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Quota Allocation to Distributors of the Supply Chain under Distributors' Uncertainty and Demand Uncertainty by Using Fuzzy Goal Programming

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Abstract

The supply chain consists of multiple components including suppliers, manufacturing centers, and distributors. The decision-making on quota allocation to distributors under Distributors' uncertainty and demand uncertainty are important parts of supply chain of many firms. In this paper, fuzzy goal programming approach is applied for quota allocation to distributors of the supply chain. Customers are assumed as a random variable, and distribution is continuous and normal. Due to surge effect, the demand through supply chain is varied, which is calculated for a particular inventory policy for reserve stock, and sudden rise and fall of demand to the Std limits of normal distribution. The maximum and minimum values of demand at the distributors' stage are considered for various Std limits. And they formulate a fuzzy goal programming by considering linear member ship function. Commercially available LINDO software is used to solve the fuzzy goal-programming problem for quota allocation to the distributors. From the results, it is found that maximized sales revenue, minimized transportation cost, minimized late deliveries, and minimized defective items are increased from maximum STD limit to the minimum STD limit. Change in minimized late deliveries and minimized defective items are negligible to consider. Moreover, difference of maximized sales revenue and minimized transportation cost is significant to consider, and it is increased from maximum STD limit to the minimum Std limit of demand. This means maximum difference can be obtained at low fluctuation of the demand than the high fluctuation of the demand. The formulated Fuzzy Goal Program can be used to solve actual problems.

Keywords: Normal continuous distribution, Linear membership function, Fuzzy goal programming, and Decision-making.

1. Introduction

Supply chain is a set of facilities, supplies, customers, products; and methods of controlling inventory, purchasing, and distribution. In a supply chain, flow of goods between a supplier and a customer passes through several echelons, and each echelon may consist of many facilities.

This paper focuses mainly on distribution performance and quota allocation to distributors and its selection under distributors' uncertainty and demand uncertainty. In designing a supply chain, a decision maker must consider decisions regarding the selection of the right distributors and their quota allocation. The choice of the right distributor is a crucial decision with extensive implications. By nature, distributor selection is a multi-criterion decision-making problem. A supply chain decision faces many constraints. Some of these are related to distributors' internal policy and externally imposed system requirements. In such decision making situations, high degree of fuzziness and uncertainties are involved. Fuzzy set theory provides a framework for handling the uncertainties of this type [6].

In this paper, a fuzzy goal programming approach is used to solve the multi-objective-optimization problems for quota allocation to the distributors in supply chain. Since crisp set assign a value of either 1 or 0. Whereas in fuzzy set is not assigned such a value. But the value of any set lies between 1 and 0. A function can be generalized such that the value assigned to the element of the universe set fall within specified range and indicated member ship grade of this element in the set. Such a function is called a

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fuzzy membership function, and set defined by it is called a fuzzy set [9].

Membership functions which can be used in fuzzy goal programming problem are linear membership functions or no linear member ship functions. Fuzzy mathematical programming which is defined by a non-linear membership function results in a non-linear programming. Usually a linear membership function is employed for linear programming in order to avoid non-linearity [7]. Therefore, in this paper, a linear membership function is employed for both objectives and constraints, which is having fuzziness.

The central four fuzzy goals considered in this paper are maximization of sales revenue, minimization of transportation cost, minimization of defective items rejected and minimization of late deliveries with constrain such as minimum and maximum capacity of distributors, maximum budget allocated to distributors, and maximum flexibility of distributors and maximum sales value of distributors. First, the problem is formulated as multi-objective linear programming, and then it is reformulated as fuggy goal programming by using variable λ. Commercial available LINDO software is used to solve the fuzzy goal-programming problem for quota allocation to distributors.

2. Review of Literature:

Complex and dynamic interactions between supply chain entities lead to considerable uncertainty in planning. Uncertainty tends to propagate the supply chain up and down. And it undergoes with surge or bullwhip effect [1]. Many proposed strategies for mitigating the bullwhip effect have a history of successful application [2]. This effect leads to inefficiencies in supply chains since it increases the cost for logistics, and it lowers its competitive ability. Particularly, the bullwhip effects negatively influence a supply chain in dimensioning of capacities, variation of demand, and high level of safety stock. Julija Petuhova and Yuri Merkuryev [8] carried out simulations to measure distributor's performance before and after applying the supply-side collaboration. Their results show that the supply-side collaboration can improve distributor's performance in terms of more accurate service level realization and better stabilizing effect.

