JJMIE

Volume 2, Number 1, Mar. 2008
ISSN 1995-6665
Pages 65 - 69

Jordan Journal of Mechanical and Industrial Engineering

Expected Delays in Completing Projects under Uncertainty

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Abstract

A method is proposed to estimate, rationally, a contingency covering probable delays in completing projects. Project completion time is assumed random variable distributed normally with given expected value and variance. Two pure strategies and a mixed strategy for decreasing expected delay, if it turns out to be unacceptable, are presented. A new function SDF(Z_C), as a function of a standard normal variable Z_C corresponding to contract time T_C, is derived and given also in a table form for calculation simplicity. The mixed strategy of compressing both expected project completion time and its variance, to achieve a desirable level of expected project delay, leads to a nonlinear program and solved by Solver/Excel.

Keywords: Project Delay Period; Time-Over-Run; Standard Delay Function (SDF); Nonlinear Minimization Problem; Consequences of Project Delay;

Nomenclature

C_T	Cost of 1 % compression of expected project completion time
C_σ	Cost of 1 % compression of standard deviation of project completion time
d	Delay period in project completion
E(d)	Expected value of delay period
SDF(Z_C)	Standard Delay Function.
TE	Expected (average) project time
T_C	Contract time
TC	Total cost of compression
t	Project completion time
f(t)	Probability density function of project completion time (normal distribution)
X_T	Percentages of compression of expected project completion time
X_σ	Percentages of compression of standard deviation
γ	Desired reduction ratio of the expected delay period
σ_P	Standard deviation of the project completion time
υ	Standard deviation reduction ratio
τ	Reduction ratio of reducing the expected project completion time
$\Phi(Z)$	Cumulative probability function of Normal distribution
$Z = \dfrac{t - T_E}{\sigma_P}$	Standard normal variable

1. Introduction

Time-over-run is one of the most important issues facing project managers and business executives in their endeavor to undertake responsibilities of completing and delivering projects .The risk of Time-over-run could be so extremely high that cannot be taken by contractors. Risk, generally is evaluated as the product of two terms: the probability of not completing the project by the time prescribed in the contract – as the risk-initiating event- and the consequences of delay.

At first, consider the second term i.e. the consequences of delay. Almost all contracts include articles prescribing delay penalties. Usually, delay penalties are prescribed as rates ($ / unit time of delay). There is an upper limit of the sum to be charged as delay penalties. This upper limit is defined by law not to exceed a specified percentage of the contract value (say 10%). As delay penalties start to exceed that limit, the contract is automatically cancelled and the remaining uncompleted work of the project could be assigned to another competent contractor (usually the next in the bidders list). Any extra costs resulting from calling a new contractor will be charged to the delayed contractor. This is just the monetary part of the consequences of the risk of time-over-run. Another part of these consequences- that is more painful- the loss of reputation of delayed contractors and hence loosing their competitive power.

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The first term of the product evaluating the risk of time-over-run is the probability of the event of completing the project in a time exceeds that prescribed in the contract (contract time). This statement implies that the completion time of a project is a random variable. This is true, since completion time of a project is completely determined by durations of its constituting activities and their precedence order. Durations of activities and tasks of a project cannot be defined as certain quantities because of the following reasons:

- All projects are unique undertakings rarely to be repeated.
- Environmental conditions- in which activities are performed - are uncertain and can be only described by parameters of random nature.
- Labor and equipment productivities cannot be assumed deterministic quantities but rather random variables because of the clear variation in their values.
- Reworks of some activities or part of activities are due to project owner refusals and/or faulty executions. The time increase in these situations is random.
- Other pertinent reasons.

Uncertainties in project planning have been recognized since the fifties of the last century. Project Evaluation and Review Technique-known as PERT- was the first model accounting for the randomness of activity durations (assuming beta distribution for activity durations). Based on the Central Limit theorem, in PERT, project completion time is assumed to be normally distributed [1]. The normality of the project completion time enables project planners to easily evaluate the probability of delay as the probability of completing a project in a time exceeding contract time.

Naturally, all contractors are concerned with avoidance or at least mitigation of the risk of time-over-run beside other risks such as cost-over-run [2]. As a method for mitigation of the risk of time-over-run is to include a rough estimate to cover such risks in the bid cost estimations. This is usually included in the estimates of contingencies.

However, problems of rational evaluation of contingencies in bidders' proposals have been considered in several researches [3], [4].

In the present work, a rational approach is proposed not only to evaluate the expected delay period and thereby the amount of expected delay penalty but also to provide a method to test different alternative strategies of reducing the expected delay period.

