

A Study on the Performance of Hydromagnetic Squeeze Film between Two Conducting Truncated Conical Plates

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Abstract

We make an effort to analyze the behavior of hydromagnetic squeeze film between two conducting porous truncated conical plates. The plates are considered electrically conducting and the clearance space between them is filled by an electrically conducting lubricant. A uniform transverse magnetic field is applied between the plates. An attempt has been made to solve the associated Reynolds' equation with appropriate boundary conditions to obtain the pressure distribution which in turn, is used to get the expression for load carrying capacity which is then used to calculate the response time. The results are presented graphically as well as in tabular form. The results suggest that the bearing system registers an enhanced performance as compared to that of a bearing system working with a conventional lubricant. Further, it is seen that aspect ratio tends to increase the load carrying capacity substantially. It is observed that the combined effect of the semi-vertical angle and the magnetization parameter is relatively better than that of the combined effect of the aspect ratio and porosity. Besides, the conductivity increases the load carrying capacity significantly. The analysis incorporated in this paper presents ample scope for improving the performance of the bearing system considerably by choosing a suitable combination of magnetization parameter, the aspect ratio, semi-vertical angle and the conductivities of the plates.

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Keywords: Hydromagnetic squeeze film; truncated conical plates; Conductivities; Reynolds' equation; Load carrying capacity;

Nomenclature

a, b	Radii (a > b) (m)
B ₀	Uniform transverse magnetic field applied between the plates.
c ²	$= 1 + \frac{KM^2}{h^2 m}$
h	Lubricant film thickness (m)
H	Magnetic field component (gauss)
H ₀	Thickness of the porous wall (m)
h ₀ '	Surface width of the lower plate (m)
h ₁ '	Surface width of the upper plate (m)
k	Aspect ratio (a/b)
K	Permeability (col ² kgm/s ²)
m	Porosity of the porous matrix
M	$= B_0 h \left(\frac{s}{\mu} \right)^{1/2} = \text{Hartmann number}$
P	Pressure distribution (N/m ²)
P	Non-dimensional pressure
S	Electrical conductivity of the lubricant (mho)
s ₀	Electrical conductivity of lower surface (mho)
s ₁	Electrical conductivity of upper surface (mho)

W	Load carrying capacity (kgm/s ²)
W	Dimensionless load carrying capacity
Δt	Response time (s)
ΔT	Non-dimensional response time
φ ₀ (h)	$= \frac{s_0 h_0'}{sh}$
φ ₁ (h)	$= \frac{s_1 h_1'}{sh}$
ψ	$= \frac{KH_0}{h^3}$
μ	Viscosity (kg/ms)
$\bar{\mu}$	Magnetic susceptibility (m ³ /kg)
μ ₀	Permeability of the free space (N/A ²)
ω	Semi-vertical angle of the cone (°)

1. Introduction

Due to the large electrical conductivity of liquid metals such as Mercury and Sodium, the possibilities of electromagnetic pressurization from the application of an external magnetic field have been explored and investigated. This electromagnetic pressurization comes into force when a large external electromagnetic field

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through the electrically conducting lubricant is applied to induce circulating currents which in turn, interacts with the magnetic field to create a body force which pumps the fluid between the bearing surfaces. As the liquid metals are good electrical conductors it becomes possible to increase the load carrying capacity by making use of electromagnetic force thereby, overcoming the defect associated with lubricants at high temperature and thus alleviating the drawbacks of low viscosity. Considerably, high increase in load carrying capacity is possible with the use of super conducting magnets while very little power is required to provide the magnetic field. Many investigators have conducted theoretical and experimental studies on the hydromagnetic lubrication for porous as well as plane metal bearings. Elco and Huges [1] discussed magneto hydrodynamic pressurization in liquid metal lubrication. Kuzma [2] and Kuzma et al. [3] analyzed the behavior of magneto hydrodynamic squeeze films. Shukla [4] dealt with the hydromagnetic bearing of squeeze films for conducting lubricants between two non-conducting non-porous surfaces in the presence of a transverse magnetic field. Shukla and Prasad [5] investigated the performance of hydromagnetic squeeze films between two conducting non-porous surfaces and studied the effect of the conductivities of surfaces on the performance of the bearing system. A number of theoretical and experimental studies (Dodge et al. [6]; Maki et al. [7]; Snyder [8]) have been devoted to magneto hydrodynamic lubrication. Sinha and Gupta [9], [10] discussed the study of hydromagnetic effect on the porous squeeze films wherein they considered annular plates and rectangular plates. Patel and Hingu [11] studied this effect for squeeze films between circular disks. Patel and Gupta [12] used Morgan-Cameron approximation and simplified the analysis for hydromagnetic squeeze films between parallel plates for a number of geometrical shapes. Patel [13] analyzed the behavior of hydromagnetic squeeze films between porous annular disks with tangential velocity slip.

