A Computer Based Approach for the Design of the Orifice-Compensated with Feeding Pocket Annular Hydrostatic Thrust Bearings

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Abstract

The orifice-compensated with feeding pocket annular hydrostatic (OCFPAH) thrust bearing is a unique type of gas bearing with many distinguishing characteristics. It is finding many applications in ultra high-speed rotors. A computer code was written to overcome the difficulties involved in the design of such a bearing. Initially, the design charts were converted into the alternative equations using a curve-fitting technique. The program, which is based on these equations, was designed so that mechanical and physical properties of the lubricant and the main dimensions of the bearing can be entered in a user-friendly manner. Many runs of the code have been carried out successfully. The code has proven to be fast, compatible with CAD and CAD/CAM packages as well as the ability of linking it with data banks and the Internet.

Keywords: Design; Hydrostatic thrust bearings; Stiffness;

Nomenclature

Alphabetic Symbol

a : Orifice radius.[mm **or** inch]
B : Axial damping.[N sec m^{-1} **or** lbs sec/inch]

$$ (B = \overline{B}\, \mu R_o \left(\frac{R_o}{C} \right)^3) $$

C : Film thickness.[mm **or** inch]
    (C $\cong 0.5 \times 10^{-3}$ to 2×10^{-3} inch $\cong 12.5$ to $50\ \mu$m)
d : Feeding hole diameter.[mm **or** inch]
K : Axial stiffness.[N m sec^{-1} **or** lbs/inch]

$$ (K = \overline{K}\, (\frac{1}{C}\pi(R_o^2 - R_i^2)(p_s - p_a)^{1+2/3\delta^2}\!\big/_{1+\delta^2})) $$

n : Number of holes.
    ("nξ" must be greater than "nξ"$_{min}$)
n_p : Number parameter.(n_p= d / 2ξR_c)
p_a : Ambient pressure [Bar **or** psi]
p_s : Supply pressure.[Bar **or** psi]
Q : Mass flow rate.[cfm **or** lbs sec/inch]

$$ (Q = \overline{Q}\, \Pi C^3 p_s^2 \big/_{6\mu\, \Re\, T}) $$

R_c : Hole radius [mm **or** inch]

$$ (R_c = \sqrt{R_o R_i} \qquad \pm 20\%) $$

R_i : Inner radius [mm **or** inch]
R_o : Outer radius.[mm **or** inch]
T : Temperature.[° R **or** K]
W : Bearing load [N **or** lb]

$$ (W = \overline{W}\, \pi \left(R_o^2 - R_i^2 \right)(p_s - p_a)) $$

V_i : Feeder hole volume [(mm **or** inch)^3]

Greek symbols

Λ_S : The restrictor coefficient

$$ (\Lambda_s = 6\mu n a^2 \sqrt{\Re T} \big/_{p_s C^3 \sqrt{1+\delta^2}}) $$

μ : Gas viscosity [N sec m^{-2} **or.** lb sec/inch^2]
σ : Squeeze number ($\sigma = 12\mu\nu(R_o/C)^2/p_a$)
ν : Vibratory frequency [rad/sec]
δ : Ratio of orifice area to the clearance
ξ : Logarithmic radii ratio ($\xi = 0.5 \log_e (R_o / R_i)$)

Notes:

    [….] Unit: *'Imperial' units are used only in the intermediate stages to be compatible with the available design equations and charts, whilst SI units are used as input and output of the design.*

    (…..) Governing equation: *Using the dash ' ‾ ' as a superscripts means that this value is a dimensional one and can be extracted from the design charts in the conventional method or direct by the code with the new approach.*

