JJMIE

Volume 2, Number 4, December. 2008
ISSN 1995-6665
Pages 175 - 182

Jordan Journal of Mechanical and Industrial Engineering

Fuzzy Genetic Prioritization in Multi-Criteria Decision Problems

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Abstract

A new method is presented to derive a priority vector $W = (w_1 , w_2 , \ldots w_n)^T$ defining the ranking of competing alternatives and factors or criteria from fuzzy pairwise judgments. The pairwise comparisons are accepted as linguistic evaluations or assessments expressing relative importance of pairs (i , j) .These evaluations are quantified in the form of trapezoidal fuzzy numbers expressed as quadruples (a_{ij}, b_{ij}, c_{ij}, d_{ij}) in order to model the vagueness and imprecision in linguistic evaluations. The problem of finding components of priority vector W, given n (n-1) / 2 quadruples (a_{ij}, b_{ij}, c_{ij}, d_{ij}) is formulated in an optimization model with a newly introduced objective function. The problem is solved by means of a genetic algorithm. The complications and inappropriateness in finding inconsistencies of fuzzy pairwise comparisons, as presented in existing literatures, are treated and resolved in the present work by introducing ratios of inconsistency index to index of inconsistency of random fuzzy comparisons. The proposed method is illustrated by numerical examples and compared with some of the existing methods in literatures.

Keywords: priority vector, trapezoidal fuzzy numbers, genetic algorithm, pairwise comparisons, inconsistency ratio.

Nomenclature

a_{ij}, b_{ij}, c_{ij}, d_{ij}	: a trapezoidal fuzzy number expressing linguistic pair-wise Judgment relating alternatives i to j
EV i j	: pair wise comparison judgments comparing i with j.
ICI	: Inconsistency Index
ICR	: Inconsistency Ratio
l i j, m i j, u i j ,	: triangular fuzzy number expressing linguistic pair-wise Judgment relating alternatives i to j
n	: size of the pairwise comparisons (reciprocal) matrix
RI	: Random Index
W	: (w1 , w2 , …. w n)T a priority vector giving weight of importance w i to each

Competing Alternative

δ_A	: Indicator function
$\delta_A = 1$	: when event A occurs
$\delta_A = 0$	: otherwise
λ	: a variable denoting to the degree of membership of the ratio w_i / w_j
$\mu_{ij}(w_i / w_j)$	: membership functions of fuzzy ratios w_i / w_j of relative importance

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1. Introduction

Selecting the optimum alternative in multi-criteria decision problems is one of the most crucial challenges facing decision makers in engineering and management in different industries and businesses. These challenges are:

1. The natural limitations of human capability to compare or to decide on among more than two factors or alternatives. It becomes more intricate if the comparison is made on the basis of multiple criteria.
2. To capture and assess possible inconsistencies in comparison judgments of more than two factors or alternatives for the purpose of discarding heavily inconsistent judgments.
3. The uncertainty, imprecision and vagueness of human comparison judgments.

Over the last thirty years, numerous valuable contributions to the study and analysis of these problems were elaborated. These endeavors started with the development of Analytical Hierarchy Process (AHP) by T.L. Saaty [12] as a mathematical model built to derive priority vectors, which arrange competing alternatives, factors and/or criteria from pair-wise comparison judgments. The process of deriving an analytical hierarchy starts usually with forming a pair-wise comparison matrix with elements a_{ij} $(i < j)$ carrying values of relative importance of alternative (factor, criterion) ***i*** as compared to alternative (factor, criterion) ***j***. The number of elements necessary and sufficient to form a pairwise comparison matrix of size ***n*** is equal to *n (n-1) / 2,* since elements a_{ij} equal to $1/ a_{ji}$.

AHP uses a 9 point scale of importance in finding a_{ij}. After building a reciprocal matrix, AHP proceeds further to show that priority vector $W = (w_1, w_2, \ldots, w_n)^T$ is the principal eigenvector of the reciprocal matrix. The corresponding Eigen value is shown to be equal to n in case of perfectly consistent pair-wise comparison judgments.

For nearly consistent and inconsistent judgments, Eigen values are larger than n. Other methods of deriving priorities are considered in several literatures. Direct Least Square (DLS) method [5] and Logarithmic Least Square method (LLS) [6] formulated the problem of deriving priorities in the form of nonlinear programs. DLS and LLS methods have drawbacks - they have multiple solutions, and they lack explicit measures of inconsistency of pair-wise comparison judgments similar to that clearly adopted in the AHP.

