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Mathematical modelling for precisely improving inputs supply for crop production

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Abstract. Although farm size may make a difference in access to all precision agriculture techniques, farms including small-scale traditional crop cultivation will likely have access to some of them in the long term. For this farm sector, a mathematical model is being developed to assist decision-making for improved dosage of nutrients and pesticides for crops or feed for animals. The objective was to find out the maximum allowed permissible deficiencies in dosing of inputs compared with the number of repetitions for improving precision dosage each time it is spread to the field. The model is based on a number of specified repetitions and it calculates the amount of deficiency to be obtained. It is possible to find that, depending on the rate of application, there is a wide range of choices among different fertilizer formulae and their concentration of available nutrients. The higher the number of applications, the more precision could be achieved. This will make it possible to arrive at optimum application rates for each field point or for supplying a more precise rate of feed to the animals.

Key words: mathematical model, site-specific crop management, precision agriculture, inputs spreading design

INTRODUCTION

Traditionally, agricultural fields have been treated as one unit and the same inputs have been applied over the entire field area. Treating a field as a single unit will often result in areas that are under- or over-supplied with inputs.

Gradual improvement of the crop production system should be given top priority for rural socio-economic development and should be an evolutionary process. A series of appropriate technologies should be integrated to support the improvement of traditional cultivation technology in accordance with local conditions (Letey, 1985; Dreyer, 2004).

What appear to be of particular interest are applications which, without reducing end output and by using the available technology at all levels, allow increasing precision by delivering total dosage during several supply repetitions (Rohrbach et al., 1971).

A pragmatic alternative to actual measurement is to realize that a uniform decision, which fails to recognize actual delivery variation, introduces error (Dräxler, 1999). Careful design planning on discharge input delivery specifications to be used in every field, on every farm are crucial for the successful and economical implementation of site-specific crop management (Hegert et al., 1997). Proper mathematical function and design for these experiments have yet to be developed.

Research is important for human life today and is especially vital for the subsistence and development of a country's rural zone (Wang, 2003). Therefore, in order to start developing the mathematical model in this work, it is important to refer to the Lisbon Commitment, with special attention to the following aspects:

1. To assist man to build up a sound and commercial agriculture.
2. To assist the selling of farm produce during occasional shortcomings in a competitive market.
3. To increase human welfare, income and employment.
4. To assist in the formation and growth of small enterprises.
5. To facilitate the usage of innovative results from research and development.
6. To assist dynamic entrepreneurs.
7. To improve the organization of the agri-food production chain.
8. To assist the installation and use of pioneering communication and transport systems.
9. To undertake any opportunity to improve local infrastructure, environmental management and development of rural zones.

However, all the above must follow the rules driven by the economy. Therefore, it is very important to undertake a cost-benefit analysis in order to determine any profit (Werner, 2006).

Nowadays, subsidies must accomplish environmental protection and at the same time provide acceptable economic development for rural zones. These regions have to be considered as a whole unit, therefore tailoring farming activities to preserve natural resources within a rural area. The objective is to achieve encouraging socio-economic development and to learn the productive potential that rural zones have for national employment (Klenke, 2006). In this sense, farming must learn to use precision agriculture technologies (Morgan & Ess, 1997). Thus, aided by precision agriculture definition, the purpose of this study is to demonstrate a mathematical approach to aid decision-making for improving input supply dosage in small farming situations.

DEVELOPMENT OF MATHEMATICAL APPROACH

(1) Theory

Precision agriculture's goal is to increase yield and human productivity and to reduce factors that eventually will become pollutants to the environment. This is achieved by reducing the effect natural variations have on forage for feeding, fertilizer, etc., including those inconsistencies due to their chemical, biological and physical elements (Ortiz-Laurel & Rössel, 2003).

For the specific case of agrochemical spreading on a field or when providing forage to livestock, dosing can have several differences, such as:

a) Material delivery through the spout (output from the machine) Q_m.

$$Q_m = Q_v * \delta \quad (1)$$

Where:
δ = density (kg m^{-3})
Q_v = flow of volume (m^3 s^{-1})

b) Mass of material supplied due to delivery of fertilizer, for instance, can be illustrated as:

$$M_F = Q_m * t_D \quad (2)$$

Where:
t_D = time taken for flowing (s)
M_F = mass of fertilizer left above the ground (kg)

c) Thus, amount of nitrogen provided through the mass of fertilizer being spread over the land, can be expressed as:

$$M_N = M_F * \delta_N \quad (3)$$
$$M_N = Q_v * \delta * t_D * \delta_N \quad (4)$$

Where:
δ_N = density of nitrogen in the mass of fertilizer (for this analysis, percentage of nitrogen in the fertilizer = concentration of nitrogen in the fertilizer)

For all the above, it has been assumed that nitrogen supplied can come from three different fertilizers' formulae, each one with different concentration (densities) of nitrogen and from variations in the concentrations in the three products (fertilizers). This initial data is used for calculating the errors (variations) in the real spreading of the material on the field, thus;

d) $$M_N = M_{N1} + M_{N2} + M_{N3} \quad (5)$$

Each mass in the above expression incorporates an error that can be originated from chemical concentration, temperature, impurities, moisture, structure and texture of the material, etc.