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1. Introduction

Thrust bearings are widely used in many engineering applications. Sternlicht [1] in his key paper, presented the different fields of use of these bearings that include aerospace, nuclear, refrigeration, textile, chemical, medical, food and pharmaceutical industries. He also, listed the machines such as turbines, compressors, generators, motors, pumps, centrifuges, blowers, and expanders in which these bearings were used. The main function of thrust bearings is to support the axial load and position of the rotors. Considerable work has been carried out towards their development of various types of thrust bearings over the past decades and the recent technological advance make this an opportune time for any study in their science and development. Characterization and classification of different types of thrust bearings can be found in many references [1-6]. Figure 1 shows an orifice-compensated with feeding pockets annular hydrostatic (OCFPAH) thrust bearing which was selected for this project. The selection procedure was based on the fact that gas bearings have the ability to operate over a wide range of temperatures with low friction and low power loss. Their construction and service is simple as no oil lubrication system is required in addition to their cleanliness and stability of lubricant. Hydrostatic bearings have a number of inherent advantages such as; high load carrying capacity without the influence of the rotating speed, film thickness can be controlled by varying the supply and orifice geometry without major changes in the overall bearing and starting and stopping are not considered critical problems.
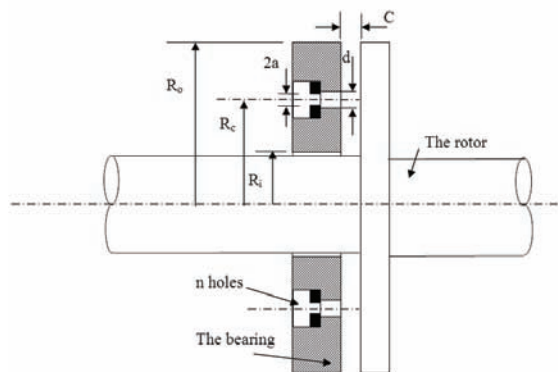

Fig. 1. Schematic of the (OCFPAH) thrust bearing

Above all, the (OCFPAH) thrust bearing has other special features like its relatively high stiffness, easy construction and its reliability in operation. Full details can be found in many references [1-6].

It is important to note that the design procedure of this type of bearing is rather lengthy and complicated as the procedure must go through a sequence of designing charts and equations analogous to that found in Cheng [5]. Fuller [7] in his review paper mentions that the designer needs to have a sharper pencil when working with gas-lubricated bearings. They are less forgiving of errors in estimating loads or of deviations from specifications during manufacture and installation and less forgiving of distortions that may find their way into the rotor, the bearing components or the housing. Further complications

will arise particularly if the stiffness of the bearing is to be optimized then controlling the dimensions of the bearing and the supply pressure can alter the stiffness. Henceforth, any changes in the above(e.g. supply pressure, dimensions, temperature and etc.) would require a repetition of the whole procedure and the manual calculations becoming impractical. Therefore, the main aim of this paper is the development of a computer code for the design of this type of bearing. This is realized with Visual Basic code with a user-friendly environment in order to include the effect of a wide variety of design parameters.

2. Problem Definition And Theoretical Background

Fig. 2 shows design procedures that must be conducted in a number of steps namely; entering or specifying the dimensions, calculating the governing parameters, calculating the characteristic quantities, checking stage and calculating the final output. These steps are quite lengthy and laborious, but the third step that deals with calculating the characteristic quantities, however, is the hardest because the design charts must be accessed at all times. As an example, Figures 3, 4, 5 show only one set of charts related to one value of radius ratio (R_o/R_i = 1.25). This indicates that each one of these charts contains six different curves related to six values of pressure ratios (P_o/P_i = 2, 4, 7, 10, 15 and 20) giving a wide range of pressure values. Different radii ratios (R_o/R_i = 1.5, 2 and 3) obtained from Cheng paper [5] will also be used in this work. Figure 6 shows design chart that can be used to find the minimum number of feed holes required. The checking stage also contains a few important points that need to be examined in order to prove the withstand ability of its operating conditions. The final stage also includes calculations of output properties of such bearing using a complicated set of equations similar to the ones illustrated in the appendix. It is necessary to note that the proof or derivation of these well-established equations is beyond the scope of this paper and can be found in many references like Cheng [5 and 6], Vohr [8] and Lund [9].
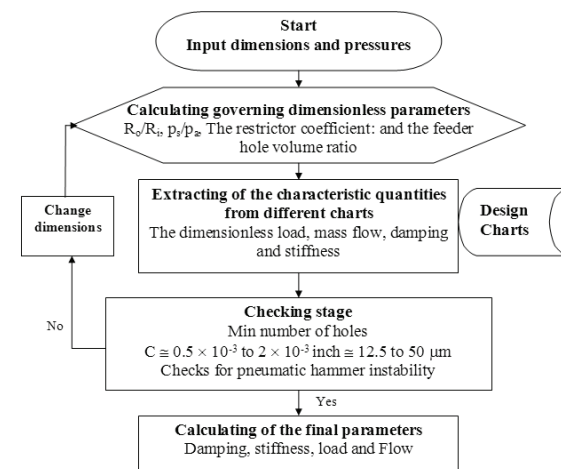

