due to the fact that filtering is of both theoretical and practical importance in signal processing. For L_2-L_∞ filter designs, there have been fruitful results in terms of the linear matrix inequality (LMI) approach[1−3]. Recently, considerable attention has been devoted to singular systems due to the fact that they can describe physical systems better than state-space ones and have extensive applications. It should be pointed out that the analysis and synthesis for singular systems are much more complicated than those for state-space systems because it requires to consider not only stability, but also regularity and absence of impulses (for continuous singular systems) and causality (for discrete singular systems) at the same time[3], and the latter two need not be considered in regular systems. For more details on singular systems, we can refer to [4]. Recently, the filtering problem for singular systems has been studied in terms of LMI approach. References [5−6] investigated the H_∞ filtering issues for singular systems while the L_2-L_∞ filtering results for singular systems were presented in [7]. When time-delays appear, [8−10] discussed the H_∞ filtering problem for singular time-delay systems. So far, to the best of our knowledge, there has been no delay-dependent method reported on the L_2-L_∞ filtering problem for singular time-delay systems.

This paper is concerned with the problem of delay-dependent L_2-L_∞ filtering problem for singular time-delay systems. A delay-dependent condition is proposed, which guarantees the considered singular time-delay system to be exponentially admissible with a prescribed L_2-L_∞ performance level. Based on it, an LMI method for designing an linear full-order filter is proposed.

1 Problem formulation

Consider the singular time-delay systems

$$\begin{aligned} E\dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + A_d\boldsymbol{x}(t-d) + B_\omega\boldsymbol{\omega}(t) \\ \boldsymbol{y}(t) &= C\boldsymbol{x}(t) + C_d\boldsymbol{x}(t-d) + D_\omega\boldsymbol{\omega}(t) \\ \boldsymbol{z}(t) &= L\boldsymbol{x}(t) \\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t),\ t\in[-d,0] \end{aligned} \tag{1}$$

where $\boldsymbol{x}(t)\in\mathbf{R}^n$ is the state, $\boldsymbol{y}(t)\in\mathbf{R}^s$ is the measurement, $\boldsymbol{z}(t)\in\mathbf{R}^q$ is the signal to be estimated, and $\boldsymbol{\omega}(t)\in\mathbf{R}^p$ is the disturbance input that belongs to $L_2[0,\infty)$. d is an unknown but constant time delay, $\bar{d}$ is a known constant satisfying $0\le d\le\bar{d}$, and $\boldsymbol{\phi}(t)\in C_{n,d_2}$ is a compatible vector valued initial function. The matrix $E\in\mathbf{R}^{n\times n}$ may be singular and it is assumed that $\operatorname{rank}E = r\le n$. A, A_d, B_ω, C, C_d, D_ω, and L are known real constant matrices with appropriate dimensions.

We consider a filter with the following form:

$$\begin{aligned} E_f\dot{\hat{\boldsymbol{x}}}(t) &= A_f\hat{\boldsymbol{x}}(t) + B_f\boldsymbol{y}(t) \\ \hat{\boldsymbol{z}}(t) &= C_f\hat{\boldsymbol{x}}(t) \end{aligned} \tag{2}$$

where the constant matrices E_f, A_f, B_f, and C_f are the filter matrices with appropriate dimensions, which are to be designed. Augmenting (1) to include the states of the filter, we obtain the following filtering error system

$$\begin{aligned} \bar{E}\dot{\bar{\boldsymbol{x}}}(t) &= \bar{A}\bar{\boldsymbol{x}}(t) + \bar{A}_d\bar{\boldsymbol{x}}(t-d) + \bar{B}_\omega\boldsymbol{\omega}(t) \\ \bar{\boldsymbol{z}}(t) &= \bar{L}\bar{\boldsymbol{x}}(t) \end{aligned} \tag{3}$$

where $\bar{\boldsymbol{z}}(t) = \boldsymbol{z}(t) - \hat{\boldsymbol{z}}(t)$, $\bar{\boldsymbol{x}}(t) = \begin{bmatrix}\boldsymbol{x}(t)^{\mathrm{T}} & \hat{\boldsymbol{x}}(t)^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$, and

$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & E_f\end{bmatrix},\ \bar{A} = \begin{bmatrix} A & 0 \\ B_fC & A_f\end{bmatrix},\ \bar{A}_d = \begin{bmatrix} A_d & 0 \\ B_fC_d & 0\end{bmatrix}$$
$$\bar{B}_\omega = \begin{bmatrix} B_\omega \\ B_fD_\omega\end{bmatrix},\ \bar{L} = \begin{bmatrix} L & -C_f\end{bmatrix}$$