Manoj Kumar , Prem Vrat and R.Shankar proposed a fuzzy multi-objective goal programming by using triangular membership function for allocating quantity to vendors; and solved this by LINDO software. The proposed approach has the capability to handle realistic situations in a fuzzy environment and provides a better decision tool for the vendor selection decision in a supply chain. Hasan Selim, Ceyhun Araz and Irem Ozkarahan[4] had developed a multi-objective production-distribution planning model in the fuzzy environment for to improve the supply chain performance. Pandian Vasant[7] attempted to model decision processes with multiple criteria in business and engineering leads to concepts of multi objective fuzzy linear programming. Hasan Selim and Irem Ozkarahan have developed a supply chain distribution network design model. The goal of the model is to select optimum numbers, locations, and capacity levels of plants and warehouses to deliver products to retailers at the least cost while satisfying desired service level. Maximal covering approach is employed in statement of service level [10].

3. Why is Simulation and Fuzzy Set Theory Used in Supply Chain?

Due to the Bullwhip effect, a poor plan can easily spread to the whole supply chain areas. The impact of a poor plan on the overall business is huge. It causes cycles of excessive inventory and severe backlogs, poor product forecasts, unbalanced capacities, poor customer service, uncertain production plans, and high backlog costs, or sometimes even lost sales. Simulation and fuzzy set theory permit the evaluation of operating performance prior to the implementation of a system and also these enable companies to perform powerfully if analyses lead them to better planning decisions.

4. A Mathematical Model to Calculate Output Required at Each Stage in The Supply Chain for Uncertainty Demand.

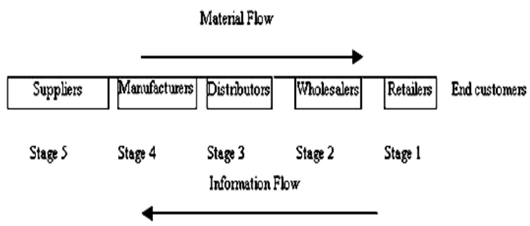

Figure 1: Five stages of simple supply chain

A mathematical model is shown below to calculate output required and safety stock required at each stage of the supply chain. For these typical five stages, simple supply chain is shown in Figure 1. The five stages are suppliers, manufacturers, distributors, wholesalers, and retailers. If the demand by End-customers is D, and the policy for reserve stock specifies that a portion (r) should be held constant, then (rD) is the safety stock for finished products at the retailer's stores. If there is a sudden rise or a sudden fall of demand at end-customers, then D (1+x) quantities of products are needed at the retailers, and also the safety stock required now should be rD (1+x). Here (x) is in terms of percentage of sudden rise or sudden fall of the demand [3]. The (x) in terms of percentage for 3Std limit of normal distribution is shown below

a.For sudden rise of the demand :

$$x = \frac{(D + 3\,Std\,) - D}{D}\ X\ 100$$

b.For a sudden fall of the demand

$$x = \frac{(D - 3\,Std\,) - D}{D}\ X\ 100$$

Similarly x in terms of percentage can be shown for all other Std limits of normal distribution. Then the output required and safety stock required at each stage will be calculated by using the following equations.

Stage 1: Retailer

- Output required = D (1+x)
- Safety stock required = rD (1+x)

Stage 2: Wholesalers

- Output required = D (1+x) for the final stage + (rD (1+x) – rD) for the Safety stock at the final stage = D (1+x (1+r))
- Safety stock required = rD (1+x (1+r))

Stage 3: Distributors

- Output required = D (1+x (1+r)) for the 2 stage + (rD (1+x (1+r)) – rD) for the safety stock at the 2 stage= D (1+x (1+r) 2)
- Safety stock required = rD (1+x (1+r) 2)

Stage 4: Factories

- Output required = $D(1+x(1+r)^2)$for the 3 stage + $(rD(1+x(1+r)^2)$– rD) for the safety stock at the 3 stage= $D (1+x (1+r)^3)$
- Safety stock required = $rD (1+x (1+r)^3)$

Stage 5: Suppliers

- Output required = $D(1+x(1+r)^3)$for the 4 stage + $(rD(1+x(1+r)^3)$– rD) for the safety stock at the 4 stage= $D (1+x (1+r)^4)$
- Safety stock required = $rD (1+x (1+r)^4)$

And so on. It can be shown that the output required at the n^{th} stage is D_n then [3] $D_n/D = 1+x(1+r)^{n-1}$

5. Multi-Objective Distributor Model Under Distributors' Uncertainty and Demand Uncertainty

Four distributors and four main objectives are considered such as: maximizing sales revenue (Z_1), minimizing transportation cost (Z_2), minimizing defective items rejected (Z_3), and minimizing late deliveries (Z_4). Besides, the constraints considered for formulation of the problem are minimum and maximum selling capacity of the distributors, maximum budget allocation to distributors, flexibility of distributors, and sales value of distributors. By considering above information, the problem is formulated as multi objective linear program as shown below [5].