Fig. 2. The design processes of the (OCFPAH) thrust bearing

The difficulty in determining the bearing dimensions and properties for any particular design case is quite

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apparent at this stage, and even becoming impractical specially if the bearing operating conditions are to be altered or changed.
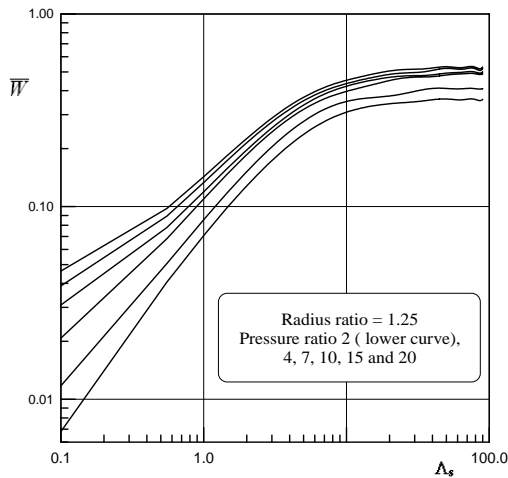

Fig. 3. The design chart for determining the dimensionless load ($\overline{W}$)of the (OCFPAH) thrust bearing for radius ratio (Ro/Ri) =1.25 (other ratios are not shown)
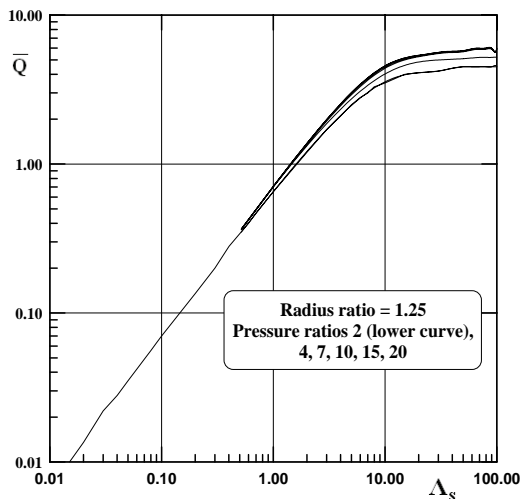

Fig. 4. The design chart for determining the dimensionless flow ($\overline{Q}$)of the (OCFPAH) thrust bearing for radius ratio (Ro/Ri) =1.25 (other ratios are not shown)
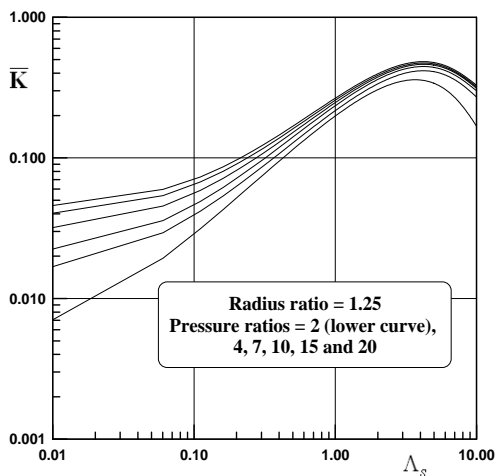