The problem to be addressed here is as follows: given scalars $\bar{d}>0$, $\gamma>0$, and system (1), design a full-order filter of the form (2) such that the filtering error system (3) with $\boldsymbol{\omega}(t)=\mathbf{0}$ is exponentially admissible and under the zero-initial condition, the L_2-L_∞ performance $\|\boldsymbol{z}(t)\|_\infty < \gamma\|\boldsymbol{\omega}(t)\|_2$ is guaranteed for all nonzero $\boldsymbol{\omega}(t)\in L_2[0,\infty)$ and for any constant time delay d satisfying $0\le d\le\bar{d}$, where $\|\boldsymbol{z}(t)\|_\infty = \sup_t\sqrt{\boldsymbol{z}(t)^{\mathrm{T}}\boldsymbol{z}(t)}$ and $\|\boldsymbol{\omega}(t)\|_2 = \sqrt{\int_0^\infty\boldsymbol{\omega}(t)^{\mathrm{T}}\boldsymbol{\omega}(t)\mathrm{d}t}$.

Lemma 1. If the singular time-delay system

$$\begin{aligned} E\dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + A_d\boldsymbol{x}(t-d) \\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t),\ t\in[-d,0] \end{aligned} \tag{4}$$

is regular and impulse free, then there exists a scalar $\kappa>0$ such that

$$\sup_{-d\le s\le d}\|\boldsymbol{x}(s)\|^2 \le \kappa\|\boldsymbol{\phi}(t)\|_{\bar{d}}^2 \tag{5}$$

Proof. Note that the regularity and the absence of impulses of the pair (E, A) imply that there always exist two nonsingular matrices M and N such that $MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0\end{bmatrix}$ and $MAN = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r}\end{bmatrix}$. Write $MA_dN = \begin{bmatrix} A_{d1} & A_{d2} \\ A_{d3} & A_{d4}\end{bmatrix}$ and $\boldsymbol{\xi}(t) = \begin{bmatrix}\boldsymbol{\xi}_1(t) \\ \boldsymbol{\xi}_2(t)\end{bmatrix} = N^{-1}\boldsymbol{x}(t)$. Then, the system (4) can be written as

$$\begin{aligned} \dot{\boldsymbol{\xi}}_1(t) &= A_1\boldsymbol{\xi}_1(t) + A_{d1}\boldsymbol{\xi}_1(t-d) + A_{d2}\boldsymbol{\xi}_2(t-d) \\ -\boldsymbol{\xi}_2(t) &= A_{d3}\boldsymbol{\xi}_1(t-d) + A_{d4}\boldsymbol{\xi}_2(t-d) \\ \boldsymbol{\xi}(t) &= \boldsymbol{\psi}(t) = N^{-1}\boldsymbol{\phi}(t),\ t\in[-\bar{d},0] \end{aligned} \tag{6}$$

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Then, for any $0 \le t \le d$, we have

$$\|\boldsymbol{\xi}_1(t)\| \le (2k_1\bar{d}+1)\|\boldsymbol{\psi}\|_{\bar{d}} + k_1\int_0^t \|\boldsymbol{\xi}_1(\alpha)\|\mathrm{d}\alpha \tag{7}$$

where $k_1 = \max\{\|A_1\|, \|A_{d1}\|, \|A_{d2}\|\}$. Applying the well known Gronwall Lemma, we obtain from (7) that for any $0 \le t \le d$, $\|\boldsymbol{\xi}_1(t)\| \le (2k_1\bar{d}+1)\|\boldsymbol{\psi}\|_{\bar{d}}\mathrm{e}^{k_1\bar{d}}$. Thus, $\sup_{0\le s\le d}\|\boldsymbol{\xi}_1(s)\|^2 \le (2k_1\bar{d}+1)^2\|\boldsymbol{\psi}\|_{\bar{d}}^2\mathrm{e}^{2k_1\bar{d}}$. It is easy to get from (6) that $\sup_{0\le s\le d}\|\boldsymbol{\xi}_2(s)\|^2 \le 4k_2^2\|\boldsymbol{\psi}\|_{\bar{d}}^2$, where $k_2 = \max\{\|A_{d3}\|, \|A_{d4}\|\}$. Hence, there exists a scalar $\kappa > 0$ such that (5) holds. □

2 Main results

Theorem 1. For prescribed scalars $\bar{d} > 0$ and $\gamma > 0$, the singular time-delay systems

$$\begin{cases} E\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + A_d\boldsymbol{x}(t-d) + B_\omega\boldsymbol{\omega}(t) \\ \boldsymbol{z}(t) = L\boldsymbol{x}(t) \\ \boldsymbol{x}(t) = \boldsymbol{\phi}(t),\ t\in[-d, 0] \end{cases} \tag{8}$$

is exponentially admissible with L_2-L_∞ performance γ for any constant time delay d satisfying $0 \le d \le \bar{d}$, if there exist matrix P and symmetric positive-definite matrices Q_1, Q_2, Z_1, and Z_2 such that