5.1. Decision Variable

x_i order quantity from the distributors I, where i=1,2…..N

5.2. Parameters

D Max. = Upper bound of aggregate demand of the item over a fixed planning period
D Min. = Lower bound of aggregate demand of the item over a fixed planning period
N = Number of distributors competing for selection
p_i = Price of a unit item at distributors i
t_i = Transportation cost of a unit item of the ordered quantity x_i for the distributors i
ld_i = Percentage of the late delivered units by the manufacturers to distributors i
C_i Max. = Upper bound of the quantity can be taken by distributors I
C_i Min. = lower bound of the quantity can be taken by distributors i
B_i = Budget allocated to each distributors
d_i = Percentage of rejected units delivered by distributors i
F_i = Supplier quota flexibility for distributors i
F Max = Upper bound of total flexibility in quota that a distributors should have
F Min = Lower bound of total flexibility in quota that a distributors should have
R_i = distributors rating value for distributors i
PV Max.= Upper bound to total purchasing value that a distributors should have
PV Min.= Lower bound to total purchasing value that a distributors should have:

Min. $Z = (Z_1, Z_2, Z_3, Z_4)$
Subjected to
$\sum x_i \le D$, D is varying form D min. to D max.
$x_i \le C_i$, C_i is varying form C_i min. to C_i max
$x_i \le B_i$

$\sum F_i x_i \ge F$, F is varying form F min. to F max
$\sum R_i x_i \ge P$, P is varying form P min. to P max
$x_i \ge 0$ and $Z_1 = \sum p_i x_i$, $Z_2 = \sum t_i x_i$, Z_3 $\sum d_i x_i$, $Z_4 = \sum ld_i x_i$

6. Fuzzy Multi-Objective Model

Consider the fowling linear multi-objective model,

Opt Z = CX
s.t. AX ≤ b

Where $Z = (z_1, z_2, \ldots\ldots\ldots z_k)$ is the vector of objectives, C is a K*N of constants, X is a an N*1 vector of the decision variables, A is an M*N matrix of constants and b is an M*1 vector of constants. This model can be applied to solve many real problems [4].To solve above problem a linear membership function can be used for each goal $\mu_{1k}(C_k X)$, where

$$\mu_{1k}(C_k X) = \begin{cases} 1 & \text{if } CkX >= Z_k \\ 1 - \dfrac{(Z_k - C_k X)}{dlk} & \text{if } Z_k - d_{1k} <= CkX <= Z_k \\ 0 & \text{if } CkX <= Z_k - d_{1k} \end{cases}$$

And another linear membership function is $\mu_{2i}(a_i X)$, for the i^{th} constraint in the system constraints AX ≤ b, where

$$\mu_{2i}(a_i X)) = \begin{cases} 1 & \text{if } a_i X <= b_k \\ \dfrac{1 - (a_i X - b_k)}{d_{2i}} & \text{if } b_k <= a_i X <= b_i + d_{2i} \\ 0 & \text{if } a_i X >= b_i + d_{2i} \end{cases}$$