Fig. 5. The design chart for determining the dimensionless stiffness ($\overline{K}$) of the (OCFPAH) thrust bearing for radius ratio (Ro/Ri) =1.25 (other ratios are not shown).
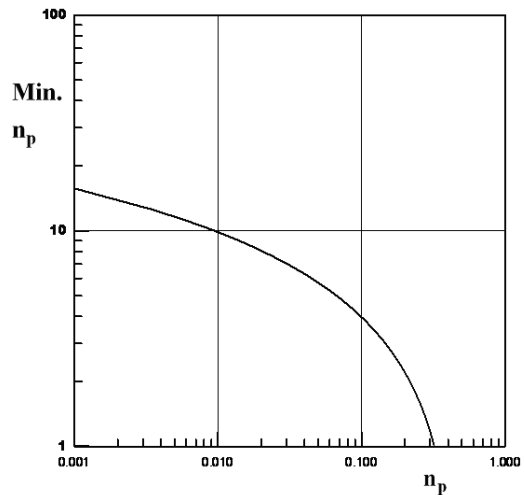

Fig. 6. The design chart for determining the number of feeder holes of the (OCFPAH) thrust bearing

3. The Methodology Used

The conversion of the equations into a computer code is an easy task with special care to the programming language syntax. The more complicated task, however, is how to make the computer understand the different charts. Using curve fitting techniques in converting each chart into an accurate equivalent formula solved this problem. A standard procedure was used in converting of the required design charts into very accurate equations with the aids of a graphic software called Grapher. The required types of equations for usage in the calculation process were accurately chosen from a list of multi-models in order to find the exact equivalent solution. The statistical data extracted like the coefficient of determination and the residuals proved that the accuracy of this process is very high.

The above equations have been converted into a Visual Basic Code. Visual Basic has been chosen because of its super characteristics as it contains strong software's instructions, perfect compatibility with windows applications and easy to operate. The code was designed to be as general as possible. In practice, the most important step is the calculation of load, flow, stiffness and damping. The mechanical and physical properties can be entered by the user or can be extracted directly from the available database in the program. The user will enter the main dimensions of the bearings. Many other dimensions, parameters and characteristics will be calculated by the code itself. One more function of the code is the large amount of information that is built-in the program. These include information about thrust bearing classifications, characteristics, key references, the full statistical data of the formulae used, and help utilities. This data was collected from different sources in the literature and distributed in different branches of the main menu of the program. The output of the code can be displayed on the screen, and the data can be saved on a file or printed on a hard copy.

The program, at this stage, contains a frame that can be filled with many more information and calculations. The

code, also, can be connected to other software that work on the complementary sides of it, like, rotor dynamics and other bearing types.

4. Curve Fittings

It is common practice for engineers to start fitting the data linearly, as it is the simplest model. Failure of the linear model will lead to representing the data by plotting these paired data on various types of graph paper. This is carried out in order to determine whether a suitably transformed scale and their corresponding points will fall close to a straight line or not. In this way, any transformation that suggests a straight line regression equation, the constant parameter(s) can be estimated or determined using the well-known method called least square method [10-14]. It can be said, however, that if there is no clear indication about the functional form of the regression of Y on X, we often assume that the underlying relationship is at least "well-behaved" to the extent that it has a Taylor series expansion and that the first few terms of this expansion will yield a fairly good approximation. The data, thus, is fit on a polynomial equation, where its degree can be determined by direct inspection of the results. An alternative method can also be used, that is by minimizing the sum of the squares of the vertical distances from the point to the curve and taking the partial derivatives with respect to all parameters and equating their partial derivatives to zero [11].

The above procedure has been used with the following general notes and guidelines:
1. The risk inherent in extrapolation.
2. The search actually seek a polynomial of the lowest possible degree that adequately describes the data.
3. Determining the degree of the polynomial by inspection or by a more rigorous method. Essentially, by first fitting a straight line as well as a second-degree polynomial and test whether the case gained by including the quadratic term. If this hypothesis was rejected, then one must fit a third-degree polynomial and the effect of the cubic term is then tested. This procedure is continued until the hypothesis cannot be rejected in two successive steps and there is, thus, no

apparent advantages in carrying out the extra terms [12-14].