Where:
M_{N1} = original mass of nitrogen inside the fertilizer “1”
M_{N2} = original mass of nitrogen inside the fertilizer “2”
M_{N3} = original mass of nitrogen inside the fertilizer “3”

e) Here, it is important to define the amount of nitrogen that is provided through the mass for the three different fertilizers. It can be considered as:

$M_{N1} = 0.3 * M_N$
$M_{N2} = 0.5 * M_N$
$M_{N3} = 0.2 * M_N$

Therefore, mass M_N is:

$$M_N = M_{N1} = 0.3 * M_N + M_{N2} = 0.5 * M_N + M_{N3} = 0.2 * M_N$$

f) Error due to the way nitrogen is supplied (spread on the field). It is calculated using the law of quadratic transmission for errors, i.e. there is a superposition of deliveries, where each error in each fertilizer is squared, (ordinary discharge delivery with inaccuracies), thus;

$$U_{MN} = \sqrt{U^2{}_{MN1} + U^2{}_{MN2} + U^2{}_{MN3}} \qquad (6)$$

Where:
U_{MN1} = error (statistical variation) due to the variation of nitrogen in the fertilizer “1”
U_{MN2} = error due to the variation of nitrogen in the fertilizer “2”
U_{MN3} = error due to the variation of nitrogen in the fertilizer “3”
U_{MN} = error due to the variation in total nitrogen provided

Each of these errors have their origin in small inaccuracies due to several factors, such as delivery volume ($m^3\ s^{-1}$), density of mass ($kg\ m^{-3}$), time taken for flowing (s) and nitrogen concentration in the mass of fertilizer ($kg\ kg^{-1}$)

Every error for each element in the process altogether form the total error for total amount of nitrogen spread in the field (large and small areas = different t_D (by definition of precision agriculture = application for small areas depending on the conditions)) which can have a positive or negative value.

U_1 = error for volume delivery ($m^3\ s^{-1}$),
U_2 = error for the density of mass ($kg\ m^{-3}$),
U_3 = error for the time taken for volume delivery (s),
U4 = error from the concentration of nitrogen in the mass of fertilizer ($kg\ kg^{-1}$)

$$M_N = Q_v * \delta * t_D * \delta_N \qquad (7)$$

U_{MN1} or U_{MN2} or U_{MN3} for the three different fertilizers =

$$\pm \sqrt{(U_1 * \delta * \delta_N * t_D)^2 + (U_2 * Q_V * \delta_N * t_D)^2 + (Q_V * \delta * t_D * U_3)^2 + (Q_V * \delta * \delta_N * U_4)^2}$$

Depending on the technology utilised for nitrogen spreading, errors U_1 are U_3 are errors due to the technique and the manner of application. Errors U_2 and U_4 are characteristics inherent to the fertilizer, which means it cannot be changed by the user. Therefore, it is important to use the above methodology to operate the model, by using data from three fertilizers, to calculate the reduction in the total error for the whole process.

(2) Example

A field is to have an application of 3000 g of fertilizer with an expected inaccuracy of 30%. It can be assumed to have four controlled deliveries.

Hence, for one controlled delivery;

3000 g * 30% = $\pm$ 900 g
1000 g * 30% = $\pm$ 300 g

$U_{MN} = \sqrt{300^2 + 300^2 + 300^2} = \pm \sqrt{270000} = \pm$ 519.6 g = 17.32%

With two controlled deliveries;

1500 g * 30% = $\pm$ 450 g

$U_{MN} = \sqrt{450^2 + 450^{2^2}} = \pm \sqrt{405000} = \pm$ 636.39 g = 21.21%

With four controlled deliveries.