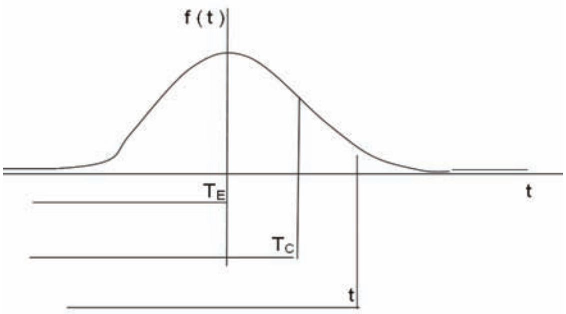

Fig. 1 Project time normal distribution and its parameters

2. Expected Delay Period

2.1. Derivation of a mathematical expression for Expected Delay Period

The delay in project completion is defined as follows

$$d = (t \mid t > T_C) - T_C$$

$$E(d) = E(t \mid t > T_C) - T_C$$

$$E(d) = \frac{\int_{T_C}^{\infty} t.f(t)dt}{\int_{T_C}^{\infty} f(t)dt} - T_C$$

$$f(t) = \frac{1}{\sigma_P \sqrt{2\pi}} .e^{-(1/2)\left(\frac{t-T_E}{\sigma_P}\right)^2}$$

$$\frac{t - T_E}{\sigma_P} = z$$

$$dz = \left(\frac{1}{\sigma_P}\right) dt$$

$$t = z.\sigma_P + T_E$$

Then,

$$E(d) = \frac{\int_{z_C}^{\infty} (z\sigma_P + T_E)\ \sigma_P\ \exp(-z^2/2)\ dz}{\sigma_P \sqrt{2\pi}\left[1 - \Phi(z_C)\right]} - T_C$$

$$E(d) = \frac{\sigma_P\ \exp(-z_C^2/2)}{\sqrt{2\pi}\left[1 - \Phi(z_C\right]} - \sigma_P\ z_C$$

$$E(d) = \sigma_P\ \text{SDF}(z_C)$$

$$SDF(z_C) = \frac{\exp(-z_C^2/2)}{\sqrt{2\pi}\left[1 - \Phi(z_C\right]} - z_C$$

The Standard Delay Function SDF(Z_C) is a newly introduced function of the standard normal variable Z_C . For ease and simplicity of practical calculations, SDF(Z_C) is tabulated in a similar way as the cumulative normal function $\Phi(Z)$

Finally, we obtain for the expected value of delay period in completing a project with completion time normally distributed with mean T_E and standard deviation σ_P having T_C as contract time in the following form:

$$E(d) = \sigma_P\ SDF\ (Z_C) \qquad (1)$$

The standard Delay function SDF (Z_C) is given by

$$SDF\ (Z_C) = \frac{e^{-Z_C^2/2}}{\sqrt{2\pi}\left[1 - \Phi(Z_C)\right]} - Z_C \qquad (2)$$

The table of SDF (Z_C) is given in table 1

2.2. Strategies of decreasing Expected Delay Period

While preparing bids, contractors may find the expected delay period as evaluated by expression (1) and table (1) excessively large. This would entail undesirable increase in bid price in case of accounting for potential penalties in bid estimates and hence diminishing chances of winning. What could be the strategies adopted by contractors in order to reduce the expected delay period to enhance their situation in bidding?.

Consider at first two pure strategies: either by reducing expected project time by means of compressing durations of activities keeping their standard deviations unchanged, or by reducing the standard deviation of project completion time by reducing the variation in estimated activities durations keeping expected durations unchanged.

2.2.1. Pure strategy of compressing T_E only, keeping σ_P unchanged

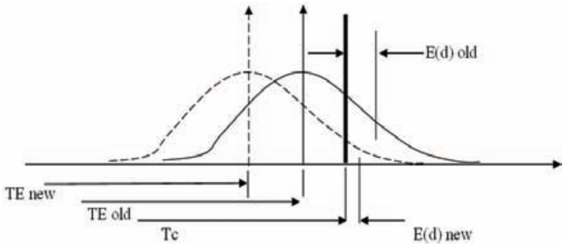

Fig. 2 Pure Strategy of decreasing delay by compressing expected project time only

Table 1. Standard Delay Function SDF (Z_C)