The possibility of having inconsistent judgments is a pivotal issue calling for serious concerns since the validity and the credibility of prioritization are, to a great extent, dependant on consistency of decision maker's judgments. Decision makers, usually, express their relative evaluations linguistically rather than in exact numbers given by a nine-point scale in the standard AHP. Linguistic evaluations such as: extremely important, moderately important and more or less of the same importance are characterized by inherent uncertainty, imprecision, and vagueness. T. L. Saaty and L. Vargas [13] treated the problem of uncertainty and its effect on the stability of rank order of competing alternatives. They considered pair-wise comparison estimations and the resulting priorities as random variables with given probability distributions. However, these quantities are extremely subjective; and differ from person to another and therefore they cannot be considered random, and cannot be treated statistically by collecting data and deriving probability distributions describing the behavior of their populations.

Fuzzy numbers are considered as the most appropriate model to express uncertainty, imprecision, and vagueness of decision makers' judgments. First approaches to solving the problem of fuzzy prioritization are given in Van Laarhoven et al [14], J. Buckley [2], C. Boender et al [1] and others.

These approaches followed similar procedures as adopted in the standard Eigen Vector method developed by Saaty [12] in the standard AHP. However, performing multiple arithmetic operations such as addition, multiplication, and division on fuzzy numbers result in fuzzy priorities with wide spreads due to propagation of fuzziness. Obtained fuzzy priorities have almost no practical meaning and sometimes they are irrational (Mikhailov [10]). Researches who tried to follow similar procedures as in standard AHP fit in the work of D.Y. Chang [4].

Chang determined crisp priority vector by performing fuzzy ordering and evaluating the truth value of the assertion that a fuzzy number I is greater than fuzzy number J. The error in this approach is that components of a priority vector may have zero values, which may result in infinite relative importance. This is in a total contradiction with assumed finite scale of relative importance upon which the problem is formulated and solved. Chang's approach does not permit evaluations of inconsistencies of pair-wise judgments. This is why Erensal et al [7] did not detect this high inconsistency; and proceeded to find a hierarchy; and reached to a conclusion which is naturally questionable. To overcome such complications, Mikhailov [10] proposed two approaches: 1) Fuzzy Preference Programming (FPP) and 2) Modified Fuzzy Preference Programming (MFPP). In the first approach (FPP), the defuzzification technique known as α – cuts is used. Crisp priorities rather than fuzzy priorities are derived from interval judgments corresponding to different α – cut levels. Priorities of the same alternative (factor or criterion) at different α – cut levels are then aggregated to obtain resultant priority. In the second approach (MFPP), a nonlinear optimization model has been obtained, and thereby avoiding the need for using α – cuts which requires a great deal of computations. The MFPP model formulates the problem in a nonlinear program given as follows:

Maximize λ Subject to:

$$\left(m_{ij} - l_{ij}\right)\lambda\ w_j - w_i + l_{ij} w_j \le 0$$

$$\left(u_{ij} - m_{ij}\right)\lambda\ w_j + w_i - u_{ij} w_j \le 0$$

$$\sum_{i=1}^{n} w_i = 1 \qquad (1)$$

(i = 1, 2,…..n-1), (j = 2,3,……n)

The optimum solution of the above nonlinear program (1) is a vector W*, represents the optimum priority vector that leads to a maximum possible degree of membership λ*. As indicated by L. Mikhailov [10], λ* is the degree of satisfaction and is a natural indicator of the inconsistency of decision makers' judgments. But however, since λ* may accept negative values as well as positive values, it becomes inappropriate to act as a natural indicator of the inconsistency and to measure the degree of satisfaction. In order to render λ* positive for inconsistent ratios as well, Mikhailov proposed to introduce tolerance parameters - thus complicating problem.

The rationality and tangibility of inconsistency ratio (ICR) as adopted by T.L. Saaty in the standard AHP disappear when replaced by the indicator λ. The absence of a reference to measure relatively how severe inconsistency is in pair-wise comparisons adds to the inappropriateness of λ. The formulation in (1) is a nonlinear program which can be solved numerically by commercial software such as "Lingo" with declaration of λ as an unrestricted variable. But however this formulation is not amenable to direct application of nontraditional optimization techniques, for instance, Genetic Algorithms since the objective (fitness) function λ is an implicit function of the decision variables ($w_1, w_2, \ldots, w_n$). Recently, these nontraditional optimization techniques, like Genetic Algorithms, are widely used because of their simplicity, ease of their implementations, capability to deal with nonlinearities, scalability, and their proven validity. The present work is mainly concerned with overcoming the above-mentioned complications and shortcomings in deriving priority vectors from fuzzy pair-wise comparison judgments. This, in principle, will be accomplished by:

1. Reformulating the problem by introducing an explicit objective function and using trapezoidal fuzzy numbers.
2. Solving the reformulated problem by use of one of the most powerful search techniques- Genetic Algorithms.
3. Determining inconsistency index RI for different sizes of pair-wise comparisons matrices.