Figure 2: Membership function for maximization of fuzzy goal

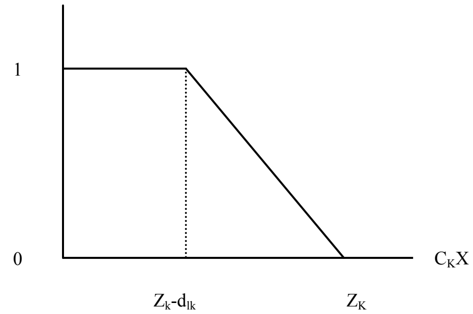

Figure 3: Membership function for minimization of fuzzy goal

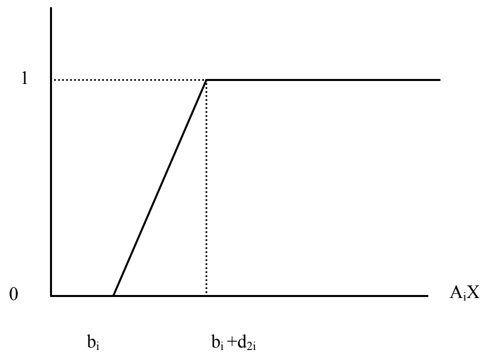

Figure 4: Membership function of fuzzy constraints

These membership functions are illustrated in Figure 2, Figure 3, and Figure 4 respectively. Where d_{1k} (k=1,2,….K) and d_{2i} (i=1,2,….M) are chosen constants of admissible violations, and a_i is the i^{th} row of matrix A. $\mu_{1k}(C_kX)$ and $\mu_{2i}(a_iX)$ denote to the degree of membership of goals and constraints respectively. Degree of membership of goals and constraints express satisfaction of the decision maker with the solution. So, values of membership functions must be maximized [4].

In one of the fuzzy set theorems, membership function of intersection of any two (or more) sets is the minimum membership function of these sets. After eliciting linear membership functions and by applying this theorem, objective function of multi-objective linear programming model incorporating the fuzzy goals and fuzzy constraints can be formulated as follows [4].

$\text{Max}_x \min(\mu_{11}(C_1X), \ldots, \mu_{1k}(C_kX), \mu_{21}(a_1X), \ldots, \mu_{2i}(a_iX))$

By introducing the auxiliary variableλ, this problem can be equivalently transformed as.

max λ

$\mu_{1k}(C_kX) \geq \lambda$ k=1,2,….K

$\mu_{2i}(a_iX) \geq \lambda$ i=1,2,….M

According to above descriptions fuzzy linear program can be rewritten as following:

Max λ

$$\lambda \leq \frac{1-(Z_k - C_kX)}{d_{1k}} \qquad k=1,2,\ldots K$$

$$\lambda \leq \frac{1-(a_iX - b_i)}{d_{2k}} \qquad i=1,2,\ldots M$$

$0 \leq \lambda \leq 1$ and $X \geq 0$

7. Fuzzy Goal Programming

Fuzzy Goal programming is one of the most powerful multi-objective decision making approach. If there are no priorities and also no relative importance assigned to objectives, formulation of fuzzy goal-programming model is similar to formulation in general fuzzy linear programming model. The main difference between Fuzzy goal programming and fuzzy linear programming is that fuzzy linear programming uses definite intervals determined by solution of linear programming models. And accordingly, the solution does not change from decision maker to another decision maker. Whereas in fuzzy goal programming, aspiration levels are specified by decision maker and reflect relative flexibility [4].

8. Methodology

The following methodology is used in three steps for quota allocation to distributors of the supply chain under Distributors' uncertainty and demand uncertainty by using fuzzy goal programming. In the first step, demand at end customer is assumed as random variable and distributed as continuous normal distributed pattern. The Monte Carlo simulation method and Excel are used to simulate the demand for various random numbers. The 20 random numbers are generated by using Excel with command Rand ().The mean demand and standard deviation are calculated for demands of 20 random numbers.

In the second step, output required or demand for five stages of simple supply chain is simulated for various Std limits of normal distribution. Output required from down stream to up stream (stage-1 to stage-5) is increased for sudden raise of the demand and decreased for sudden fall of the demand at particular inventory policy for reserve stock r. Therefore, this variability of demand from stage -1 to stage-5 is called as surge effect. So that surge effect is simulated here for five stages supply chain.

In the third step, an illustration for quota allocation to distributors under distributors' uncertainty is taken, and maximum and minimum outputs required for all STD limits at the stage of distributor are taken for demand uncertainty at distributors. For this, first multi objective linear programming problem is formulated; and is solved for individual objectives. With these results of individual objectives, fuzzy aspiration levels are fixed. And finally fuzzy goal programming problem is formulated and solved by using commercial available software LINDO.

9. Simulation of Demand Uncertainty at End Customer

The demand at end customer is assumed as random variable and distributed as continuous normal distributed pattern. In simulation process, mean of demand is assumed as (500) units and standard deviation Std is assumed as (25) units. The demand is calculated for the normal distribution limits such as ±3Std, ±2Std, ± 1Std and ±0.67Std and cumulative probabilities are taken from normal distribution tables to the Z values such as –3, -2, -1, 0, 1, 2 and 3 [3]. A graph has been plotted is shown in Figure 5 for calculated values of demand and cumulative values of probability distribution.