Zc	SDF(Zc)									
	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.798	0.794	0.791	0.787	0.784	0.780	0.777	0.773	0.770	0.766
0.1	0.763	0.759	0.756	0.753	0.749	0.746	0.743	0.739	0.736	0.733
0.2	0.730	0.726	0.723	0.720	0.717	0.714	0.711	0.708	0.705	0.701
0.3	0.698	0.695	0.692	0.689	0.686	0.683	0.681	0.678	0.675	0.672
0.4	0.669	0.666	0.663	0.661	0.658	0.655	0.652	0.649	0.647	0.644
0.5	0.641	0.639	0.636	0.633	0.631	0.628	0.626	0.623	0.620	0.618
0.6	0.615	0.613	0.610	0.608	0.605	0.603	0.600	0.598	0.596	0.593
0.7	0.591	0.588	0.586	0.584	0.581	0.579	0.577	0.575	0.572	0.570
0.8	0.568	0.566	0.563	0.561	0.559	0.557	0.555	0.552	0.550	0.548
0.9	0.546	0.544	0.542	0.540	0.538	0.536	0.534	0.532	0.530	0.528
1	0.526	0.524	0.522	0.520	0.518	0.516	0.514	0.512	0.510	0.508
1.1	0.506	0.504	0.502	0.501	0.499	0.497	0.495	0.493	0.492	0.490
1.2	0.488	0.486	0.484	0.483	0.481	0.479	0.478	0.476	0.474	0.472
1.3	0.471	0.469	0.467	0.466	0.464	0.463	0.461	0.459	0.458	0.456
1.4	0.455	0.453	0.451	0.450	0.448	0.447	0.445	0.444	0.442	0.441
1.5	0.439	0.438	0.436	0.435	0.433	0.432	0.430	0.429	0.427	0.426
1.6	0.425	0.423	0.422	0.420	0.419	0.418	0.416	0.415	0.414	0.412
1.7	0.411	0.410	0.408	0.407	0.406	0.404	0.403	0.402	0.400	0.399
1.8	0.398	0.397	0.395	0.394	0.393	0.392	0.390	0.389	0.388	0.387
1.9	0.386	0.384	0.383	0.382	0.381	0.380	0.378	0.377	0.376	0.375
2	0.374	0.373	0.372	0.370	0.369	0.368	0.367	0.366	0.365	0.364
2.1	0.363	0.362	0.361	0.359	0.358	0.357	0.356	0.355	0.354	0.353
2.2	0.352	0.351	0.350	0.349	0.348	0.347	0.346	0.345	0.344	0.343
2.3	0.342	0.341	0.340	0.339	0.338	0.337	0.336	0.335	0.334	0.333
2.4	0.333	0.332	0.331	0.330	0.329	0.328	0.327	0.326	0.325	0.324
2.5	0.323	0.323	0.322	0.321	0.320	0.319	0.318	0.317	0.316	0.316
2.6	0.315	0.314	0.313	0.312	0.311	0.311	0.310	0.309	0.308	0.307
2.7	0.307	0.306	0.305	0.304	0.303	0.303	0.302	0.301	0.300	0.299
2.8	0.299	0.298	0.297	0.296	0.296	0.295	0.294	0.293	0.293	0.292
2.9	0.291	0.290	0.290	0.289	0.288	0.287	0.287	0.286	0.285	0.285
3	0.284	0.283	0.283	0.282	0.281	0.280	0.280	0.279	0.278	0.278
3.1	0.277	0.276	0.276	0.275	0.274	0.274	0.273	0.272	0.272	0.271
3.2	0.270	0.270	0.269	0.269	0.268	0.267	0.267	0.266	0.265	0.265
3.3	0.264	0.264	0.263	0.262	0.262	0.261	0.261	0.260	0.259	0.259
3.4	0.258	0.258	0.257	0.256	0.256	0.255	0.255	0.254	0.253	0.253
3.5	0.252	0.252	0.251	0.251	0.250	0.250	0.249	0.248	0.248	0.247
3.6	0.247	0.246	0.246	0.245	0.245	0.244	0.244	0.243	0.243	0.242
3.7	0.241	0.241	0.240	0.240	0.239	0.239	0.238	0.238	0.237	0.237
3.8	0.236	0.236	0.235	0.235	0.234	0.234	0.233	0.233	0.232	0.232
3.9	0.231	0.231	0.230	0.230	0.229	0.229	0.229	0.228	0.228	0.227
4	0.227	0.226	0.226	0.225	0.225	0.224	0.224	0.223	0.223	0.223

Given the desired reduction ratio γ of reducing the expected delay period, required to evaluate the corresponding reduction ratio τ of reducing the expected project completion time.