Prakash and Vij [14] investigated the load carrying capacity and time height relation for squeeze film between porous plates. Various geometries like circular, annular, elliptic, rectangular, conical and truncated conical plates were incorporated in this article. Prajapati [15] considered the performance of hydromagnetic squeeze film between two conducting porous conical plates. Patel and Deheri [16] dealt with the behavior of magnetic fluid based squeeze film between porous conical plates and it was established that the magnetic fluid and the angle of the cone played a central role for enhancing the performance of the bearing system. Besides Vadher et al. [17] studied the hydromagnetic squeeze film between conducting porous transversely rough triangular plates and it was found that the negative effect introduced by the roughness could be neutralized up to certain extent by the positive effect of magnetization parameter in the case of negatively skewed roughness. Here an endeavor has been made to study and analyze the behavior of a hydromagnetic squeeze film between porous conducting truncated conical plates.

2. Analysis

The configuration of the bearing system is shown below.

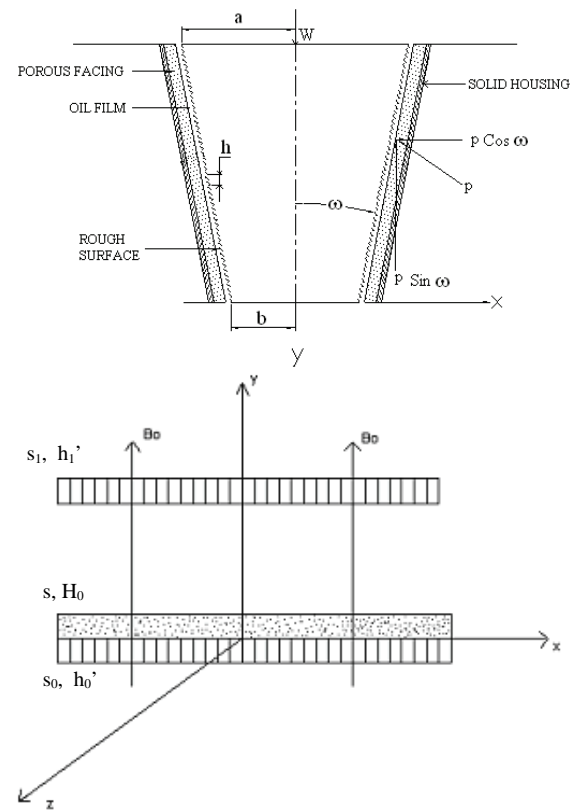


Figure: Configuration of the bearing system

The lower plate with a porous facing is assumed to be fixed while upper plate moves along its normal towards the lower plate. The plates are considered electrically conducting and the clearance space between them is filled by an electrically conducting lubricant. A uniform transverse magnetic field is applied between the plates. The flow in the porous medium satisfies the modified form of Darcy's law. (cf. Prajapati [15]), while in the film region the equations of hydromagnetic lubrication theory hold. Then, under usual assumptions of hydromagnetic lubrication the modified Reynolds' equation for the lubricant film pressure is (c.f. Prajapati [15]; Pakash and Vij [14]).

$$\frac{1}{x} \frac{d}{dx} \left(x \frac{dp}{dx} \right) = \frac{h'}{\left[\frac{2h^3}{\mu M^3} \left(\tanh \frac{M}{2} - \frac{M}{2} \right) - \frac{\nu h^3}{\mu c^2} \right]} \cdot \frac{1}{\left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)}} \right]} \tag{1}$$

where

$$M = B_0 h \left(\frac{s}{\mu} \right)^{1/2}$$

Solving this equation with the use of boundary conditions

$$p(a \operatorname{cosec} \omega) = 0 ; p(b \operatorname{cosec} \omega) = 0 \tag{2}$$

gives the pressure distribution in dimensionless form as

$$P = \frac{-ph^3}{\mu h \pi (a^2 - b^2) \operatorname{cosec} \omega} \cdot \frac{\operatorname{cosec} \omega \cdot \left[\frac{\ln(x \sin \omega / b)}{\ln(a/b)} - \frac{(x \sin \omega / b)^2 - 1}{(a/b)^2 - 1} \right]}{4\pi \left[\frac{2}{M^3} \left(\tanh \frac{M}{2} - \frac{M}{2} \right) - \frac{\psi}{c^2} \right]} \cdot \left[\frac{\varphi_0 + \varphi_1 + 1}{\varphi_0 + \varphi_1 + \frac{\tanh(M/2)}{(M/2)}} \right] \tag{3}$$