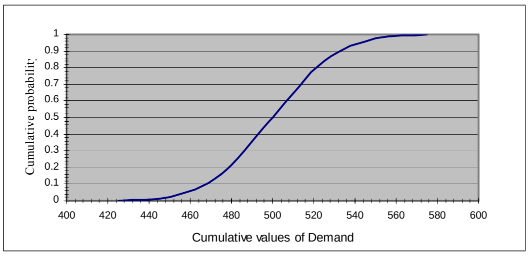

Figure 5: calculated values of demand and cumulative values of probability distribution.

The Monte Carlo simulation method Microsoft Excel is used to simulate the demand for various random numbers. The 20 random numbers are generated by the command Rand () and 20 numbers are considered as the probability of occurrence of the demand for 20 months. The value of demands for 20 months are taken from graph (From Figure 5) for corresponding random numbers and tabulated in Table 1. The mean and standard deviation are calculated for demands of 20 numbers.

10. Simulation of Surge Effect in Supply Chain

Surge effect in five stages supply chain is simulated for minimum and maximum fluctuation of the demand at end costumer. Five stages supply chain is shown in Figure1.

Here the demand is considered as two different cases for the limits of 0.67Std, 1Std, 2Std, 3Std of the normal distribution curve. In the first case, sudden rise of demand is considered from mean to upper limit of the demand. In the second case, sudden fall of demand is considered from mean to lower limit of demand. The (x) in terms of percentage for both the cases is calculated using the formulas, shown in the section-5.

Table 1: The value of demands for 20 months

Months	Random numbers	Simulated demand (X)	D - X	$(D-X)^2$
1	0.30236	485	16.2	262.44
2	0.99278	572	-70.8	5012.64
3	0.047325	455	46.2	2134.44
4	0.662034	510	-8.8	77.44
5	0.293601	480	21.2	449.44
6	0.901139	530	-28.8	829.44
7	0.125719	468	33.2	1102.24
8	0.749202	515	-13.8	190.44
9	0.694485	510	-8.8	77.44
10	0.184959	472	29.2	852.64
11	0.901331	530	-28.8	829.44
12	0.367548	490	11.2	125.44
13	0.459338	492	9.2	84.64
14	0.696288	512	-10.8	116.64
15	0.406118	490	11.2	125.44
16	0.619101	508	-6.8	46.24
17	0.566073	504	-2.8	7.84
18	0.188001	477	24.2	585.64
19	0.323692	490	11.2	125.44
20	0.263223	482	19.2	368.64
		9972	52	13404

*Mean (D)=498.6 Mean (D) = $\frac{\sum_{i}^{n} Xi}{n} = 498.6$

*Standard deviation (Std) = $\sqrt{\frac{\Sigma (D - X)^2}{n}} = 25.88822$

By using this (x) in terms of percentage and inventory policy for reserve stock (r) in terms of percentage, quantity required at various stages through supply chain from downstream to upstream is calculated by using equation presented in section-5. The results are tabulated in table-2 and a graph is drawn and shown in Figure 6 for policy decision r = 20% and x for sudden rise and for sudden fall of demand.

Table 2 : Quantity required at various stages in supply chain for sudden rise and sudden fall of demand and Inventory policy for reserve stock r =20%

	Upper standard limits			
	0.67std	1std	2std	3std
	X% of sudden rising			
	0.034788	0.051922	0.103844	0.155765
Retailers	515.9453	524.4883	550.3766	576.2644
Whole -salers	519.4144	529.666	560.7319	591.7973
Distributors	523.5772	535.8792	573.1583	610.4368
Factories	528.5727	543.335	588.07	632.8041
Suppliers	534.5672	552.282	605.964	659.645
	Lower standard limits			
	.0.67std	1std	2std	3std
	X% of sudden falling			
	0.034788	0.051922	0.103844	0.155765
Retailers	481.2547	472.7117	446.8234	420.9356
Whole -salers	477.7856	467.534	436.4681	405.4027
Distributors	473.6228	461.3208	424.0417	386.7632
Factories	468.6273	453.865	409.13	364.3959
Suppliers	462.6328	444.918	391.236	337.555

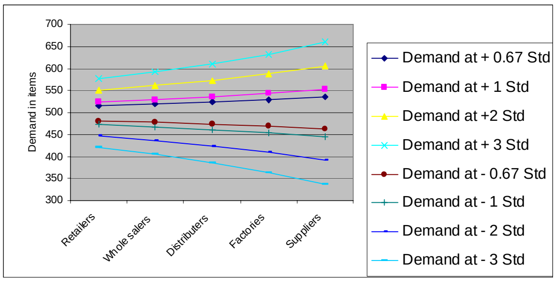

Figure 6: for policy decision r=20% and x for sudden rise and for sudden fall of demand.