4.1. The load Capacity of the thrust bearing

The load of the (OCFPAH) thrust bearing may be influenced by many parameters. Investigations of such parameters can be carried out by plotting the load at different values of the restrictor coefficient whilst the pressure ratio and radius ratio are maintained constant. Other values of pressure ratio or radius ratio need other design charts similar to those explained above. The entire effect of all these parameters can be observed in a family of design charts like those found in Cheng [5], where the required statistical data of the first value of pressure ratio($R_o/R_i=2$) and radius ratio ($R_o/R_i =1.25$) for the curve fitting is extracted from the original design charts. The data are then converted into pairs of mathematical (the restrictor coefficient) and random (the dimensionless load) variables. Drawing the variables on different graph papers (normal, log-log, semi-log, etc.) proves that it cannot be straightened out by any standard technique and, thus, the linear model cannot be reached. Hence, a polynomial model must be used in representing the data. Starting from the second-degree polynomial and increasing its degree step by step and keep inspecting the effect of that leads to a very confident result with the tenth degree polynomial. Other similar trials were carried out for different values of pressure ratios (namely 4,7,10,15 and 20). The results obtained from curve fitting for these trials for different conditions are presented in Table 1 and in Figure 3.

4.2. The flow through the thrust bearing

The flow through the (OCFPAH) thrust bearing can be affected by many parameters. These effects, as in the case of the load calculation, can be seen with a family of design charts. The previously described procedure can also be used here to collect the required statistical data and to convert them into pairs of mathematical (the restrictor coefficient) and random (the dimensionless flow) variables. The polynomial model is again used to represent the data. The results obtained for curve fitting for the different conditions are presented in Table 2 and in Figure 4.

Table 1: The results of curve fitting of the load of the (OCFPAH) thrust bearings with different pressure ratios and with radius ratio = 1.25

Pressure ratio		2	4	7	10	15	20
Polynomial coefficients	Degree 0	0.0012E-10	0.00259993	0.00951562	0.0197223	0.0267031	0.0339925
	Degree 1	0.08062601	0.0925192	0.114349	0.112973	0.122239	0.124698
	Degree 2	0.00920107	0.0101155	-.0152103	-.0142421	-.0167423	-.0169637
	Degree 3	0.00064731	0.00062018	0.0011961	0.00108646	.00138093	0.0013883
	Degree 4	2.9877E-5	-.34942E-5	-5.7754E-5	-.23343E-5	:-10252E-5	-7.0775E-5
	Degree 5	9.2136E-7	5.8632E-7	1.7716E-6	1.6286E-6	2.3326E-6	2.3043E-6
	Degree 6	1.8918E-8	-.9399E-9	-.51488E-8	-.30382E-8	-4.9444E-8	-.84804E-8
	Degree 7	2.53506	1.1414E-10	4.492E-10	4.3289E-10	6.7123E-10	6.5425E-10
	Degree 8	2.1187E-12	-.5137E-13	-.5677E-12	-.5252E-12	-5.623E-12	-5.457E-12
	Degree 9	9.9909E-15	3.7138E-15	1.6012E-14	1.6201E-14	2.6435E-14	2.5567E-14
	Degree 10	2.0255E-17	-.1626E-18	-.1010E-17	-3.207E-17	-5.328E-17	-5.141E-17
Residuals*		0.00042545	0.00079808	0.00022594	0.00074087	0.00091901	0.00126009
R^2**		0.999195	0.998802	0.999744	0.999177	0.999022	0.998692

* Sums of Squares of Residuals about Polynomial
** Coefficient of Determination (R-squared)

Table 2: The results of curve fitting of the flow through the (OCFPAH) thrust bearing with different pressure ratios & with radius ratio = 1.25

Pressure ratio		2	4	7	10	15	20
Polynomial coefficients	Degree 0	-0.0011197	-0.0173592	-0.0158591	-0.0196556	-0.0182668	-0.0182668
	Degree 1	0.703616	0.77185	0.748359	0.759607	0.749732	0.74973
	Degree 2	-0.0511522	-0.0541708	-0.0298796	-0.028081	-0.0214549	-0.0214549
	Degree 3	0.00216023	0.00224434	-0.0011824	-0.0016026	-0.0024620	-0.0024620
	Degree 4	-6.5382E-5	-5.9771E-5	0.00017008	0.00020174	0.00025446	0.00025446
	Degree 5	1.6798E-6	1.0591E-6	-7.7213E-6	-8.9793E-6	-1.0831E-5	-1.0831E-5
	Degree 6	-3.5515E-8	-1.2601E-8	1.9174E-7	2.2146E-7	2.6149E-7	2.6149E-7
	Degree 7	5.3264E-10	1.0033E-10	-2.8491E-9	-3.2808E-9	-3.8232E-9	-3.8232E-9
	Degree 8	-5.011E-12	-5.264E-13	2.525E-11	2.9031E-11	3.3524E-11	3.3524E-11
	Degree 9	2.6124E-14	1.7258E-15	-1.231E-13	-1.415E-13	-1.623E-13	-1.623E-13
	Degree 10	-5.733E-17	-2.817E-18	2.544E-16	2.9231E-16	3.3357E-16	3.3357E-16
Residuals*		0.0118048	0.0646438	0.0686515	0.0940048	0.106286	0.106286
R2**		0.999906	0.999618	0.999676	0.999566	0.99952	0.99952