Then the load carrying capacity given by

$$w = 2\pi \int_{b \operatorname{cosec} \omega}^{a \operatorname{cosec} \omega} p \cdot x dx$$

is obtained in dimensionless form as

$$W = - \frac{wh^3}{\mu h \pi^2 (a^2 - b^2)^2 \operatorname{cosec}^2 \omega} \cdot \frac{\operatorname{cosec}^2 \omega \cdot \left[\frac{(a/b)^2 + 1}{(a/b)^2 - 1} - \frac{1}{\ln(a/b)} \right]}{8\pi \left[\frac{2}{M^3} \left(\tanh \frac{M}{2} - \frac{M}{2} \right) - \frac{\psi}{c^2} \right]} \cdot \left[\frac{\varphi_0 + \varphi_1 + 1}{\varphi_0 + \varphi_1 + \frac{\tanh(M/2)}{(M/2)}} \right] \tag{4}$$

Lastly, if the time Δt taken for the plate to move from the film thickness $h = h_0$ to $h = h_1$ then the dimensionless squeeze time ΔT is obtained from equation (4) as

$$\Delta T = \int_0^{t_1/t_0} \frac{W h_0^2}{\mu \pi^2 (a^2 - b^2)^2} dt$$

this means

$$\Delta T = \frac{1}{8\pi} I \tag{5}$$

where I is given by

$$I = -h_0^2 \int_1^{h_1/h_0} \left[\frac{1}{\frac{2h^3}{M^3} \left(\tanh \frac{M}{2} - \frac{M}{2} \right) - \frac{KH_0}{c^2}} \cdot \frac{1}{\frac{\varphi_0 + \varphi_1 + 1}{\varphi_0 + \varphi_1 + \frac{\tanh(M/2)}{(M/2)}}} \right] dh$$

3. Results and Discussion

Equations (3), (4), and (5) determine the dimensionless pressure, the load carrying capacity and response time respectively. These performance characteristics depend on various parameters such as $M, \psi, \phi_0 + \phi_1, k$ and ω . The above parameters respectively describe the effect of magnetization, porosity, conductivities, aspect ratio and semi-vertical angle. Setting M and $\phi_0 + \phi_1$ to be zero this investigation reduces to the observations of Prakash and Vij [14]. Further, taking $\psi=0$, this study leads to the behavior of squeeze film between two non-porous truncated conical plates. Setting the roughness parameters to be zero the present study reduces to the contributions of [5], [6] and [10] under special situations.

It is easily noticed that as $\phi_0 + \phi_1$ increases, W increases for fixed values of M, ψ, k and ω . Besides, the effect of conductivity on the pressure distribution, load carrying capacity W and the response time ΔT comes through the factor

$$\left(\frac{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)}}{\phi_0 + \phi_1 + 1} \right)$$

For large values of M , this tends to $\frac{\phi_0 + \phi_1}{\phi_0 + \phi_1 + 1}$ as

$\tanh M \sim 1, 2/M \sim 0$. One can easily see that both of these functions are increasing functions of $\phi_0 + \phi_1$. It may be observed from the mathematical analysis also that as $\phi_0 + \phi_1$ increases the pressure, load carrying capacity and the response time increase. It is also noticed that the bearing with magnetic field can support a load even when there is no flow.

In Fig. 1- 4, we have the variation of load carrying capacity with respect to the magnetization parameter M for various values of porosity parameter ψ , conductivity parameter $\phi_0 + \phi_1$, semi-vertical angle, ω and aspect ratio k respectively. It is clearly seen from these figures that load carrying capacity increases significantly with respect to the magnetization parameter M wherein semi-vertical angle ω is the dominant partner. Further, the increase in load carrying capacity for the combination M and $\phi_0 + \phi_1$ is relatively less as compared to the other three cases.

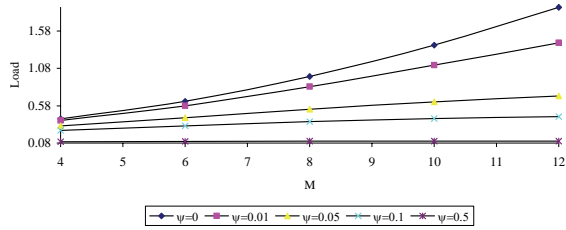


Figure 1: Variation of load carrying capacity with respect to M and Ψ

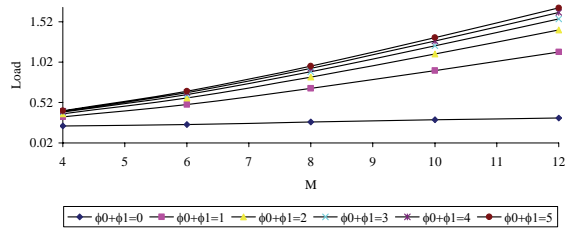


Figure 2: Variation of load carrying capacity with respect to M and $\Phi_0 + \Phi_1$