To find τ, we proceed as follows:

$$E(d)_{New} = (1-\gamma).E(d)_{old}$$
$$\sigma_P .SDF(Z_{Cnew}) = (1-\gamma).E(d)_{old}$$
$$SDF(Z_{Cnew}) = \frac{(1-\gamma).E(d)_{old}}{\sigma_P}$$

From table 1, we find the value of Z_{Cnew} corresponding to the value of $SDF(Z_{Cnew})$

$$T_{Enew} = T_C - \sigma_P .Z_{Cnew}$$
$$\tau = \frac{T_{Eold} - T_{Enew}}{T_{Eold}}$$

T_C, T_{Eold}, σ_P, γ are given and $SDF(d)_{old}$ is found from table 1.

Illustrative Example:

Given: $T_{E\ old}$ = 300 days; σ_P = 40 days; T_C = 350 days, then Z_C = 1.25. From table 1, SDF(1.25)= 0.479, the original expected delay period is evaluated: $E(d)_{old}$ = 40 x 0.479 = 19.16 days. It is required to reduce this expected delay by 40%, what would be the corresponding percentage of compression of T_E

$$SDF(Z_{Cnew}) = \frac{(1-\gamma).E(d)_{old}}{\sigma_P} = 0.6 \text{ x } 19.16 / 40 = 0.2874$$

Again from table 1, the corresponding $Z_{C\ new}$ = 2.95, then $T_{E\ new}$ = 350 – 40 x 2.95= 232 The corresponding compression ratio τ will be τ = (300 – 232) / 300 = 0.2267.

The expected project time should be compressed by about 23% in order to reduce the expected delay period by 40%. The decision whether to adopt this strategy or not, would be taken on the basis of tradeoff between the cost of compression of the expected project completion time from 300 days to 232 days and the expected penalty of 8 days delay.

2.2.2. Pure strategy of decreasing σ_P only, keeping T_E unchanged

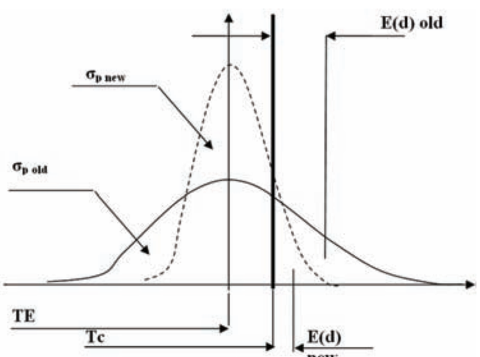

Fig. 3 Pure strategy of decreasing delay by compressing project time variance only

Given the desired reduction ratio γ of reducing the expected delay period. Required to evaluate the corresponding reduction ratio υ of reducing the standard deviation of project completion time. To find υ, we proceed as follows:

$$E(d)_{New} = (1-\gamma).E(d)_{old}$$
$$\sigma_{Pnew}.SDF(Z_{Cnew}) = (1-\gamma).E(d)_{old} \quad (3)$$

Since σ_{Pnew} is unknown, it is necessary to solve the transcendental equation in (3) to find σ_{Pnew}. This equation may be solved by trial and error or by using one of the tools of the Excel “Goal Seek.”

Illustrative example:

Consider the above solved example but by employing the strategy of reducing σ_P rather than T_E. The transcendental equation in (3) will take the form:

$$\sigma_{Pnew}.SDF(Z_{Cnew}) = 0.6x19.16 = 11.496$$

The solution by trial and error in a table form is found by giving values to σ_{Pnew} and evaluating the left hand side (LHS)of the equation and repeat until we approach the value of the right hand side.

σ_P	Z_{Cnew}	$SDF(Z_{Cnew})$ From table 1	LHS
35	1.428571	0.45	15.75
32	1.5625	0.43	13.76
30	1.666667	0.415	12.45
29	1.724138	0.408	11.832
28.5	1.754386	0.404	11.514

σ_{Pnew} = 28.5 days. The standard deviation reduction ratio υ will be

υ = (40 – 28.5)/ 40 = 0.2875.

The standard deviation of project completion time must be reduced by about 29% in order to have 40% reduction in expected delay period. In absolute figures, the standard deviation of the project completion time should reduce by about 12 days in order to gain about 8 days reduction in expected delay. The same argument as regards to whether to go with the reduction in standard deviation or not, would be decided by tradeoff between the cost of reduction of variance of project completion time and the value of penalties of the 8 days delay.

2.2.3. Mixed strategy of decreasing both T_E and σ_P with different ratios

The objective of adopting a mixed strategy is to minimize the total cost of compressing both expected project completion time and its standard deviation. Next, we formulate this minimization problem.

Decision variables in this problem are X_T and X_σ percentages of compression of expected project completion time and of its standard deviation respectively.