Table 3: The results of curve fitting of the stiffness of the (OCFPAH) thrust bearings with different pressure ratios & with radius ratio = 1.25

Pressure ratio		2	4	7	10	15	20
Polynomial coefficients	Degree 0	0.00457279	0.0143095	)0.019739	0.0291722	0.0376087	0.0429851
	Degree 1	0.248417	0.253803	0.270389	0.273506	0.275645	0.280146
	Degree 2	-0.0601906	-0.0567191	-0.0608826	-0.0612723	-0.0620105	-0.0632268
	Degree 3	0.00631848	0.00573283	0.00623178	0.00623401	0.00633225	0.00647457
	Degree 4	-0.0003648	-0.0003253	-0.0003574	0.00035574	-0.0003625	-0.0003713
	Degree 5	1.27054E-5	1.12487E-5	1.24514E-5	1.23414E-5	1.26079E-5	1.29242E-5
	Degree 6	-2.778E-7	2.45462E-7	2.73016E-7	-2.6972E-7	-2.7611E-7	-2.8307E-7
	Degree 7	3.83601E-9	-3.3918E-9	-3.7837E-9	3.72822E-9	.82306E-9	3.91805E-9
	Degree 8	-3.245E-11	2.875E-11	-3.213E-11	-3.159E-11	-3.244E-11	-3.323E-11
	Degree 9	1.534E-13	1.363E-13	1.5242E-13	1.4963E-13	1.5383E-13	1.5745E-13
	Degree 10	-3.102E-16	2.764E-16	-3.093E-16	-3.032E-16	-3.12E-16	-3.191E-16
Residuals		0.0059766	0.00628429	0.00990691	0.0106458	0.0123008	0.01262
R²		0.989296	0.991683	0.988507	0.988228	0.986709	0.986828

4.3. The Stiffness of the thrust bearing

Many parameters may also affect the stiffness of the (OCFPAH) thrust bearing. These effects, as in the case of the load and flow calculation, can be seen with a family of design charts. The same procedure was used in collection of the required statistical data and converting them into pairs of mathematical (the restrictor coefficient) and random (the dimensionless stiffness) variables. Similarly, the polynomial model is again used to represent the data. The results obtained for curve fitting with the different conditions are presented in Table 3 and in Figure 5.

4.4. Number of holes

The minimum number of holes is an important design output and it is a function of the number parameter that is defined in the appendix. The data proved that the function is intrinsically linear. Graphing the relation between log (Number parameter) in X axis and Log (Minimum Number parameter) in Y axis results in a straight line with the equation:

*Log (nξ) = -2.54638 * log (np) - 1.89372*

with the following statistics:
- Residual sum of squares = 6.63503
- Coefficient of determination, R-squared = 0.985326
- Residual mean square, sigma-hat-sq'd = 0.315954

This equation is shown in Fig. 6 and it will be used at latter stage in the program.