$$E^{\mathrm{T}}P = P^{\mathrm{T}}E \ge 0 \tag{9a}$$

$$\begin{bmatrix} \Xi_{11} & P^{\mathrm{T}}A_d & E^{\mathrm{T}}Z_1E & P^{\mathrm{T}}B_\omega & \bar{d}^2A^{\mathrm{T}}Z \\ * & \Xi_{22} & E^{\mathrm{T}}Z_2E & 0 & \bar{d}^2A_d^{\mathrm{T}}Z \\ * & * & \Xi_{33} & 0 & 0 \\ * & * & * & -I & \bar{d}^2B_\omega^{\mathrm{T}}Z \\ * & * & * & * & -4\bar{d}^2Z \end{bmatrix} < 0 \tag{9b}$$

$$\begin{bmatrix} E^{\mathrm{T}}P & L^{\mathrm{T}} \\ * & \gamma^2 I \end{bmatrix} \ge 0 \tag{9c}$$

where $Z = (Z_1+Z_2)$, $\Xi_{11} = P^{\mathrm{T}}A + A^{\mathrm{T}}P + Q_1 - E^{\mathrm{T}}Z_1E$, $\Xi_{22} = -Q_2 - E^{\mathrm{T}}Z_2E$, and $\Xi_{33} = -Q_1 + Q_2 - E^{\mathrm{T}}ZE$.

Proof. Under the and condition of the theorem, we first show the exponential admissibility of the singular time-delay system (8) with $\boldsymbol{\omega}(t) \equiv 0$. Noting that $\operatorname{rank} E = r \le n$, we can always find two nonsingular matrices G and H such that $GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$. Denote $GAH = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$ and $G^{-\mathrm{T}}PH = \begin{bmatrix} \bar{P}_1 & \bar{P}_2 \\ \bar{P}_3 & \bar{P}_4 \end{bmatrix}$. From (9a), it is easy to obtain that $\bar{P}_2 = 0$. Then, pre- and post-multiplying $\Xi_{11} < 0$ by H^{T} and H, respectively, we have $A_4^{\mathrm{T}}\bar{P}_4 + \bar{P}_4^{\mathrm{T}}A_4 < 0$, which implies A_4 is nonsingular and thus the pair (E, A) is regular and impulse free. Now, pre- and post-multiplying (9b) by $\Pi = \begin{bmatrix} I & I & I & 0 & 0 \end{bmatrix}$ and Π^{T}, respectively, we get $P^{\mathrm{T}}(A+A_d) + (A+A_d)^{\mathrm{T}}P < 0$. Considering this and (9a), it can be deduced that the pair $(E, A+A_d)$ is regular and impulse free, and the matrix P is nonsingular. Thus, the singular time-delay system (8) with $\boldsymbol{\omega}(t) \equiv 0$ is regular and impulse free for any constant time delay d satisfying $0 \le d \le \bar{d}$.

Next, we will show the exponential stability of system (8). To this end, define the following Lyapunov candidate for system (8) with $\boldsymbol{\omega}(t) \equiv 0$ as

$$\begin{aligned} V(\boldsymbol{x}_t, t) = {}& \boldsymbol{x}(t)^{\mathrm{T}}E^{\mathrm{T}}P\boldsymbol{x}(t) + \int_{t-\frac{d}{2}}^{t}\boldsymbol{x}(s)^{\mathrm{T}}Q_1\boldsymbol{x}(s)\,\mathrm{d}s + \\ & \int_{t-d}^{t-\frac{d}{2}}\boldsymbol{x}(s)^{\mathrm{T}}Q_2\boldsymbol{x}(s)\,\mathrm{d}s + \\ & \frac{d}{2}\int_{-\frac{d}{2}}^{0}\int_{t+\beta}^{t}\dot{\boldsymbol{x}}(\alpha)^{\mathrm{T}}E^{\mathrm{T}}Z_1E\dot{\boldsymbol{x}}(\alpha)\,\mathrm{d}\alpha\mathrm{d}\beta + \\ & \frac{d}{2}\int_{-d}^{-\frac{d}{2}}\int_{t+\beta}^{t}\dot{\boldsymbol{x}}(\alpha)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\dot{\boldsymbol{x}}(\alpha)\,\mathrm{d}\alpha\mathrm{d}\beta \end{aligned} \tag{10}$$

where $\boldsymbol{x}_t = \boldsymbol{x}(t+\theta)$ and $-2\bar{d} \le \theta \le 0$. Calculating the derivative of $V(\boldsymbol{x}_t, t)$ along the solutions of system (8) with $\boldsymbol{\omega}(t) \equiv 0$ yields for any $t \ge d$, we have