Parameters : C_T and C_σ costs of 1 % compression of expected project completion time and its standard deviation respectively. TC total cost of compression. T_E , σ_P expected project completion time and its standard deviation respectively, T_C contract time, $E(d)_O$ initial expected delay period, evaluated as explained below

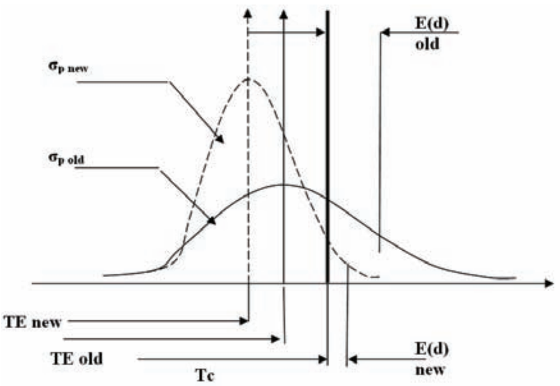

Fig. 4 Mixed strategy of decreasing delay by compressing both expected project time and its variance

Objective Function:

Minimize

$$TC = C_T . X_T + C_\sigma . X_\sigma$$

Subject to

$$\frac{\exp\left[-0.5.\left(\frac{T_C-(1-0.01.X_T)T_E}{(1-0.01.X_\sigma).\sigma_P}\right)^2\right]}{\sqrt{2\pi}.\left[1-\Phi\left(\frac{T_C-(1-0.01.X_T)T_E}{(1-0.01.X_\sigma).\sigma_P}\right)\right]} - \left[\frac{T_C-(1-0.01.X_T)T_E}{(1-0.01.X_\sigma).\sigma_P}\right] .. \le (1-\gamma).E(d)_O$$

This nonlinear program has a linear objective function but the constraint is nonlinear. This program can be easily solved by SOLVER under EXCEL

Illustrative example

Given T_E = 100 days, σ_P = 20 days, T_C = 110 days, γ = 0.4, C_T = 4000 $, C_σ = 3500 $

Initially evaluate the expected delay period $E(d)_O$

ZC = (110 – 100) / 20 = 0.5

From table 1, we get SDF(0.5) = 0.641, then

$E(d)_O = \sigma_P$ *SDF(0.5) = 12.82 days*

Solving the nonlinear program by Solver/ Excel, we find the following compression percentages that necessary to be applied on both T_E and σ_P respectively to get the desired 40% reduction of the expected delay at a minimum cost.

XT = 14.162 %
Xσ = 15.1 %
TC = 109 467 $

3. Quality and Quantity of resources allocated to project activities and their effects on activities durations

Resources, considered in the present work, are mainly labor and equipment. By quality resources, we mean highly skilled and experienced labor and/or high quality equipment with minimum failure rates. The use of such quality resources helps to minimize the uncertainty in predicting productivity and availability of resources and eliminate or at least minimize the expected amount of rework. Hence, planners could assume small variations (variances) in activities durations. On the other hand, as the quality of resources rises the direct cost of activity would increase. This is the cost of reducing variance of activity duration. Quantitative increase of resources leads to compression of activities expected durations but without a noticeable reduction in their variances. Of course, quantitative and qualitative increase of resources allocated to different activities would result in decrease of their expected durations and variances as well. The problem of defining the best way to compress project completion time with minimum increase in direct cost is a well-known problem formulated and solved, in deterministic framework, in Operations Research. But, a similar problem of defining the best way to reduce variance (standard deviation) of project completion time, to writer's knowledge, is not existing and needs to be treated in future works.

The decrease in indirect cost of projects as completion times get shorter should be considered in evaluations of total cost increase due to compression of project completion times and their variances.

4. Conclusion and Future work

Estimation of expected delay in completing projects contributes significantly to the reliable assessment of risks of both time-over-run and cost-over-run. The method proposed in this work is simple and amenable to practical computations. A model is required to be built in future works to enable planners to select the optimum alternatives of decreasing project completion time variance alone and along with decrease of expected project completion time aiming at decreasing expected project delay.

References

[1] Hamdy A. Taha, Operations Research, 7th ed, Prentice Hall, 2003

[2] Ali Touran, "Calculation of Contingency in Construction Projects", IEEE, Transactions on Engineering Management, Vol 50, No 2, May 2003

[3] R. De Neufville, E.N. Hani and V. Lesage. " Building Models: Effects of bidder Risk aversion", Journal of Construction Div. ASCE, Vol. 103, No. 2, 1977

[4] M. Ranasinghe, "Contingency allocation and management for building projects", Construction Management Econ.., vol. 12, pp 233-243, 1994