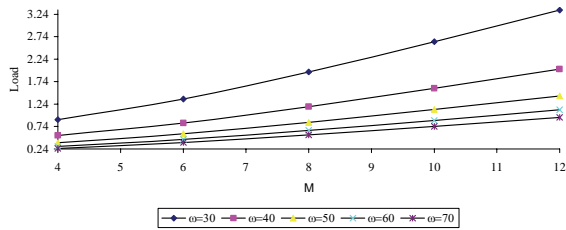


Figure 3: Variation of load carrying capacity with respect to M and ω

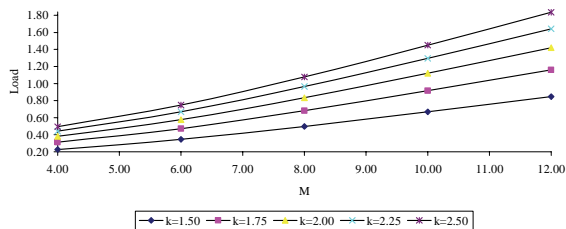


Figure 4: Variation of load carrying capacity with respect to M and k.

Fig. 5-7 depict the distribution of load carrying capacity with respect to conductivity $\phi_0 + \phi_1$ for several values of the parameters ψ , ω and k respectively. It is noticed that the conductivity tends to increase the load carrying capacity. Here the combined effect of the conductivities and the aspect ratio is relatively better than the combined effect of conductivity and porosity.

Fig. 8 and Fig. 9 give the load profile with respect to porosity for various values of the aspect ratio and semi-vertical angle associated with the cone. It is transparent from these two figures that the bearing suffers on account of porosity as the load carrying capacity decreases considerably. However, Fig. 10-13 make it clear that porosity effects are negligible up to $\psi \approx 0.001$.

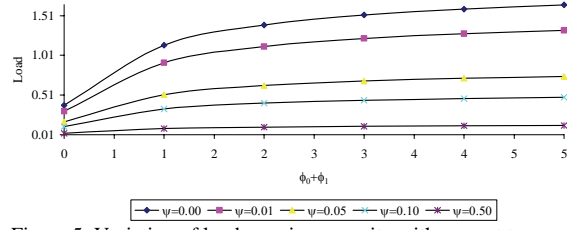


Figure 5: Variation of load carrying capacity with respect to $\Phi_0 + \Phi_1$ and Ψ

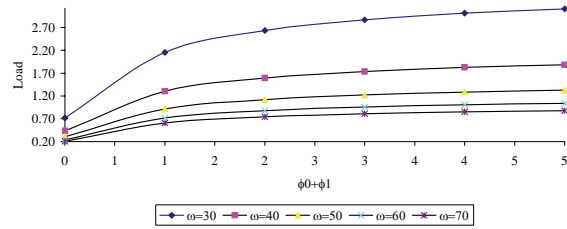


Figure 6: Variation of load carrying capacity with respect to $\Phi_0 + \Phi_1$ and ω

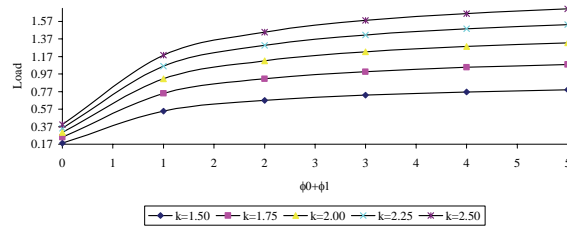


Figure 7: Variation of load carrying capacity with respect to $\Phi_0 + \Phi_1$ and k