Reformulating the problem enables the writer to restore back the concept of inconsistency ratio *ICR* with reference to inconsistencies of random fuzzy pair wise comparisons.

2. Problem Statement and Mathematical Formulation

Given **n (n-1)/ 2** pair-wise comparison judgments EV_{ij} of the relative importance of n alternatives, factors or criteria named afterwards as factors. The pair-wise comparison judgments are linguistic. It is required, at first, to prioritize these n factors and determine priority vector $W (w_1, w_2, \dots w_n)^T$ and then to evaluate the relative inconsistency ICR of judgments of each decision maker (DM).

2.1. Modeling of Linguistic Judgments

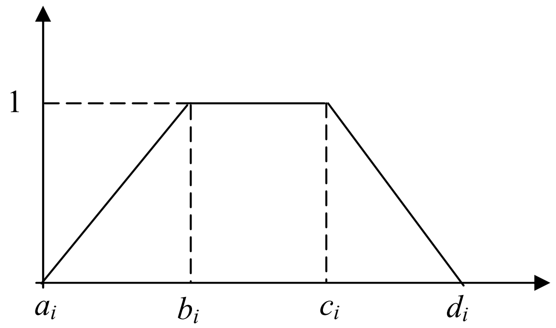

Figure 1: A membership function of a trapezoidal fuzzy number

Linguistic evaluations are usually characterized by their vagueness, imprecision, and uncertainty. They cannot be expressed by crisp numbers as already adopted in standard AHP. Random numbers are also not the proper model because of subjectivity of judgments and absence of statistical data necessary to derive probability distributions describing uncertainty. Fuzzy numbers are the most appropriate model to quantify linguistic judgments. Quantification of linguistic judgments is necessary for proceeding further with the solution of the stated problem and finding the weights w_i of the n competing factors. Fuzzy numbers are normal convex fuzzy sets first introduced by L. Zadeh [15]. Membership functions are used to express the degree of belonging of each element to the fuzzy set. Membership functions of fuzzy numbers may be taken as triangular or trapezoidal. In the present work, trapezoidal membership functions are considered as shown in Figure 1 - since triangular are special cases of the trapezoidal. Usually, trapezoidal fuzzy numbers are given in the form of quadruples $EV_{ij} (a_{ij}, b_{ij}, c_{ij}, d_{ij})$. EV_{ij} is evaluating the importance of factor *i* relative to factor *j* as judged by a decision maker *DM*. A scale quantifying linguistic judgments into trapezoidal fuzzy numbers is given in the following table 1.

Table 1: Scale translating linguistic judgments into fuzzy numbers

Linguistic judgments When i compared with j	Fuzzy numbers			
	a_{ij}	b_{ij}	c_{ij}	d_{ij}
Extremely Important	7	8	9	10
Moderately Important	5	6	7	8
Important	3	4	5	6
Slightly Important	1	2	3	4
Nearly of Equal Importance	1/2	1	1	3/2

Since $EV_{ij} = 1 / EV_{ji}$ the matrix of pair-wise comparisons is sufficiently defined by *n (n-1)/* 2, which are the number of the elements in the upper triangle of the matrix for which (i < j), (*i* = 1,2,…, n-1), (*j* = 2,3,…*n*).

2.2. Mathematical Formulation

2.2.1. Decision Variables:

w_i - weight of importance of factor i (i = 1,2,…. n), ($0 < w_i < 1$)

Our objective is to find values of weights w_i similar to values of their ratios w_i / w_j, which will be maximizing their membership in corresponding fuzzy sets EV_{ij}. Maximum contributions to the value of the objective or the fitness function occur when ratios w_i / w_j lie in the interval from b_{ij} to c_{ij}. On the other hand, contributions diminish as ratios w_i / w_j get values less than b_{ij} or greater than c_{ij}. Negative contributions to the objective function or fitness function occur as ratios w_i / w_j take values less than a_{ij} or greater than d_{ij}. Next function **G** ($w_1, w_2, \dots w_n$) is introduced in the present work, to represent the behavior of the fitness of a solution $w_1, w_2, \dots w_n$.