5. Construction Of The Program

Figure 7 shows the main structure of the program without showing any sub forms. The drop lists that are located at the top of this form allow the user to perform many functions directly. This includes File, Air properties, Dimensions, Calculations, Results, Information, References and Help. Most functions can be performed by other methods like clicking above the designed icons that are shown below the drop list menu in the main tool bar. All other forms of the code have been designed as child forms of the above form. The main choice among all of these varieties for this paper is the calculations sub form shown in Fig. 8 after performing a numerical example. From this calculations form, it can be seen that, many other sub forms can be easily addressed. After completing the entries of the input properties and dimensions, the calculation process can be started by clicking above the calculating icon. The result will be displayed directly at proper locations as illustrated in Fig. 8.
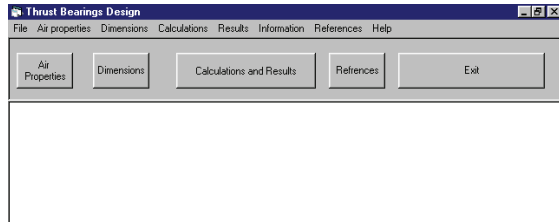
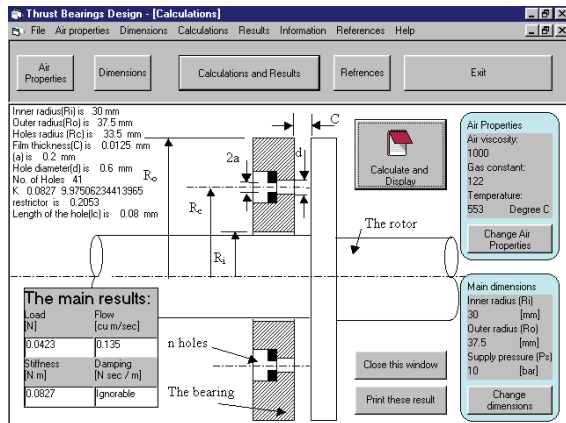Fig. 7. The main form of the program for the design of (OCFPAH) thrust bearing.

Fig. 8. The calculation form including the result of the numerical example.

The code is user-friendly and can be used like any other window software. When starting the program, the user will enter the main menu after passing through the welcoming form. The input data should be entered from the dimensions and air properties forms. All output data is obtained from the calculations and results form. Any change of the input parameters will immediately cause a change in the results. One more feature of the program is the huge amount of information about thrust bearings, which is built-in the code and can be seen from the Information drop list. Many other features of the program will not be explained here because they are from the standard style of many other window applications.

6. Results And Discussion

The code has been used for many trials in the debugging stage until the final edition has been reached. Using this final version is very easy and clear results come out rapidly. Many tasks have been carried out with the code and the results proved that it is work probably as expected. A couple of numerical examples are shown here. The first was worked out previously using the traditional method. This example represents a design of a thrust bearing for a very high speed [around 50000 rpm] rotating tube. Table 4 shows the bearing dimensions which are chosen according to application data. This data is used as inputs to the design chart. Table 4 also shows the main check points that are used as an indicator of the validity of the design chart. The second part of the table shows the dimensionless output quantities and the real value of the load, flow, damping and stiffness. The calculated results by the code, as shown in figure 8, are almost the same as that of the traditional method, used by the author in

previous work [2]. The main difference is in the time consumed and the expertise of the designer. The time consumed in using the traditional method takes many working hours, compared to that of a few minutes in operating or using the procedure. The traditional method requires from the designer to be an expert in the field whilst any person with a little knowledge of the subject can use the code. The second example was carried out to find the effect of changing the pressure supply (from 1-15 bar) on the load, flow and stiffness of the same bearing in the previous example. Figure 9 shows the results of these runs that were performed within a very short time (of less than 30 minutes). Whereas if we did the computations using the traditional method (hand calculations) we would need a very long time (many days), which is impractical.
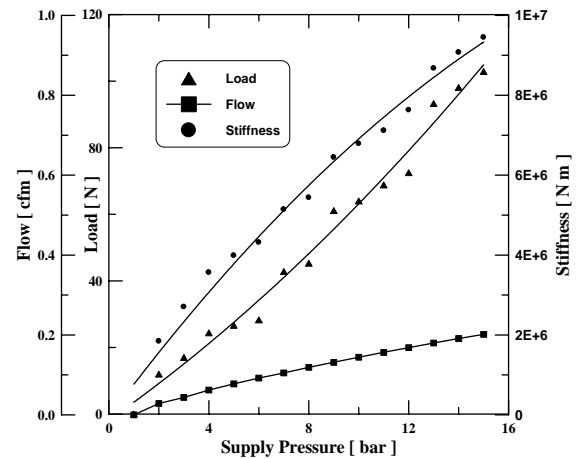

Fig. 9. The graphical results of the effect of supply pressure variance on Load, flow and stiffness.