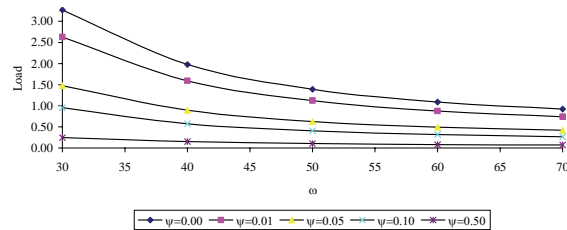


Figure 8: Variation of load carrying capacity with respect to ω & Ψ

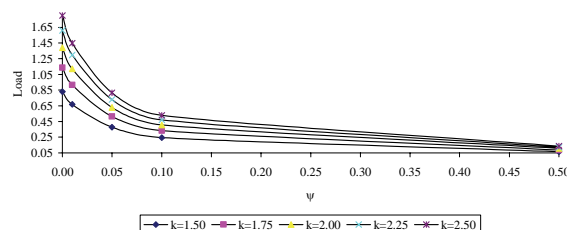


Figure 9: Variation of load carrying capacity with respect to Ψ & k

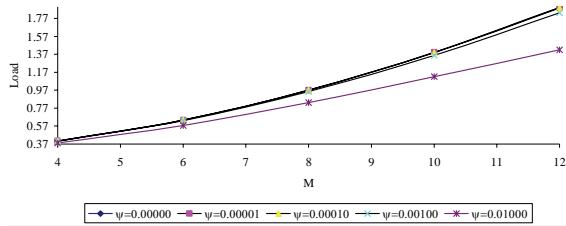


Figure 10: Variation of load carrying capacity with respect to M and Ψ

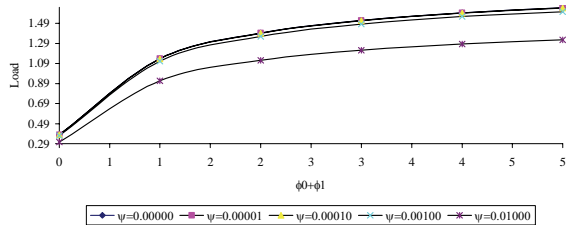


Figure 11: Variation of load carrying capacity with respect to $\Phi_0+\Phi_1$ and Ψ

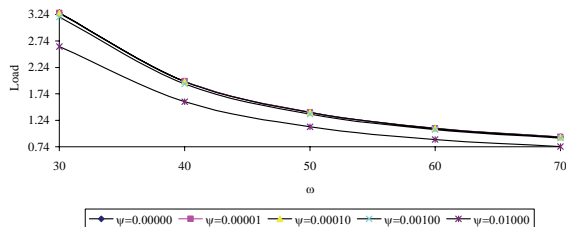


Figure 12: Variation of load carrying capacity with respect to ω and Ψ

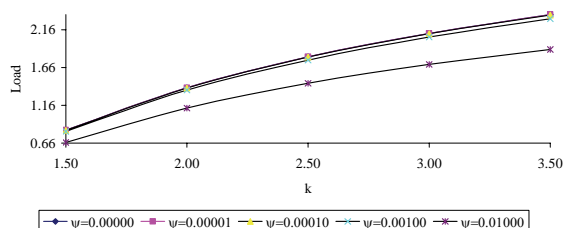


Figure 13: Variation of load carrying capacity with respect to k and Ψ

Lastly, we have Fig. 14, which describes the distribution of load carrying capacity with respect to the aspect ratio for various values of the semi-vertical angle. From this figure one can easily notice that the load carrying capacity decreases substantially due to the semi-vertical angle even if the aspect ratio is taken suitably.

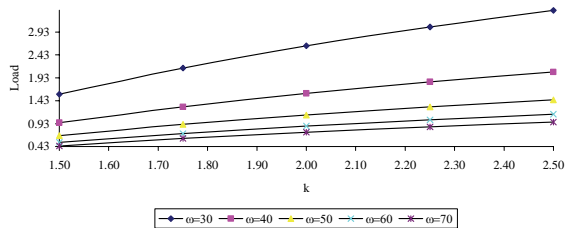


Figure 14: Variation of load carrying capacity with respect to ω and k

The combined effect of the semi vertical angle and magnetization parameter is relatively better than that of the combined effect of aspect ratio and porosity in the sense that the load distribution is more in the former case. Interestingly, the response time follows the trends of load carrying capacity (Table : (1) – (14)). However, this article suggests that the combined effect of magnetization parameter, aspect ratio and semi-vertical angle prevents the response time to fall rapidly.

Table: 1 Variation of Response Time with respect to M and Ψ .

	$\Psi=0$	$\Psi=0.01$	$\Psi=0.05$	$\Psi=0.1$	$\Psi=0.5$
M=4	0.11380	0.10717	0.08691	0.07030	0.02779
M=6	0.17938	0.16241	0.11782	0.08771	0.02881
M=8	0.27335	0.23460	0.14971	0.10309	0.02953
M=10	0.39152	0.31525	0.17718	0.11450	0.02939
M=12	0.53303	0.39944	0.19947	0.12269	0.03008

Table: 2 Variation of Response Time with respect to M and $\Phi_0+\Phi_1$

	$\Phi_0+\Phi_1=0$	$\Phi_0+\Phi_1=1$	$\Phi_0+\Phi_1=2$	$\Phi_0+\Phi_1=3$	$\Phi_0+\Phi_1=4$	$\Phi_0+\Phi_1=5$
M=4	0.06427	0.09644	0.10717	0.11253	0.11575	0.11790
M=6	0.06960	0.13921	0.16241	0.17401	0.18097	0.18561
M=8	0.07320	0.19550	0.23460	0.25415	0.26533	0.27370
M=10	0.08598	0.25793	0.31525	0.34391	0.36110	0.37256
M=12	0.09218	0.32262	0.39944	0.43785	0.46089	0.47625

Table:3 Variation of Response Time with respect to M and ω .