11. Quota Allocation to Distributors Under Distributors' Uncertainty and Demand Uncertainty by Using Fuzzy Goal Programming Approach

11.1. Distributors' Uncertainty

Distributors' uncertainty is such that distributors con not take fixed quantity from manufacturers. Quantity taken by uncertain distributors is varying from time to time. Generally this type of distributors can give good performance in certain range of quantity taken from the manufacturers i.e. between minimum quantities which the distributors can receive to maximum quantity distributors can be received from manufactures or factories. When distributors get below minimum quantity, they cannot give good performance due to loss of their business. Besides, distributors cannot get beyond maximum quantity due to their maximum constraint.

11.2. Demand Uncertainty

Similarly, demand of any item at end customers is dynamics; and is not fixed to a certain value. This also varies from time to time. Demand is assumed as continuous random variable and distribution as continuous normal distribution at end customers. Plus and minus Std limits of normal distribution are considered for variable demand. By using this end customers demand, demand at the distributors stage is calculated for each Std limit. This calculated demand at distributors' stage is used in the multi objective linear program. The overall flexibility and overall purchase value rating variability are also considered for distributors of the supply chain.

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11.3. Variability of The Demand

The variability of demands (i.e. $D_{min.}$ and $D_{max.}$) at stage of distributor in supply chain for all Std limits are taken form Table 2 and tabulate separately in the following Table 3.

Table 3 Variability of Demand for r=20%and Various Std Limits at Stage of Distributes in Supply Chain

STD limits of Normal distribution	Variability of demand at the stage of distributors ($D_{min.}$ - D_{max})
0.67 Std	474 - 524
1 Std	461 – 536
2 Std	424 – 573
3 Std	387 – 610

Table 4: Distributors Source Data of The Illustrative Example

Distributors No.	s_i Rs.	t_i Rs	d_i (%)	ld_i (%)	C_i Min. Units
1	100	15	3	2	50
2	300	10	4	3	15
3	250	5	2	9	25
4	350	20	7	5	30
Distributors No.	C_i Max. Units	B_i Rs	Fi (%)	R_i (%)	Distributors No.
1	200	20000	99.6	86	1
2	235	70500	99.9	92	2
3	225	56250	9.98	98	3
4	220	77000	9.97	88	4

11.4. An Illustration of Quota Allocation to Distributors

Distributors' source data of the illustrated example is prepared based on available data, where data is presented for the supplier selection problem [6]. Distributors source data of the illustrated example shown in Table 4[6] represents the data source for the sale price quoted (s_i in rupees per unit); transportation cost (t_i in rupees per unit); the percentage defective items rejections d_i; the percentage of late deliveries to distributors ldi; minimum distributors capacity limitations $C_{i;}$ min. units, maximum distributors capacity limitations $Ci_{;}$ max. units; the budget allocations for distributors B_i; distributors' quota flexibility F_i and distributors sales value rating R_i on a scale of 0–1. The least value of flexibility in distributors' quota and least total purchase value of distributors' items are policy decisions; and depend on the demand. The least and maximum value of flexibility in distributors' quota is given as $F = F_oD$, and the least and maximum total purchase value of distributed items is given as PV =RD: If overall least flexibility (F_o) is 0.05 and maximum flexibility (F_o) is 0.95 on the scale of 0–1, the overall least distributors' rating (R) is 0.10 and maximum rating (R) 0.90 on the scale of 0–1, and the mean demand (D) of normal distribution is 500. Then the least and maximum value of flexibility in distributors' quota (F) and the least and maximum total purchase value of distributed items (PV) are 25, 475, 50 and 450, respectively. By using variability demand from Table 4 and distributors' source data from Table 5, a multi-objective linear programming problem for quota allocation to distributors can be formulated as:

Maximize $Z_1 =100x_1 + 300x_2 + 250x_3 + 350x_4$

Minimize $Z_2 = 15x_1 + 10x_2 + 5x_3 + 1x_4$

Minimize $Z_3 = 0.03x_1 + 0.04 x_2 + 0.02 x_3 + 0.07 x_4$

Minimize $Z_4 = 0.02x_1 + 0.03x_2 + 0.09x_3 + 0.05x_4$

Subjected to

$474 \geq x_1 + x_2 + x_3 + x_4 \leq 524$ (0.67 Std)

$50 \geq x_1 \leq 200,\ 15 \geq x_2 \leq 235,\ 25 \geq x_3 \leq 225,\ 30 \geq x_4 \leq 220$

$100x_1 \leq 20000,\ 300x_2 \leq 70500,\ 250x_3 \leq 56250,$

$350x_4 \leq 77000$

$25 \leq 0.996x_1 + 0.999 x_2 + 0.998 x_3 + 0.997 x_4 \geq 475$

$50 \leq 0.86x_1 + 0.92 x_2 + 0.98 x_3 + 0.88 x_4 \geq 450$

$x_1, x_2, x_3, x_4 \geq 0$

11.5. Aspiration Levels or Fuzzy Range:

In the Table 5, LPP Results of the individual objectives for minimum and maximum bound of constraints are shown for 0.67 Std limit.