$$\begin{aligned} \dot{V}(\boldsymbol{x}_t, t) = {}& 2\boldsymbol{x}(t)^{\mathrm{T}}E^{\mathrm{T}}P\dot{\boldsymbol{x}}(t) + \boldsymbol{x}(t)^{\mathrm{T}}Q_1\boldsymbol{x}(t) - \\ & \boldsymbol{x}(t-d)^{\mathrm{T}}Q_2\boldsymbol{x}(t-d) + \boldsymbol{x}(t-\frac{d}{2})^{\mathrm{T}}(Q_2-Q_1)\boldsymbol{x}(t-\frac{d}{2}) + \\ & \frac{d^2}{4}\dot{\boldsymbol{x}}(t)^{\mathrm{T}}E^{\mathrm{T}}(Z_1+Z_2)E\dot{\boldsymbol{x}}(t) - \frac{d}{2}\int_{t-\frac{d}{2}}^{t}\dot{\boldsymbol{x}}(\alpha)^{\mathrm{T}}E^{\mathrm{T}}Z_1E \times \\ & \dot{\boldsymbol{x}}(\alpha)\,\mathrm{d}\alpha - \frac{d}{2}\int_{t-d}^{t-\frac{d}{2}}\dot{\boldsymbol{x}}(\alpha)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\dot{\boldsymbol{x}}(\alpha)\,\mathrm{d}\alpha \end{aligned}$$

According to Jensen integral inequality[11], the following equations are true:

$$\begin{aligned} & -\frac{d}{2}\int_{t-\frac{d}{2}}^{t}\dot{\boldsymbol{x}}(\alpha)^{\mathrm{T}}E^{\mathrm{T}}Z_1E\dot{\boldsymbol{x}}(\alpha)\,\mathrm{d}\alpha \le \\ & \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-\frac{d}{2}) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -E^{\mathrm{T}}Z_1E & E^{\mathrm{T}}Z_1E \\ * & -E^{\mathrm{T}}Z_1E \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{x}(t-\frac{d}{2}) \end{bmatrix} - \\ & \frac{d}{2}\int_{t-d}^{t-\frac{d}{2}}\dot{\boldsymbol{x}}(\alpha)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\dot{\boldsymbol{x}}(\alpha)\,\mathrm{d}\alpha \le \\ & \begin{bmatrix} \boldsymbol{x}(t-d) \\ \boldsymbol{x}(t-\frac{d}{2}) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -E^{\mathrm{T}}Z_2E & E^{\mathrm{T}}Z_2E \\ * & -E^{\mathrm{T}}Z_2E \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t-d) \\ \boldsymbol{x}(t-\frac{d}{2}) \end{bmatrix} \end{aligned}$$

Thus, through algebraic manipulations, we have that when $t \ge d$, $\dot{V}(\boldsymbol{x}_t, t) \le \boldsymbol{\eta}(t)^{\mathrm{T}}\Psi\boldsymbol{\eta}(t)$, where

$$\Psi = \begin{bmatrix} \Xi_{11} & P^{\mathrm{T}}A_d & E^{\mathrm{T}}Z_1E \\ * & \Xi_{22} & E^{\mathrm{T}}Z_2E \\ * & * & \Xi_{33} \end{bmatrix} + \frac{d^2}{4}\begin{bmatrix} A^{\mathrm{T}} \\ A_d^{\mathrm{T}} \\ 0 \end{bmatrix} Z \begin{bmatrix} A^{\mathrm{T}} \\ A_d^{\mathrm{T}} \\ 0 \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{\eta}(t) = \begin{bmatrix} \boldsymbol{x}(t)^{\mathrm{T}} & \boldsymbol{x}(t-d)^{\mathrm{T}} & \boldsymbol{x}(t-\frac{d}{2})^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

Now, applying the Schur complements and from (9b), it is easy to see that there exits a scalar $\lambda_0 > 0$ such that $\dot{V}(\boldsymbol{x}_t, t) \le -\lambda_0\|\boldsymbol{x}(t)\|^2$. Moreover, by the definition of $V(\boldsymbol{x}_t, t)$, there exist positive scalars λ_1, λ_2, and λ_3 such that for any $t \ge d$, $V(\boldsymbol{x}_t, t) \le \lambda_1\|\boldsymbol{x}(t)\|^2 + \lambda_2\int_{t-d}^{t}\|\boldsymbol{x}(s)\|^2\,\mathrm{d}s + \lambda_3\int_{t-d}^{t}\|\boldsymbol{x}(s-d)\|^2\,\mathrm{d}s$. Then, according to Lemma 1, it is easy to get that there exists a scalar $\lambda_4 > 0$ such that $V(\boldsymbol{x}_d, d) \le \lambda_4\|\boldsymbol{\phi}(t)\|_{\bar{d}}^2$. Set $\hat{G} = \begin{bmatrix} I_r & -A_2A_4^{-1} \\ 0 & A_4^{-1} \end{bmatrix}G$. It is easy to get $\hat{G}EH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ and $\hat{G}AH = \begin{bmatrix} \hat{A}_1 & 0 \\ \hat{A}_3 & I \end{bmatrix}$, where $\hat{A}_1 = A_1 - A_2A_4^{-1}A_3$ and $\hat{A}_3 = A_4^{-1}A_3$. Denote $\hat{G}A_dH = \begin{bmatrix} A_{d1} & A_{d2} \\ A_{d3} & A_{d4} \end{bmatrix}$, $\hat{G}^{-\mathrm{T}}PH = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}$, and $H^{\mathrm{T}}Q_1H = \begin{bmatrix} Q_{11} & Q_{21} \\ * & Q_{22} \end{bmatrix}$. Considering (9a) and nonsingularity of P, we can deduce that $P_1 > 0$ and $P_2 = 0$. Define $\boldsymbol{\zeta}(t) = \begin{bmatrix} \boldsymbol{\zeta}_1(t) \\ \boldsymbol{\zeta}_2(t) \end{bmatrix} = H^{-1}\boldsymbol{x}(t)$; then the system (8) with