2.2.2. Fitness function(expression 2), Where (expression 3):

Conditions $a_{ij} < b_{ij}$ and $c_{ij} < d_{ij}$ are necessary to render the fitness function G($w_1, w_2, \dots w_n$), which is finite. Triangular membership functions could be accepted as special cases where, $b_{ij} = c_{ij}$.

$$G(w_1, w_2, \dots, w_n) = \underset{i<j}{Min}\,(\mu_{12}, \mu_{13}, \dots, \mu_{ij}, \dots, \mu_{(n-1)n}) \quad (2)$$

$$\mu_{ij} = \frac{w_i / w_j - a_{ij}}{b_{ij} - a_{ij}} \delta_{(w_i/w_j) < b_{ij}} + \delta_{b_{ij} < (w_i/w_j) < c_{ij}} + \frac{d_{ij} - w_i / w_j}{d_{ij} - c_{ij}} \delta_{(w_i/w_j) > c_{ij}} \quad (3)$$

In these Special cases the indicator functions:

$$\delta_{b_{ij} < (w_i / w_j) < c_{ij}} = 0$$

Fitness function :

G(w_1 , w_2 , …. w_n) as given in (2) and (3) is obviously nonlinear function in decision variables

w_i (i = 1, 2, …..n). Therefore, the problem of deriving a priority vector $W (w_1 , w_2 , …. w_n)^T$ from pair-wise comparison judgments are expressed in the form of $n (n-1)/2$ trapezoidal fuzzy evaluations EV_{ij} (a_{ij}, b_{ij}, c_{ij}, d_{ij}) and can be readily given in the following form:

Maximize G (w_1, w_2, w_n)

Subject to

$$\sum_{i=1}^{n} w_i = 1 \qquad (4)$$

Where G (w_1, w_2, …. w_n) is completely defined by expressions (2) and (3).

3. Solution of The Formulated Problem by a Genetic Algorithm

The optimization problem formulated in (4) could be effectively and efficiently solved by applying one of the most powerful search techniques- Genetic Algorithms (*GA*) originally developed by J.H. Holland [9] and later refined by D.E. Goldberg [8] and others. *GA* outperforms Directed Search Techniques because of their capability to explore wider spaces of feasible solutions aiming at a global optimum solution and thus have higher effectiveness. *GA* also outperforms methods of Random Search since they exploit obtained good solutions to arrive at better solutions and thus they are more efficient. *GA* starts with devising a special coding set of decision variables known as chromosomes as inspired by theories of evolution and genetics in biology. Each chromosome represents a solution to the problem by coding all variables of decision. The chromosome representation, as shown in **Error! Reference source not found.**, consists of *n* boxes (genes). Each gene carries a value for one of the *n* decision variables w_i . An initial population of 30 to 50 chromosomes is randomly generated. The fitness of each *feasible* chromosome (solution in which the *n* decision variables sum to one) is evaluated by (2) and (3). Consecutive generations are evolved from the initial population by applying *GA* operators: crossover, mutation, and copying. In biological systems, nature mercilessly selects only the fittest for longer lives and further reproduction. In a similar manner, chromosomes are selected for crossover and copying in accordance with and proportional to their fitness values. Crossover of two parent chromosomes is operated aiming at having better offspring. Crossover is generally operated with a predetermined probability P*xov*er raging from 90% to 95%. Copying is a complementary with crossover event having probability of 5% to 10% and is operated with the purpose of enriching the population with good solutions.

Mutation is a random change of gene values in order to widen the exploration front. This operator is necessary to avoid premature convergence and falling into a local optimum. However, it is recommended to apply mutation with low probability **P*mut*** from 1% to 10% in order to preserve the exploitation capability of *GA,* and not becoming just random searches. The Genetic Algorithm adopted in the present work, named as *Fuzzy Genetic Prioritization* (***FGP***), is described in the following steps:

w_1	w_2			w_i			w_n

Figure 2: A chromosome of n genes carrying values of n decision variables

1. Build a chromosome, by generating ***n*** random numbers uniformly distributed between 0 and 1, and normalize these ***n*** random numbers to render them - summing to one. These ***n*** normalized random numbers represent one of the feasible solutions.
2. Substitute for the values of the priorities w i , where(i = 1,2,….n) in (2) by the values of n normalized random number obtained in step 1 in order to evaluate the fitness of the chromosome.
3. Repeat steps 1 and 2 until having an initial population of 30 chromosomes (feasible solutions).
4. Decide on probabilities of crossover Pxover and mutation Pmut to start reproduction.
5. Select two chromosomes from the population by giving higher chances to those Chromosomes with higher fitness
6. Generate **y** - a continuous random number [0,1] uniformly distributed. If y <= **P*xov*er**. Then the next operation will be crossover, otherwise it will be copying. In case of having crossover apply steps 7 and 8, otherwise skip them and go directly to 9.
7. In case of having crossover operation, uniform crossover is applied since it is found more suitable for the problem at hand. In uniform crossover, each two corresponding genes in the two selected in step 5 parent chromosomes will exchange their values with 50% probability. Thereby two new child chromosomes are formed.
8. Generate **z** – a continuous random number [0, 1] uniformly distributed. If z <= **P*mut***, then perform mutation in the first gene in one of the parents, otherwise skip and take next gene. Repeat this step for all genes of the two child chromosomes. Normalization of the set of newly obtained gene values after crossover and mutation should be restored back and then one evaluates the fitness of each new child. Go to step 10.