Table 5: LPP Results of the Individual Objectives for Minimum and Maximum Bound of Constraints at 0.67 Std Limit

S. No	Objectives	Min. bound	Max bound	Difference	Max bound Moved to	Fuzzy range
1	Max. Sales revenue	26250.00	164750.0	138500	303250	277000
2	Min. transportation cost	150.4082	1735.217	1584.8088	3320.0258	3169.6176
3	Min. Defective items rejected	1.3895	13	12.389535	25.3895	24.3895
4	Min. Late deliveries	1.2283	14.5614	13.3331	27.8946	26.6662

In case of goal programming selection of aspiration levels or fuzzy range is most important. From Table 5 for maximization of sales revenue, the difference between 26250.00 and 164750.0 is 138500. Then the maximum bound is moved to the 164750.0 + 138500 = 3032350**.** Therefore, now the fuzzy range is 2 X 1385000 = 277000 to maximize the sales revenue. Similarly, fuzzy range for other objectives can be calculated.

11.6. Linear Membership Functions

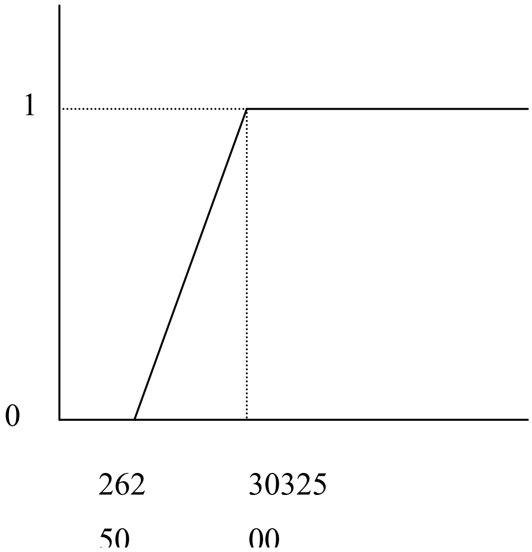

Figure 7: Linear membership function for sales revenue at 0.67 Std (Maximization)

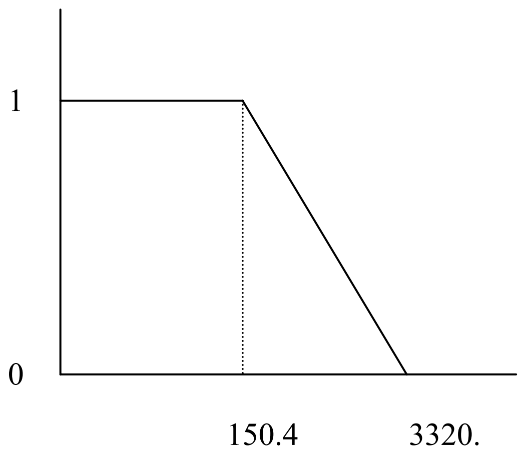

Figure 8: Linear membership function for transportation cost at 0.67 Std (Minimization)

Figure9: Linear membership function for demand variation at 0.67 Std

The Linear membership functions for other objectives like minimization of defective items rejected and minimization of late deliveries are same as Figure 8. Similarly the linear membership functions for other constraints like capacity limitation of distributors, flexibility of distributors and sales value of distributors are same as Figure 9.

11.7. Fuzzy Goal Programming

By using data from Table 6 and linear membership functions and introducing the auxiliary variable λ, the fuzzy goal programming problem can be written as follows for 0.67 Std limit.