Table 4: Numerical example of using the code.

Bearing dimensions and air properties:	R_o =37.5 mm $\quad\quad$ p_a=1e5 Pa R_i =30 mm $\quad\quad$ $\sqrt{\Re T} == 298.75$ m sec^{-1} p_s=7.1e5 Pa $\quad\quad$ μ=1.93e-5 N sec m^{-2}
Calculated dimensions	$R_c = \sqrt{R_o R_i} = 33.5$ mm $\quad$ n=41 a=0.2 mm $\quad\quad$ d=0.6 mm C=0.0125 mm $\quad\quad$ lc=0.08 mm
Governing dimensionless parameters	R_o/R_i=1.25 $\quad\quad$ p_s/p_a=7.1 σ =1.5 $\quad$ Λ_s=0.36 The feeder hole volume ratio =0.056
Dimensionless output: $\overline{W}\ \overline{Q}\ \ \overline{K}\ \ \overline{B}$ are not shown as output by code	
Check list	n>n min $\quad$ 5e-3≤C≤2e-3 Pneumatic Hammer, ok $\quad\quad$ Load, ok
Output parameters	W=43.105N $\quad\quad$ K=5.17e6 N m sec^{-1} Q=0.1065 cfm $\quad\quad$ B = Ignored

7. Conclusions

It must be emphasized that the use of this program is not a substitute for the solid understanding of basic principles of bearing design. It can, however, relieve the designer from the burden of performing many routine calculations and provide analytical capabilities far beyond those attainable using hand calculations or user-written subroutines. For these reasons, programs such as (OCFPAH) will rapidly become indispensable tools for industrial designers.

The frequent use of the new approach of the design of this important type of bearing with many distinguished characteristics proved that the proposed procedure was working very well. The new approach saves time, needs less experience and does the job accurately.Future work is planned to integrate this code with other related codes like radial bearings and rotor dynamics.

References

[1] B. Sternlicht, "Gas bearing turbomachinary". Trans. Of ASME, Journal of Lubrication Technology, Vol. 90**,** 1968, 665-679.

[2] AL-Ajlouni M., Studies of the rotating high speed separation systems, PhD Thesis, Cardiff, UK,1996.

[3] Gross W.A, Gas film lubrication. New York and London: John Wiley and sons, Inc, 1962.

[4] Ford GWK., Winfrith UKAEA. The place and scope of gas lubrication. In: Grassam NS, Powell, JW, editors. Gas lubricated bearings, London: Butterworths, 1964, 3-24

[5] Cheng HS. Hydrostatic thrust bearings. Design of gas bearings/ MTI/RPI manual. Latham, New York, 1966.

[6] Cheng HS. Basic relations. Design of gas bearings/ MTI/RPI manual. Latham, New York, 1966.

[7] DD. Fuller, "A Review of the state-of-the-Art for the Design of self-Acting Gas-Lubricated bearings". Trans. Of ASME, Journal of Lubrication Technology, Vol. 91**,** 1969, 1-16.

[8] Vohr JH. Restrictor flow. Design of gas bearings/ MTI/RPI manual. Latham, New York 1966.

[9] Lund JW. Dynamic performance and stability. Design of gas bearings/ MTI/RPI manual. Latham, New York, 1966.

[10] Johnson, RA. Miller and Freund's Probability and Statistics for Engineers. Upper Saddle River, USA: Prentice Hall, 2000.

[11] Hines WW, Montgomery DC. Probability and Statistics in Engineering and Management Science. New York and others: John Wiley and sons,1990.

[12] Dovore JL. Probability and Statistics for Engineering and Sciences. Fourth edition London: Duxbury Press, 2004.

[13] Montgomery D. C., Runger G. C. and Hubele N. F., Engineering Statistics. Fourth edition. New York and others: John Wiley and sons,2007.

[14] Montgomery D. C., Design and Analysis of Experiments. Sixth edition. John Wiley and sons, New York ,2005.