9. Copying, in the present algorithm, is performed simply by replacing the two worst chromosomes having least fitness by the two selected in step 5 chromosomes.
10. By completing the above steps, a new generation is already obtained. Go to step 5 for further reproduction.
11. Repeat until convergence is obtained or reaching at a given terminating signal.
12. The solution is then reached by taking values of the genes of the fittest chromosome in the last generation.

The described algorithm is illustrated in Figures 3, 4 and 5 and then implemented in Visual Basic forApplication (VBA) under Excel.

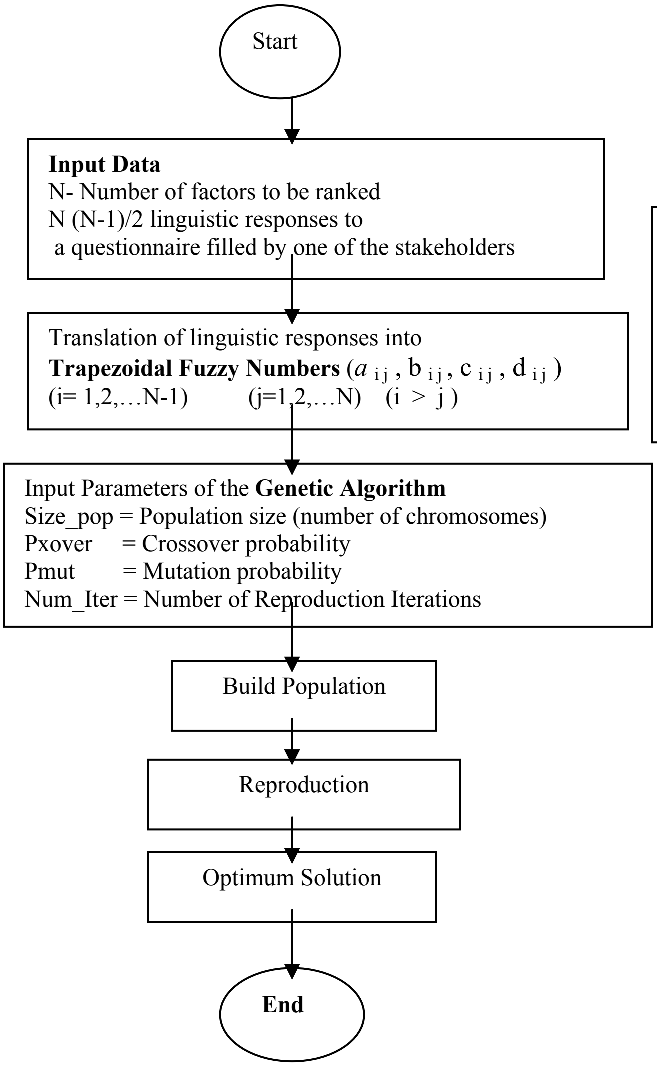

Figure 3: Main Block diagram of the Fuzzy Genetic Prioritization (FGP) Model

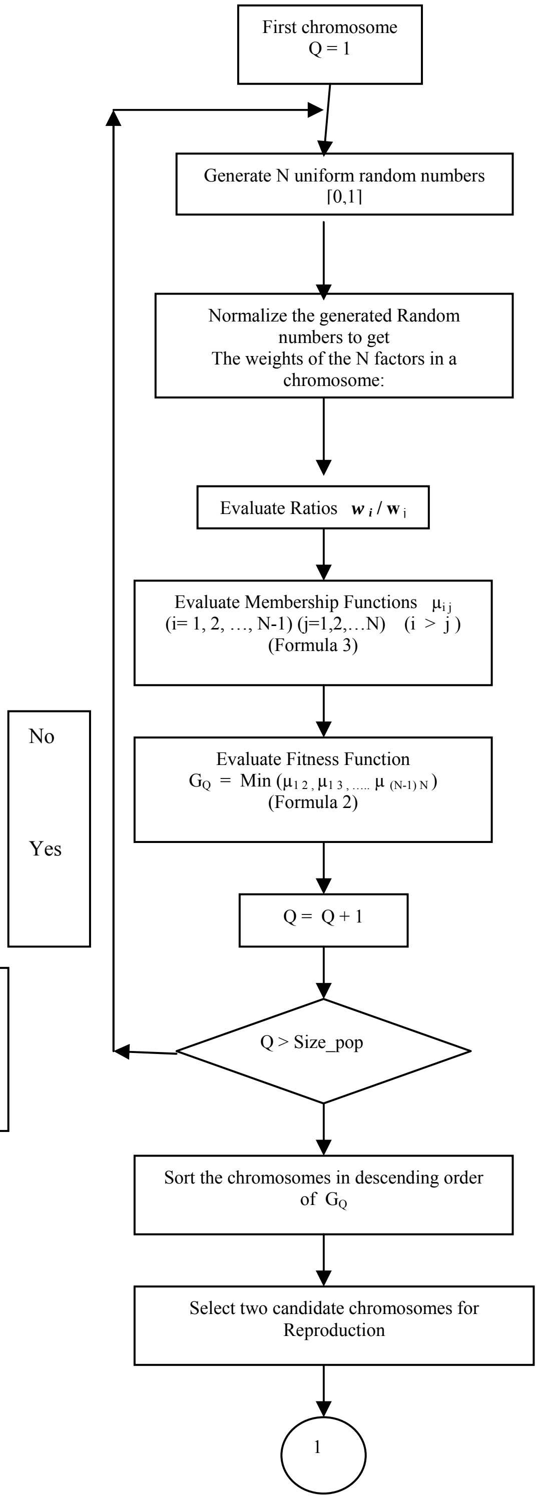

Figure 4: Flow Chart for Build Population Procedure

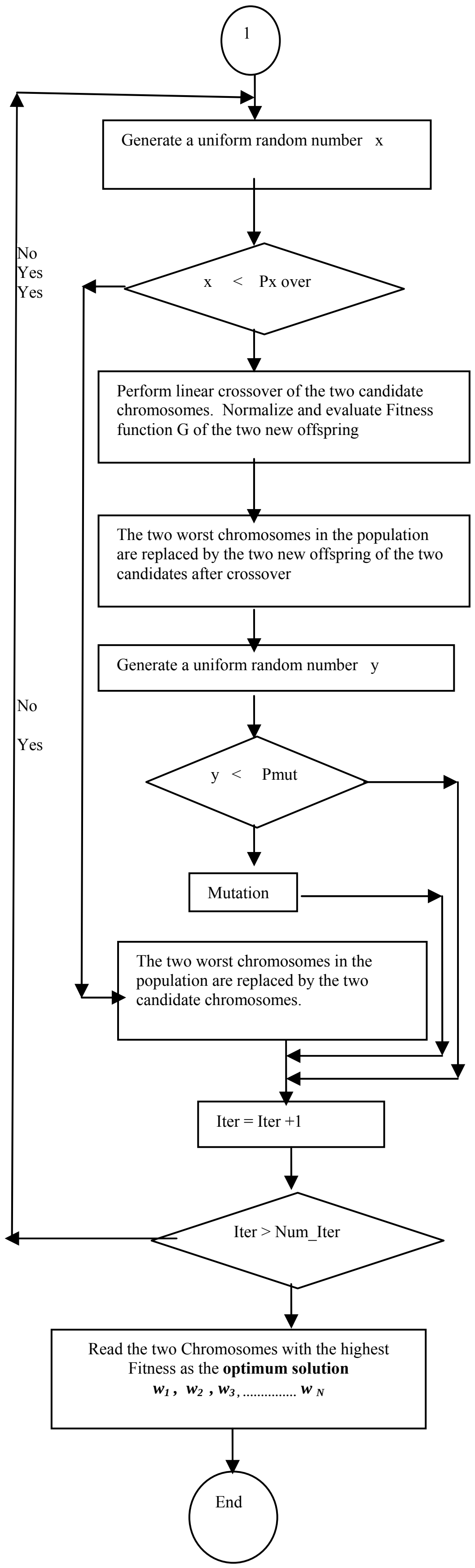

Figure 5: Flow chart for Reproduction and Optimum solution procedure

4. Evaluating Inconsistency of Fuzzy Pairwise Comparison

When constructing a judgment matrix for more than two competing factors or alternatives for the purpose of their ranking, there is a possibility of being inconsistent in decision making *DM* pair-wise comparisons.