$\boldsymbol{\omega}(t) \equiv 0$ is a restricted system equivalent to

$$\begin{aligned}\dot{\boldsymbol{\zeta}}_1(t) &= \hat{A}_1\boldsymbol{\zeta}_1(t) + A_{d1}\boldsymbol{\zeta}_1(t-d) + A_{d2}\boldsymbol{\zeta}_2(t-d)\\ -\boldsymbol{\zeta}_2(t) &= \hat{A}_{13}\boldsymbol{\zeta}_1(t) + A_{d3}\boldsymbol{\zeta}_1(t-d) + A_{d4}\boldsymbol{\zeta}_2(t-d)\\ \boldsymbol{\zeta}(t) &= \boldsymbol{\psi}(t) = H^{-1}\boldsymbol{\phi}(t),\ t\in[-d,0]\end{aligned} \tag{11}$$

To prove the exponential stability, we define a new function as

$$W(\boldsymbol{x}_t,t) = \mathrm{e}^{\varepsilon t}V(\boldsymbol{x}_t,t),\ t\geq d \tag{12}$$

where the scalar $\varepsilon > 0$. Then, we find that for any $t \geq d$

$$\begin{aligned}&W(\boldsymbol{x}_t,t) - W(\boldsymbol{x}_d,d) \leq \int_d^t \mathrm{e}^{\varepsilon s}\left[\varepsilon V(\boldsymbol{x}_s,s) - \lambda_0\|\boldsymbol{x}(s)\|^2\right]\mathrm{d}s \leq\\ &\int_d^t \mathrm{e}^{\varepsilon\alpha}\left[\varepsilon\lambda_1\|\boldsymbol{x}(\alpha)\|^2 + \varepsilon\lambda_2\int_{\alpha-d}^{\alpha}\|\boldsymbol{x}(s)\|^2\mathrm{d}s + \right.\\ &\left.\varepsilon\lambda_3\int_{\alpha-d}^{\alpha}\|\boldsymbol{x}(s-d)\|^2\mathrm{d}s - \lambda_0\|\boldsymbol{x}(\alpha)\|^2\right]\mathrm{d}\alpha\end{aligned} \tag{13}$$

By Lemma 1 and interchanging the integration sequence, we get for any $t \geq d$,

$$\begin{aligned}&\int_d^t \mathrm{e}^{\varepsilon\alpha}\mathrm{d}\alpha\int_{\alpha-d}^{\alpha}\|\boldsymbol{x}(s)\|^2\mathrm{d}s \leq d\mathrm{e}^{\varepsilon d}\int_0^t \mathrm{e}^{\varepsilon\alpha}\|\boldsymbol{x}(\alpha)\|^2\mathrm{d}\alpha \leq\\ &\quad d\mathrm{e}^{\varepsilon d}\int_d^t \mathrm{e}^{\varepsilon\alpha}\|\boldsymbol{x}(\alpha)\|^2\mathrm{d}\alpha + d^2\mathrm{e}^{2\varepsilon d}\kappa\|\boldsymbol{\phi}(t)\|_{\bar{d}}^2\\ &\quad\int_d^t \mathrm{e}^{\varepsilon\alpha}\mathrm{d}\alpha\int_{\alpha-d}^{\alpha}\|\boldsymbol{x}(s-d)\|^2\mathrm{d}s \leq\\ &\quad d\mathrm{e}^{2\varepsilon d}\int_d^t \mathrm{e}^{\varepsilon\alpha}\|\boldsymbol{x}(\alpha)\|^2\mathrm{d}\alpha + 2d^2\mathrm{e}^{3\varepsilon d}\kappa\|\boldsymbol{\phi}(t)\|_{\bar{d}}^2\end{aligned} \tag{14}$$