Max λ

Subjected to

$$\lambda \le \frac{Z_1 - 2625}{277000}, \lambda \le \frac{3320.0258 - Z_2}{3169.6176}$$

$$\lambda \le \frac{25.3895 - Z_3}{24.3895}, \lambda \le \frac{27.8946 - Z_4}{26.6662}$$

$$\lambda \le \frac{x_1 + x_2 + x_3 + x_4 - 474}{50}, \lambda \le \frac{x_1 - 50}{150}$$

$$\lambda \le \frac{x_2 - 15}{220}, \lambda \le \frac{x_3 - 25}{200} \quad \lambda \le \frac{x_4 - 30}{190}$$

$$100x_1 \le 20000, 300x_2 \le 70500$$

$$250x_3 \le 56250, \; 350x_4 \le 77000$$

$$\lambda \le \frac{0.996x_1 + 0.999x_2 + 0.998x_3 + 0.997x_4 - 25}{450}$$

$$\lambda \le \frac{0.86x_1 + 0.92x_2 + 0.98x_3 + 0.88x_4 - 50}{400}$$

$x_1, x_2, x_3, x_4 \ge 0, \; 0 \le \lambda \le 1$

11.8. Results and Discussions:

Table 6: Allocation of items for four distributors by Fuzzy goal programming

Standard limits	x_1 in No. of items	x_2 in No. of items	x_3 in No. of items	x_4 in No. of items	λ	Total of x_1, x_2,x_3 & x_4 in No. of items	Variability of demand in No. of items
Mean 500 demand	74	109	141	176	0.16	500	No variability exactly =500
o.67 Std	70	109	119	183	0.126	481	474 - 524
1 Std	72	104	116	180	0.145	472	461 – 536
2 Std	78	86	111	177	0.182	452	424 – 573
3 Std	83	70	108	174	0.214	435	387 – 610

Table 7: Optimized values of the four objectives at various std Limits

Standard limits	Max. Sales revenue in Rs. (1)	Min. transportation cost in Rs. (2)	Total cost = diff. of (1) & (2)	Min. defective items rejected in No.s	Min. Late deliveries in No. s
Mean 500 demand	136950	3081	133869	22	27
0.67 Std	133500	2918	130582	22	25
1 Std	130400	2880	127520	22	24
2 Std	123300	2557	120743	21	23
3 Std	117200	2662	114538	20	23

Formulated above goal program is solved for various Std limits of normally distributed demand. The corresponding results are in Table 6 and Table 7. In Table 6, quantity allocated to four distributors and sum of allocated quantity of distributors are present. In Table 6, the total quantity is maintained within the range of variable demand of standard limits. In the Table 7, optimized values of all four objectives are present.

Optimized values of four objectives are decreased for varying from 0.67Std limit to 3Std limit. The effect of minimized defective items rejected and minimization of late deliveries are nominal. But effect of maximization sales revenue and minimization of transportation cost are significant. Both are decreased for varying from 0.67Std limit to 3Std limit. Therefore, the difference is more at minimum fluctuation of demand than maximum fluctuation of demand. Figure 10 shows the variation of maximized revenue, minimized transportation cost and difference of two for varying from 0.67Std limit to 3Std limit. Finally, this can be said that loss of business for any supply chain is more at maximum fluctuation of demand than minimum fluctuation of demand.

12. Conclusions:

- The surge effect in supply chain is simulated for uncertainty demand. The demand of the end customer is assumed as random demand of continuous normal distribution.
- From the simulation results, it is found that the quantity required at the stages through the supply chain from down stream to upstream is increased for sudden rise of demand of the end customers, and is decreased for the sudden fall of demand of the end customers.
- Fuzzy goal programming problem is successfully formulated for multi objectives of distributors and for demand varying from maximum limit to the minimum limit at the stage of the distributors.

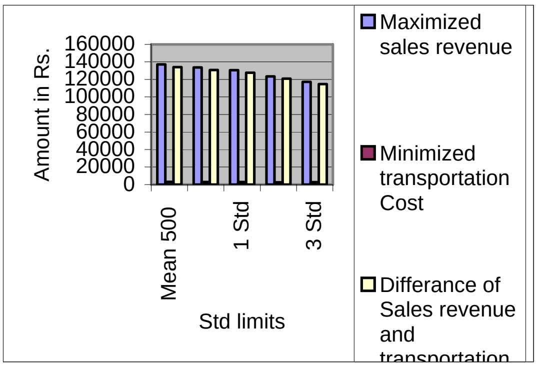

Figure 10: Sales revenue, Transportation cost and difference of two

- Neglecting the effect of optimized results of defective items rejected and late deliveries, the difference of maximization of sales revenue and minimization of transportation cost is more at minimum fluctuation of demand than the maximum fluctuation of demand.

- The total allocated items are also maintained within the range of the variable demand at the stage of distributors. At minimum fluctuation of demand, the items allocated to distributors are more, so that it has been given high difference.

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