For example comparing factor *A* with factor *B*, it is found that *A* is more important than *B* (*A* > *B*). Similarly comparing *B* with *C*, it is found that (*B*>*C*). To be consistent *A* should be rather more important than *C*. But for any reason, *DM* may enter a wrong entry (*A* < *C*). The comparison judgment becomes severely inconsistent. If the entry is correct, without appropriate evaluation of the relative importance, inconsistent comparisons are also obtained and to a lesser degree. Inconsistency increases as the number of factors to be compared and ranked. T.L. Saaty proposed a method to evaluate inconsistency in judgments; and showed that the judgment is perfectly consistent if the maximum eigen value of the reciprocal matrix is equal to the matrix size *n*. Note that the inconsistency index *ICI* is equal to (Max. eigen value – matrix size) / (matrix size – 1). In order to take a decision to discard one of *DM* matrices or not, *ICI* should be compared with the value of *ICI* of a completely random judgment *RI*. Saaty [12] evaluated *RI* for matrices of different sizes. The ratio of *ICI* of any *DM* judgment to *RI* gives the inconsistency ratio (*ICR*). Based on the value of *ICR*, it can decide whether to discard a *DM* judgment, depending on the number of available *DM* s and the criticality of the decision making problem. Acceptable values of *ICR* may take values ranging from say 0.01 to 0.1. Method of T.L. Saaty [12] is applicable only to case of *DM* comparison judgments represented by crisp (single) values.

It cannot be applied directly to the case of fuzzy representations. As already stated in the introduction, one of the main concerns of the present work is to extend the method of finding *ICR* s in the standard AHP to cover cases of fuzzy judgments. This is to overcome complications and inconveniences produced in existing methods that is adopted for evaluation of inconsistency in case of fuzzy judgments.

It is easy to show that fitness function $G(w_1, w_2, ... w_n)$ in (2) has the upper bound of unity in case of perfectly consistent judgments for which $b_{ij} <= w_i / w_j <= c_{ij}$. As ratios w_i / w_j get values less than b_{ij} or greater than c_{ij}, the value of fitness function decreases. This behavior of $G(w_1, w_2, ... w_n)$ proposes the following expression to evaluate the inconsistency index ICI as follows:

$$ICI = 1 - G(w_1, w_2, ... w_n) \geq 0 \qquad (5)$$

As in the standard AHP, inconsistency measure becomes more tangible if the index *ICI* is related to a random index *RI*. Next, values of *index RI* for different matrix sizes (n = 3 to 9) have been computed by solving the problem formulated in (4) with random fuzzy judgments.

The inconsistency ratio is given as:

$$ICR = \frac{ICI}{RI} \quad (6)$$

5. Evaluation of the Random Index (*RI*)

Fuzzy numbers $(a_{ij}, b_{ij}, c_{ij}, d_{ij})$ are generated randomly. The *GA* described in section 3 is run for seven values of n (n = 3, 4,…, 9). For each value of n, the *GA* is run 100 times in order to take an average for the fitness function $G(w_1, w_2, \ldots w_n)$ Having $G(w_1, w_2, \ldots w_n)$, *ICI* can be calculated by (5). In case of randomly generated fuzzy pairwise judgment, *RI = ICI.* In table 2, results of these computations are given.

Table 2: Random Index RI for different sizes n of fuzzy pairwise judgment matrices

n	3	4	5	6	7	8	9
RI	2.62	4.74	6.18	7.2	7.9	9.62	11.28

6. Numerical Examples

The first example to be considered is that presented several times by L. Mikhailov and P. Tsvetnov [11]. The pair-wise comparison judgment matrix is given in table 3.

Table 3: Pairwise comparison (reciprocal) matrixwith triangular fuzzy judgments taken from L. Mikhailov et al [11]

	A	B	C
A	1	(2, 3, 4)	(1, 2, 3)
B	(1/4, 1/3, 1 / 2)	1	(1/3, 1 / 2, 1)
C	(1/3, 1 / 2, 1)	(1, 2, 3)	1

The solution obtained by applying Fuzzy Genetic Prioritization (*FGP*), and developed in the present work, and that is obtained by (MFPP), and is developed by Mikhailov is presented in the following table 4

Table 4: Priority vector derived by FGP of the present work and MFPP of Mikhailov et al [11]

	Present work (*FGP*)	MFPP
w_1	*0.53749*	*0.538*
w_2	*0.17*	*0.17*
w_3	*0.2925*	*0.292*
$G(w_1, w_2, \ldots, w_n)$	*0.83757*	$\lambda = 0.838$
ICI	*0.1624*	
RI	*2.62*	
ICR	*0.062*	