Let the scalar $\varepsilon > 0$ small enough such that $\varepsilon\lambda_1 + \varepsilon d\mathrm{e}^{\varepsilon d}\lambda_2 + \varepsilon d\mathrm{e}^{2\varepsilon d}\lambda_3 - \lambda_0 \leq 0$. Then, substituting (14) into (13) gives that there exists a scalar $k > 0$ such that for any $t \geq d$,

$$\lambda_{\min}(P_1)\|\boldsymbol{\zeta}_1(t)\|^2 \leq V(\boldsymbol{x}_t,t) \leq k\mathrm{e}^{-\varepsilon t}\|\boldsymbol{\phi}(t)\|_{\bar{d}}^2 \tag{15}$$

Hence, for any $t \geq d$,

$$\|\boldsymbol{\zeta}_1(t)\|^2 \leq \lambda_{\min}(P_1)^{-1}k\mathrm{e}^{-\varepsilon t}\|\boldsymbol{\phi}(t)\|_{\bar{d}}^2 \tag{16}$$

Combining Lemma 1 yields for any $t > 0$, $\|\boldsymbol{\zeta}_1(t)\|^2 \leq a\mathrm{e}^{-\varepsilon t}\|\boldsymbol{\phi}(t)\|_{\bar{d}}^2$, where $a = \max\{\lambda_{\min}(P_1)^{-1}k, \kappa\|H^{-1}\|^2\mathrm{e}^{\varepsilon\bar{d}}\}$. Applying the same approach of [9], it is easy to prove from (9) that there exist scalars $\rho > 0$ and $\varepsilon_1 > 0$ such that $\|\boldsymbol{\zeta}_2(s)\|^2 \leq \rho\mathrm{e}^{-\varepsilon_1 t}\|\boldsymbol{\zeta}_2(s)\|_{\bar{d}}^2$, which means, combining (16), that system (8) is exponentially stable for any constant time delay d satisfying $0 \leq d \leq \bar{d}$.

Next, we will establish the L_2-L_∞ performance. Choose the Lyapunov functional candidate as in (10) and the index $W(t) = V(\boldsymbol{x}_t,t) - \int_0^t \boldsymbol{\omega}(s)^{\mathrm{T}}\boldsymbol{\omega}(s)\,\mathrm{d}s$ for system (8). Under zero initial condition, it easy to see that

$$\begin{aligned}W(t) = &\int_0^t \left[\dot{V}(\boldsymbol{x}_s,s) - \boldsymbol{\omega}(s)^{\mathrm{T}}\boldsymbol{\omega}(s)\right]\mathrm{d}s \leq\\ &\int_0^t \begin{bmatrix}\boldsymbol{\eta}(s)\\ \boldsymbol{\omega}(s)\end{bmatrix}^{\mathrm{T}}\Theta\begin{bmatrix}\boldsymbol{\eta}(s)\\ \boldsymbol{\omega}(s)\end{bmatrix}\mathrm{d}s\end{aligned}$$

where

$$\Theta = \begin{bmatrix}\Xi_{11} & P^{\mathrm{T}}A_d & E^{\mathrm{T}}Z_1E & P^{\mathrm{T}}B_\omega\\ * & \Xi_{22} & E^{\mathrm{T}}Z_2E & 0\\ * & * & \Xi_{33} & 0\\ * & * & * & -I\end{bmatrix} + \frac{d^2}{4}\begin{bmatrix}A^{\mathrm{T}}\\ A_d^{\mathrm{T}}\\ 0\\ B_\omega^{\mathrm{T}}\end{bmatrix}Z\begin{bmatrix}A^{\mathrm{T}}\\ A_d^{\mathrm{T}}\\ 0\\ B_\omega^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$$

Hence, by Schur complements, (9b) implies that for any $t > 0$, $W(t) < 0$ for any non-zero $\boldsymbol{\omega}(t) \in L_2[0,\infty)$. Thus, $V(\boldsymbol{x}_t,t) < \int_0^t \boldsymbol{\omega}(s)^{\mathrm{T}}\boldsymbol{\omega}(s)\,\mathrm{d}s$. On the other hand, (9c) implies $L^{\mathrm{T}}L \leq \gamma^2E^{\mathrm{T}}P$. Therefore, for any $t > 0$, $\boldsymbol{z}(t)^{\mathrm{T}}\boldsymbol{z}(t) = \boldsymbol{x}(t)^{\mathrm{T}}L^{\mathrm{T}}L\boldsymbol{x}(t) \leq \gamma^2\boldsymbol{x}(t)^{\mathrm{T}}E^{\mathrm{T}}P\boldsymbol{x}(t) < \gamma^2\int_0^t \boldsymbol{\omega}(s)^{\mathrm{T}}\boldsymbol{\omega}(s)\,\mathrm{d}s \leq \gamma^2\int_0^\infty \boldsymbol{\omega}(s)^{\mathrm{T}}\boldsymbol{\omega}(s)\,\mathrm{d}s$. Thus, L_2-L_∞ performance $\|\boldsymbol{z}(t)\|_\infty < \gamma\|\boldsymbol{\omega}(t)\|_2$ is guaranteed. Therefore, the singular time-delay systems (8) is exponentially admissible with L_2-L_∞ performance γ. □