	$\omega=30$	$\omega=40$	$\omega=50$	$\omega=60$	$\omega=70$
M=4	0.25156	0.15221	0.10717	0.08385	0.07122
M=6	0.38122	0.23067	0.16241	0.12707	0.10793
M=8	0.55068	0.33320	0.23460	0.18356	0.15591
M=10	0.73998	0.44774	0.31525	0.24666	0.20950
M=12	0.93760	0.56731	0.39944	0.31253	0.26545

Table: 4 Variation of Response Time with respect to M and k.

	k=1.50	k=1.75	k=2.00	k=2.25	k=2.50
M=4	0.06397	0.08745	0.10717	0.12401	0.13857
M=6	0.09695	0.13252	0.16241	0.18792	0.21000
M=8	0.14004	0.19143	0.23460	0.27146	0.30334
M=10	0.18818	0.25724	0.31525	0.36477	0.40761
M=12	0.23844	0.32593	0.39944	0.46219	0.51647

Table:5 Variation of Response Time with respect to $\Phi_0+\Phi_1$ and Ψ

	$\Psi=0.00$	$\Psi=0.01$	$\Psi=0.05$	$\Psi=0.10$	$\Psi=0.50$
$\Phi_0+\Phi_1=0$	0.10678	0.08598	0.04832	0.03123	0.00815
$\Phi_0+\Phi_1=1$	0.32033	0.25793	0.14497	0.09368	0.02446
$\Phi_0+\Phi_1=2$	0.39152	0.31525	0.17718	0.11450	0.02989
$\Phi_0+\Phi_1=3$	0.42711	0.34391	0.19329	0.12491	0.03261
$\Phi_0+\Phi_1=4$	0.44846	0.36110	0.20295	0.13115	0.03424
$\Phi_0+\Phi_1=5$	0.46270	0.37256	0.20940	0.13532	0.03533

Table:6 Variation of Response Time with respect to $\Phi_0+\Phi_1$ and ω

	$\omega=30$	$\omega=40$	$\omega=50$	$\omega=60$	$\omega=70$
$\Phi_0+\Phi_1=0$	0.20181	0.12211	0.08598	0.06727	0.05714
$\Phi_0+\Phi_1=1$	0.60544	0.36633	0.25793	0.20181	0.17141
$\Phi_0+\Phi_1=2$	0.73998	0.44774	0.31525	0.24666	0.20950
$\Phi_0+\Phi_1=3$	0.80725	0.48844	0.34391	0.26908	0.22855
$\Phi_0+\Phi_1=4$	0.84761	0.51286	0.36110	0.28254	0.23997
$\Phi_0+\Phi_1=5$	0.87452	0.52914	0.37256	0.29151	0.24759

Table: 7 Variation of Response Time with respect to $\Phi_0+\Phi_1$ and k

	k=1.50	k=1.75	k=2.00	k=2.25	k=2.50
$\Phi_0+\Phi_1=0$	0.05132	0.07016	0.08598	0.09948	0.11117
$\Phi_0+\Phi_1=1$	0.15397	0.21047	0.25793	0.29845	0.33350
$\Phi_0+\Phi_1=2$	0.18818	0.25724	0.31525	0.36477	0.40761
$\Phi_0+\Phi_1=3$	0.20529	0.23062	0.34391	0.39793	0.44467
$\Phi_0+\Phi_1=4$	0.21555	0.29465	0.36110	0.41783	0.46690
$\Phi_0+\Phi_1=5$	0.22240	0.30401	0.37256	0.43109	0.48173

Table: 8 Variation of response time with respect to Ψ and ω

	$\omega=30$	$\omega=40$	$\omega=50$	$\omega=60$	$\omega=70$
$\Psi=0.00$	0.91900	0.55606	0.39152	0.30633	0.26019
$\Psi=0.01$	0.44774	0.44774	0.31525	0.24666	0.20950
$\Psi=0.05$	0.17718	0.25165	0.17718	0.13863	0.11775
$\Psi=0.10$	0.08959	0.16262	0.11450	0.03959	0.07609
$\Psi=0.50$	0.01987	0.04246	0.02989	0.02339	0.01987

Table: 9 Variation of Response Time with respect to Ψ and k

	k=1.50	k=1.75	k=2.00	k=2.25	k=2.50
$\Psi=0.00$	0.23371	0.31947	0.39152	0.45302	0.50623
$\Psi=0.01$	0.18818	0.25724	0.31525	0.36477	0.40761
$\Psi=0.05$	0.10577	0.14458	0.17718	0.20502	0.22910
$\Psi=0.10$	0.06835	0.09343	0.11450	0.13249	0.14805
$\Psi=0.50$	0.01784	0.02439	0.02989	0.03459	0.03865

Table: 10 Variation of Response time with respect to M and Ψ

	M=4	M=6	M=8	M=10	M=12
$\Psi=0.00000$	0.11380	0.17938	0.27335	0.39152	0.53303
$\Psi=0.00001$	0.11380	0.17937	0.27330	0.39142	0.53285
$\Psi=0.00010$	0.11373	0.17920	0.27290	0.39057	0.53125
$\Psi=0.00100$	0.11310	0.17753	0.26891	0.38227	0.51578
$\Psi=0.01000$	0.10717	0.16241	0.23460	0.31525	0.39944