The comparison in table 4 reveals coincidence of solutions obtained from the two approaches. This evidently proves validity of the proposed Fuzzy Genetic Prioritization (*FGP*) as developed in the present work. The inconsistency ratio *ICR*, as proposed in the present work, gives a value of 0.062, which is low, and gives a tangible measure of consistency of pair-wise comparison given in table 3 rather than that of λ = 0.838 used in [11].The second example is considered by Erensal et al [7]; and is solved by the method of D. Chang [4]. As already stated in the introduction, method of Chang belongs to a category of works that extend the approach of standard AHP to be applied directly to fuzzy pair-wise comparisons. However, Chang derived a crisp priority vector by evaluating the truth value of the assertion that a fuzzy number (I) is greater than a fuzzy number (J) as pointed out in the introduction. The priority vector, derived from reciprocal matrix in table 5 given by Erensal et al [7] applying Chang's method, is given in table 6. The same example, as given in table 5, is solved by the Fuzzy Genetic Prioritization *FGP* of the present work; and is also solved by the Modified Fuzzy Preference Programming MFPP developed in [10]. The comparison of the results of the three approaches reveals the following:

- The solution of Erensal et al [7] is not an optimum solution since better solutions have been obtained by both *FGP* and MFPP for the same reciprocal matrix.
- The inconsistency ratio *ICR* as obtained by the present work for the optimum solution is 0.341. This ratio is high; and assures that pairwise judgment in table 5 is inconsistent. This result is confirmed by the findings of MFPP for which λ has negative value ($\lambda = -1.06$).

Table 5 : Pair-wise comparison reciprocal matrix with triangular fuzzy evaluations , from Erensal et al [7]

	Cost	Price	Quality	Flexibility	Time
Cost	1	1/5,1/3,1	1/7,1/5,1/3	1/5.1/3,1	1/7,1/5,1/3
Price	1,3,5	1	1/9,1/7,1/5	0.5,1,1.5	1/5,1/3,1
Quality	3,5,7	5,7,9	1	1/7,1/5,1/3	0.5,1,1..5
Flexibility	1,3,5	0.9,1,1.1	3,5,7	1	1/7,1/5,1/3
Time	3,5,7	1,3,5	0.9,1,1.1	3,5,7	1

7. Findings and Conclusions

7.1. The extension of the standard AHP method, as introduced by T.L. Saaty, in which the pairwise comparisons are given by crisp values, to be applied directly to cases of having fuzzy pair-wise judgments is not valid; and leads mostly to irrational and inconsistent solutions.

7.2. The proper handling of problems of deriving priorities from fuzzy pair-wise judgments is to formulate them as optimization problems. Fuzzy Preference Programming developed by Mikhailov [10] is the first trial in this concern.

7.3. Expressing objective functions explicitly, in optimization models, is an important prerequisite to rendering the nonlinear programs amenable to applications of the effective and widespread nontraditional search techniques such as Genetic Algorithms.

Table 6: Priority vector derived by three methods FGP of the present work, Chang [4] applied by Erensal et al [7] and MFPP of Mikhailov [10] for the pair-wise comparisons matrix given in table 5

	Present work (FGP)	Chang method applied by Erensal et al [11]	MFPP by Mikhailov Using Lingo 8
w_1	0.0473	0.00	0.0447
w_2	0.0282	0.12	0.03188
w_3	0.1223	0.31	0.1224
w_4	0.2602	0.25	0.2578
w_5	0.542	0.32	0.54314
G	-1.107	-6.8	λ = -1.06
ICI	2.107	7.8	
ICR	0.341	1.262	

7.4. It has been demonstrated that Genetic Algorithms can be easily and transparently applied to solve the formulated optimization models for deriving priorities from fuzzy pair-wise judgments after modification of the objective (fitness) function and making it explicit function of decision variables w_i. The coincidence of results obtained from the application of the developed, in the present work, Fuzzy Genetic Prioritization FGP and solutions obtained from MFPP by Mikhailov is clearly demonstrated by the two provided numerical examples.

7.5. Modeling of linguistic judgments in the form of trapezoidal fuzzy numbers is sought to be more realistic and capable of capturing fuzziness of comparisons.

7.6. To have ICR as a tangible measure of inconsistency of pair-wise judgments is crucial for making rational decisions. The measure ICR acts as a screening element by means of which seriously inconsistent responses of DM to questionnaires could be discarded. This screening is mandatory prior to getting geometric means in case of having group of decision makers. The computations of the random indexes RI for matrices of different sizes with random fuzzy entries, as elaborated in the present work, made it possible to evaluate ICR s

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