Theorem 2. For prescribed scalars $\bar{d} > 0$ and $\gamma > 0$, the filtering error system (5) is exponentially admissible with L_2-L_∞ performance γ for any constant time delay d satisfying $0 \leq d \leq \bar{d}$, if there exist matrices X, U, $\bar{A}_f$, $\bar{B}_f$, $\bar{C}_f$ and symmetric positive-definite matrices S_1, S_2, R_1 and R_2 such that

$$E^{\mathrm{T}}X = X^{\mathrm{T}}E \geq 0 \tag{17a}$$

$$E^{\mathrm{T}}U = U^{\mathrm{T}}E \geq 0 \tag{17b}$$

$$E^{\mathrm{T}}(X-U) = (X-U)^{\mathrm{T}}E \geq 0 \tag{17c}$$

$$\begin{bmatrix}\Delta_1 & \Delta_2 & \Delta_4 & E^{\mathrm{T}}R_1E & (X-U)^{\mathrm{T}}B_\omega & \bar{d}^2A^{\mathrm{T}}R\\ * & \Delta_3 & \Delta_5 & E^{\mathrm{T}}R_1E & \Psi & \bar{d}^2A^{\mathrm{T}}R\\ * & * & \Delta_6 & E^{\mathrm{T}}R_2E & 0 & \bar{d}^2A_d^{\mathrm{T}}R\\ * & * & * & \Delta_7 & 0 & 0\\ * & * & * & * & -I & \bar{d}^2B_\omega^{\mathrm{T}}R\\ * & * & * & * & * & -4\bar{d}^2R\end{bmatrix} < 0 \tag{17d}$$

$$\begin{bmatrix}E^{\mathrm{T}}(X-U) & E^{\mathrm{T}}(X-U) & L^{\mathrm{T}} + \bar{C}_f^{\mathrm{T}}\\ * & E^{\mathrm{T}}X & L^{\mathrm{T}}\\ * & * & \gamma^2I\end{bmatrix} \geq 0 \tag{17e}$$

where $R = R_1 + R_2$, $\Psi = X^{\mathrm{T}}B_\omega + \bar{B}_fD_\omega$, and

$$\begin{aligned}\Delta_1 &= A^{\mathrm{T}}(X-U) + (X-U)^{\mathrm{T}}A - E^{\mathrm{T}}R_1E + S_1\\ \Delta_2 &= \bar{A}_f^{\mathrm{T}} - E^{\mathrm{T}}R_1E + S_1\\ \Delta_3 &= A^{\mathrm{T}}X + C^{\mathrm{T}}\bar{B}_f^{\mathrm{T}} + X^{\mathrm{T}}A + \bar{B}_fC - E^{\mathrm{T}}R_1E + S_1\\ \Delta_4 &= (X-U)^{\mathrm{T}}A_d\\ \Delta_5 &= X^{\mathrm{T}}A_d + \bar{B}_fC_d\\ \Delta_6 &= -S_2 - E^{\mathrm{T}}R_2E\\ \Delta_7 &= -S_1 + S_2 - E^{\mathrm{T}}(R_1+R_2)E\end{aligned}$$

Then, a desired H_∞ filter in the form of (3) can be chosen with parameters as

$$\begin{aligned}A_f &= -U^{-\mathrm{T}}(\bar{A}_f - A^{\mathrm{T}}(X-U) - X^{\mathrm{T}}A - \bar{B}_fC)\\ B_f &= U^{-\mathrm{T}}\bar{B}_f,\ C_f = \bar{C}_f,\ E_f = E\end{aligned} \tag{18}$$

Proof. By Theorem 1 and the methods of [4, 8], the desired results can be obtained easily. □

3 Numerical examples

Example 1. Consider the singular time-delay system

(4) and

$$E=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ A=\begin{bmatrix}0.6341&0.5413\\-0.6121&-1.121\end{bmatrix}$$

$$A_d=\begin{bmatrix}-0.45&0\\0&-0.1210\end{bmatrix}$$

Comparing the result in Theorem 1 with the methods in [10,12–15], we get Table 1, which demonstrates our result has less conservatism.

Table 1 Comparison of maximum allowed delays $\bar{d}$

[12]	[13]	[10]	[14−15]	Theorem 1
2.4841	1.1576	2.1372	2.4865	2.5176

Example 2. Consider singular time-delay system (8) with the following parameters:

$$E=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ A=\begin{bmatrix}-2&0\\0&-0.9\end{bmatrix},\ A_d=\begin{bmatrix}-1&0\\-1&-0.9\end{bmatrix}$$

$$\boldsymbol{B}_\omega=\begin{bmatrix}0\\1\end{bmatrix},\boldsymbol{C}=\begin{bmatrix}1\\0\end{bmatrix}^{\mathrm{T}},C_d=0,D_\omega=1,\ \boldsymbol{L}=\begin{bmatrix}1\\1.2\end{bmatrix}^{\mathrm{T}}$$

In this example, the time delay upper bound $\bar{d}$ is assumed to be 1.1. The minimum L_2-L_∞ performance γ of the filtering system (3) achieved by [2] is 0.5460. However, applying Theorem 2 in this paper, the achieved L_2-L_∞ performances of the filtering system (3) can be calculated as 0.4872. Thus, for the above system, Theorem 2 in this paper is less conservative than that in [2].