Table: 11 Variation of Response Time with respect to $\Phi_0+\Phi_1$ and Ψ

	$\Phi_0+\Phi_1=0$	$\Phi_0+\Phi_1=1$	$\Phi_0+\Phi_1=2$	$\Phi_0+\Phi_1=3$	$\Phi_0+\Phi_1=4$	$\Phi_0+\Phi_1=5$
$\Psi=0.00000$	0.10678	0.32033	0.39152	0.42711	0.44846	0.46270
$\Psi=0.00001$	0.10675	0.32025	0.39142	0.42700	0.44835	0.46259
$\Psi=0.00010$	0.10652	0.31956	0.39057	0.42603	0.44738	0.46158
$\Psi=0.00100$	0.10425	0.31276	0.38227	0.41702	0.43787	0.45177
$\Psi=0.01000$	0.08598	0.25793	0.31525	0.34391	0.36110	0.37256

Table: 12 Variation of Response Time with respect to ω and Ψ

	$\omega=30$	$\omega=40$	$\omega=50$	$\omega=60$	$\omega=70$
$\Psi=0.00000$	0.91900	0.55606	0.39152	0.30633	0.26019
$\Psi=0.00001$	0.91878	0.55593	0.39142	0.30626	0.26012
$\Psi=0.00010$	0.91679	0.55472	0.39057	0.30560	0.25956
$\Psi=0.00100$	0.89729	0.54293	0.38227	0.29910	0.25404
$\Psi=0.01000$	0.73998	0.44774	0.31525	0.24666	0.20950

Table:13 Variation of Response Time with respect to k and Ψ

	k=1.50	k=1.75	k=2.00	k=2.25	k=2.50
$\Psi=0.00000$	0.23371	0.39152	0.50623	0.59392	0.66346
$\Psi=0.00001$	0.23365	0.39142	0.50611	0.59378	0.66330
$\Psi=0.00010$	0.23315	0.39057	0.50501	0.59249	0.66186
$\Psi=0.00100$	0.22819	0.38227	0.49427	0.57939	0.64779
$\Psi=0.01000$	0.18818	0.31525	0.40761	0.47822	0.53422

Table: 14 Variation of Response Time with respect to ω and k

	k=1.50	k=1.75	k=2.00	k=2.25	k=2.50
$\omega=30$	0.44172	0.60381	0.73998	0.85622	0.95679
$\omega=40$	0.26727	0.36535	0.44774	0.51807	0.57892
$\omega=50$	0.18818	0.25724	0.31525	0.36477	0.40761
$\omega=60$	0.14724	0.20127	0.24666	0.28541	0.31893
$\omega=70$	0.12506	0.17095	0.20950	0.24241	0.27089

It can be seen clearly from this investigation that the negative effect of porosity and semi-vertical angle can be compensated up to considerable extent by the positive effect introduced by M , $\phi_0 + \phi_1$ and k . Thus, while designing the bearing system, the radii ratio and the conductivities must be given due considerations for a better performance of hydromagnetic squeeze film even if the semi-vertical angle is chosen suitably.

In addition, this article suggests that the negative effect induced by the porosity can be neutralized sufficiently by the positive effect introduced by the magnetization parameter and the aspect ratio by suitably choosing the conductivities. It is appealing to note that the combined effect of magnetization parameter and the aspect ratio prevents the response time to fall rapidly.

4. Practical importance

The practical importance of the present study lies in describing the bearing behavior during steady state performance. It also offers understanding the bearing's behavior during the transient squeezing period.

5. Conclusion

This article makes it clear that the negative effect induced by porosity can be over come completely by the positive effect of magnetization parameter and conductivities by choosing suitably the aspect ratio and the semi vertical angle and the bearing may register a considerably enhanced performance.

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Appendix – 1

Normally the following assumptions are made:

1. The lubricant flow is considered laminar and lubricant film is assumed to be isoviscous.
2. Although, forces due to gravitational attraction and electromagnetic forces are present, these forces are small as compared to the viscous force involved (and hence can be considered negligible).
3. The flow is considered steady and temperature changes of the lubricant are neglected.
4. The bearing surfaces are assumed to be perfectly rigid so that elastic deformations of the bearing, surfaces may be neglected.
5. The surface roughness it is of very small order of magnitude in comparison with the minimum film thickness.
6. The thickness of the lubricant film is very small when compared to the dimensions of the bearing.
7. The lubricant velocity along the transverse direction to the film is considered small enough.

8. Velocity gradients and indeed the second derivatives along the direction transverse to the film are predominant as compared to those in the plane of the film.
9. The lubricant inertia is considered negligible.
10. The porous matrix of the bearing surface is assumed to be homogeneous and isotropic.
11. Darcy's law is assumed to govern the lubricant flow within the porous matrix, while no slip condition is taken at the porous matrix-film interface.