750 g * 30% = $\pm$ 225 g

$U_{MN} = \sqrt{225^2 + 225^2 + 225^2 + 225^2} = \pm \sqrt{202500} = \pm$ 450g = 15%

Finally, these results show how well inaccuracies are overcome for each tested delivery, thus;

1. One flow has an inaccuracy of 30%
2. Two flows has an inaccuracy of 21.21%, i.e. error is reduced by 8.79% from the preceding figure.
3. Three flows has an inaccuracy of 17.32%, i.e. error is reduced by 3.89% from the preceding figure.
4. Four flows has an inaccuracy of 15%, i.e. error is reduced by 2.32% from the preceding figure.

Therefore, it can be concluded that increasing the number of deliveries for the same amount of material, inaccuracy is reduced, that is, there is more precision for the distribution.

Under certain circumstances, there could be a tolerance range (T) for total error when nitrogen is spread (Viscarra Rossel & McBratney, 1998). Thus, it can be necessary to calculate those errors that could arise for the three deliveries of fertilizers, using the following equation (Klaus, 2006):

$$\pm \mathrm{T} \quad \leq \pm \sqrt{U^2{}_{MN1} + U^2{}_{MN2} + U^2{}_{MN3}} \qquad (8)$$

Finally, from that expression the error that could be generated for using fertilizer one, for instance is;

$$(\pm U_{MN1})^2 = \pm T^2 - (\pm U_{MN2})^2 - (\pm U_{MN3})^2$$

Fig. 1 shows the second example for illustrating the way the number of delivery repetitions influences the precision of supplying forage to livestock.

From Fig. 1, the following can be drawn:

1. For one discharge delivery of 9 kg m^{-1} (can also be expressed as kg m^{-2} or kg s^{-1}) which has a variation of 35% for the value of the mass or energy provided or any other physical parameter, there is a dosage error of 35%.
2. For two flow deliveries of 4.5 kg m^{-1} each and with a discharge variation of 35%, there is a dosage error of 15%.
3. For a delivery of three times and with a variation for the flow supply of 35%, there is a dosage error of 8%.

The methodology described can also be applied to any solid, liquid and gaseous material. There are also equations for supplying different discharge flows citing different inaccuracies for each one of them.

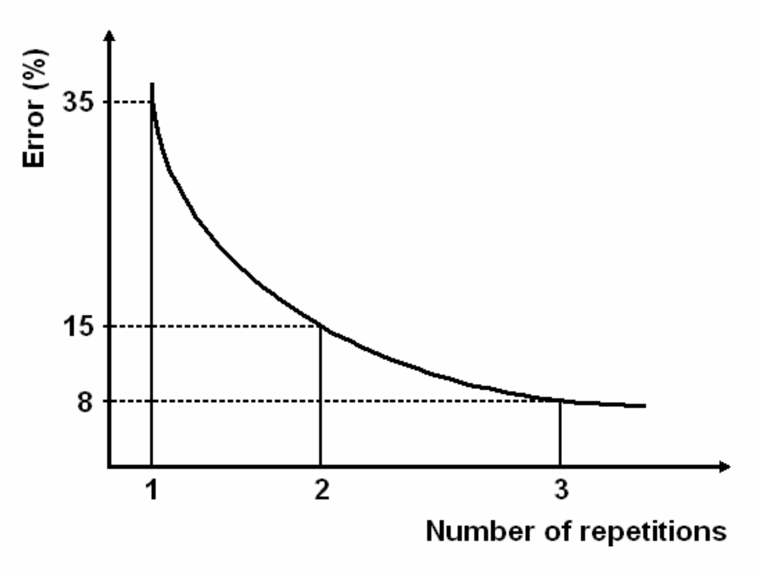

Fig. 1. Precision variations while supplying forage (silage, hay or other) to cows in a covered shed.

CONCLUSIONS

By using the superposition method it is possible to reach the goal of precisely controlling the variability of crop inputs supply in a more economical and technologically efficient manner. In this particular case, a nutritive element, by controlling its concentration, density, homogeneity, etc., expecting this nutrient is normally spread on the field by using the available technology. Assuming also, that the basic mechanism rests on the process of controlling input dosing, therefore, when targeting for the highest dosage percentage for treatment control in farming, it is possible to make precise measurements or, through the superposition of flow deliveries where the target factor is expected to have the highest variation with respect to the expected result for the end product. This method can be the basis for a technology of high precision with an economical benefit: by reducing the technical precision required and by taking advantage of natural processes that occur in agriculture, such as application of inputs throughout variable dosage during diverse periods of time and expecting different results all over plants or animal growth. In the calculated example, the superposition method analyzed the biological energy provided through forage through supplying different flow deliveries to the animals. The error inherent in the biological energy provided is reduced for each repetition for the delivery of forage and it is much smaller compared to the delivery of forage in just one application.

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