Example 3. Consider the singular time-delay system (1) with the following matrices

$$E=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ A=\begin{bmatrix}0.4&0.2\\0.1&-1\end{bmatrix},\ A_d=\begin{bmatrix}-0.9&1\\0&0.5\end{bmatrix}$$

$$\boldsymbol{B}_\omega=\begin{bmatrix}-1\\0.4\end{bmatrix},\ \boldsymbol{C}=\begin{bmatrix}1&-0.9\end{bmatrix},\ \boldsymbol{C}_d=\begin{bmatrix}0.5&0.4\end{bmatrix},$$

$$D_\omega=1,\ \boldsymbol{L}=\begin{bmatrix}-1&0\end{bmatrix}$$

For this example, the time delay upper bound $\bar{d}=0.6$ and L_2-L_∞ performance index $\gamma=3.5$. Solving the LMIs (17), we get the desired filter with the following parameter matrices as follows

$$E_f=\begin{bmatrix}1&0\\0&0\end{bmatrix},\ A_f=\begin{bmatrix}-1.2466&4.8477\\-0.3834&-1.1635\end{bmatrix}$$

$$\boldsymbol{B}_f=\begin{bmatrix}0.3736\\-0.4624\end{bmatrix},\ \boldsymbol{C}_f=\begin{bmatrix}0.3211&0\end{bmatrix}$$

The simulation result of the state response of the designed filter is given in Fig. 1, where the exogenous disturbance input $\boldsymbol{\omega}(t)$ is given as $\boldsymbol{\omega}(t)=2.5/(2+5t)$, $t\geq 0$. Fig. 2 is the simulation result of the error response of $\boldsymbol{z}(t)-\hat{\boldsymbol{z}}(t)$ of the designed filter. It can be seen that the designed L_2-L_∞ filter satisfies the specified requirements.

4 Conclusion

The problem of L_2-L_∞ filtering for singular time-delay systems is investigated in this paper. A delay-dependent sufficient condition for the solvability of the problem has been obtained in terms of a set of LMIs. Several numerical examples are provided to demonstrate the effectiveness of the proposed methods.

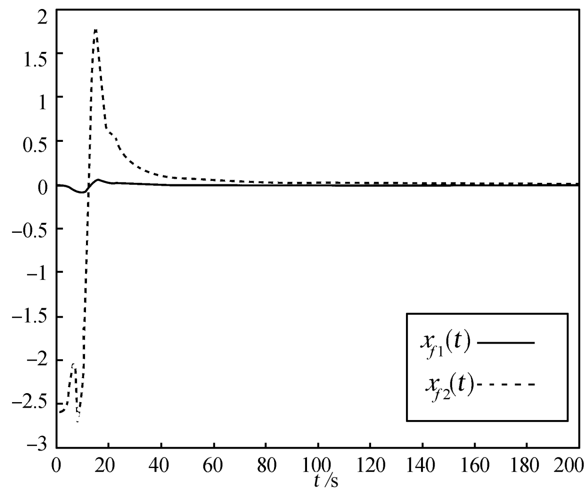

Fig. 1 State responses of $x_{f1}(t)$ and $x_{f2}(t)$

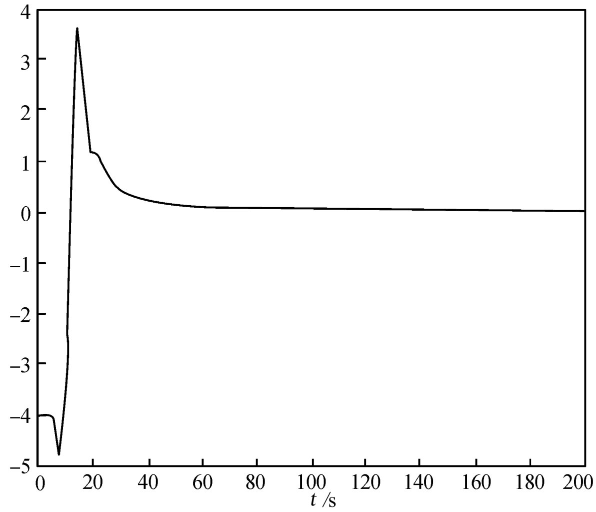

Fig. 2 Error response of $\bar{z}(t)$

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