Appendix – 2 Derivation of M H D equations:

When a large external electromagnetic field through the electrically conducting lubricant is applied, it gives rise to induced circulating currents, which in turn, interacts with the magnetic field and creates a body force called Lorentz force. This extra electromagnetic pressurization pumps the fluid between the bearing surfaces. In such a case Navier-Stokes equations for a steady, incompressible and isoviscous liquid gets modified as

$$\rho(\vec{q} \cdot \nabla) \vec{q} = -\nabla p + \mu \nabla^2 \vec{q} + \vec{J} \times \vec{B}_0 \quad (E1)$$

$$\nabla \cdot \vec{q} = 0 \quad (E2)$$

The Maxwell's equations and the Ohm's law governing electromagnetic phenomena are:

$$\nabla \cdot \vec{B} = 0 \quad (E3)$$

$$\nabla \times \vec{B} = \mu_e \vec{J} \quad (E4)$$

$$\nabla \cdot \vec{E} = 0 \quad (E5)$$

$$\nabla \times \vec{E} = 0 \quad (E6)$$

and

$$\vec{J} = \sigma(\vec{E} + \vec{q} \times \vec{B}_0) \quad (E7)$$

The equations of motion in the x- and z- directions are obtained from the equations (E1) and (E3) – (E6), take the form

$$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - J_z B_0 = 0 \quad (E8)$$

$$-\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial y^2} + J_x B_0 = 0 \quad (E9)$$

in addition, the electric currents in these directions from the equation (E7) are obtained as

$$J_x = \sigma(E_x - w B_0) \quad (E10)$$

$$J_z = \sigma(E_z + u B_0) \quad (E11)$$

Substituting these values of J_x and J_z in equations (E8) and (E9), we get,

$$\frac{\partial^2 u}{\partial y^2} - \frac{M^2}{h^2} u = \frac{1}{\mu} \frac{\partial p}{\partial x} + \frac{M}{h} \sqrt{\frac{\sigma}{\mu}} E_z \quad (E12)$$

$$\frac{\partial^2 v}{\partial y^2} - \frac{M^2}{h^2} v = \frac{1}{\mu} \frac{\partial p}{\partial z} + \frac{M}{h} \sqrt{\frac{\sigma}{\mu}} E_x \quad (E13)$$

where

$$M = B_0 h \left(\frac{\sigma}{\mu} \right)^{1/2} = \text{Hartman number.}$$

and

$$\frac{\partial p}{\partial y} = 0 \quad (E14)$$

Solving equations (E12) and (E13) with necessary boundary conditions we obtain the values of u and v given by

$$u = \frac{h^2}{\mu M^2} \frac{\partial p}{\partial x} \left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)}} \right] \left[\frac{\cosh \frac{M}{2} \left(\frac{2y}{h} - 1 \right)}{\cosh \frac{M}{2}} - 1 \right] \quad (E15)$$

$$v = \frac{h^2}{\mu M^2} \frac{\partial p}{\partial z} \left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)}} \right] \left[\frac{\cosh \frac{M}{2} \left(\frac{2y}{h} - 1 \right)}{\cosh \frac{M}{2}} - 1 \right] \quad (E16)$$

The velocity of the lubricant in the porous region satisfies the modified Darcy's law, equation of continuity and generalized Ohm's law.

In the present case, we have, for the porous region:

$$\bar{u} = \left[-\frac{K}{\mu} \frac{\partial p}{\partial x} - K \sqrt{\frac{\sigma}{\mu}} \frac{M}{h} E_x \right] \frac{1}{c^2} \quad (E17)$$

$$\bar{w} = -\frac{K}{\mu} \frac{\partial p}{\partial y} \quad (E18)$$

$$\bar{v} = \left[-\frac{K}{\mu} \frac{\partial p}{\partial z} - K \sqrt{\frac{\sigma}{\mu}} \frac{M}{h} E_z \right] \frac{1}{c^2} \quad (E19)$$

where

$$c^2 = 1 + \frac{K M^2}{m h^2}$$

Using equations (E15 - E19) in the equation of continuity and simplifying it one gets the modified Reynolds' equation as

$$\nabla^2 p = \frac{1}{\left[\frac{2h^3}{\mu M^3} \left\{ \left(\tanh \frac{M}{2} - \frac{M}{2} \right) - \frac{\psi h^3}{\mu c^2} \right\} \right]} \frac{dh}{dt} \left[\frac{\phi_0 + \phi_1 + 1}{\phi_0 + \phi_1 + \frac{\tanh(M/2)}{(M/2)}} \right] \quad (E20)$$

where

$$\psi = \frac{KH_0}{h^3}$$

