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Multi-goal pig ration formulation; mathematical optimization approach

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Abstract. Organically produced pork is characterized by high production costs, within the main part goes to ration cost. Forage must be produced under strict conditions, reflecting in high prime costs. The main challenge for farmers is how to formulate economically efficient, nutrition balanced and politically acceptable rations at the least-cost to be competitive. This challenging task demands handy tool that merges all three viewpoints. In this paper an example of such a tool, based on three step approach, is presented. In the first step, a common linear program is utilized to formulate least-cost ration. In the second step, a sub-model, based on weighted goal programming and supported by a system of penalty functions, is used to formulate a nutritionally balanced and economically acceptable ration that also fulfils conditions demanded by organic farming. The most 'efficient' energy content of the ration is searched in the last step. The obtained results confirm the benefits of the applied approach.

Key words: mathematical programming, ration costs, organic farming, pork

INTRODUCTION

Organic farming is globally characterized by higher production costs that are affected by strict organic production policy constraints; however profits are very diverse and are highly related to the market strategy in place. Because the farmer's main objective is to maximize profits, costs must be minimized. This may be accomplished through improved technical or economical efficiency. Due to high expense of ration costs and the possibility of negative externalities that might occur, it is obvious that ration formulation is a crucial task in daily pig breeding management– even more if the organic farming practice is in place.

Comparing to the conventional production the majority of fodder is usually produced at the farm gate or less common, purchased (maximum 20%) from another organic producer in the same region at the relatively high price. In this case, changes in world (cereal) markets could rapidly affect the economic outcome. However, even if the majority of the feed is produced at the farm gate, there are opportunity costs that require the decision maker to make efficient decisions in relation to breeding practices. This may allow for improved productivity, or at least may keep profitability at an acceptable level. Organic fattening confronts also with the lack of availability of pure amino acids that results in more unbalanced protein composition, increased feed cost and what is unlike with organic philosophy, increased load of excessive nitrogen from manure on the environment (Blair, 2007). In order to help breeders to deal with these challenges, many tools based on mathematical programming (MP) paradigm have been developed.

The first problem of this kind has been conducted by Waugh (1951), who applied the linear programming (LP) paradigm in order to formulate rations on a least-cost basis. This approach has been very popular in the past, especially after the rapid development of personal computers. In the 1960s, it became a classical approach to formulate animal diets as well as feed-mixes (Black & Hlubik, 1980). More recently, Castrodeza et al. (2005) stressed that the daily routine of ration formulation is one of the fields in which LP is most widely used.

Common to all LP problems is the concept of constraint optimization, which means that one tries to find the optimum of a single objective function. However, exclusive reliance just on one objective (cost function) as the only and the most important decision criteria is one of the reasons why the LP paradigm may be a deficient method in the process of ration formulation (Rehman & Romero, 1984; 1987). Lara & Romero (1994) stress that in practice decision makers never formulate rations exclusively on the basis of a single objective, but rather on the basis of several different objectives, where economic issues are only one of many concerns.

In common LP models for pig ration formulation, animal amino acid requirements are usually expressed in terms of minimal concentrations. Such models do not consider the total exceeded amount of protein or its quality as long as the minimal amounts of essential amino acids are satisfied (Bailleul et al., 2001). The same authors stress that 'economical optimal' diets are often too rich in protein, which directly burdens the environment and does not improve animal growth. This problem could partly be solved by adding additional upper or lower constraints. However, it might rapidly lead into over-constraint model that has no feasible solution. This problem is also related to the next LP drawback-rigidity of constraints (right hand side–RHS) (Rehman & Romero, 1984). This means that no constraint (e.g. given nutrition requirements) violation is allowed at all. However, relatively small deviations in RHS would not seriously affect animal welfare, but would result in a feasible solution (Lara & Romero, 1994).

Numerous methodological developments in the field of MP have eased these problems of LP paradigm (Buysse et al., 2007). For instance in the field of animal nutrition, Rehman & Romero (1984) introduced goal programming (GP) and its improvement with a system of penalty function (PF), as well as multi-objective programming (MOP) as a way to incorporate more than one objective function; Lara & Romero (1994) applied interactive methodologies where the optimal ration is achieved through 'computer dialog'; Castrodeza et al. (2005) addressed a multicriteria fractional model.

The purpose of this paper is to present a spreadsheet tool for organic pig ration formulation, designed as a three-phase optimization approach that merges two normative MP techniques. The first part of the paper provides a brief overview of weighted goal programming (WGP) and the penalty function. This is followed by a short description of the optimization tool that also involves LP in order to calculate least-cost ration formulation. Finally, the characteristics of the analysed case are presented, followed by the results and discussion.

MATERIALS AND METHODS

Weighted goal programming supported by a system of penalty functions

Common to all MP problems is the concept of constraint optimization, which means that one tries to find the optimum of a single objective function within set of constraints. Based on the approaches reported in the literature and the primary aim of the tool presented in this paper, we decided to apply the WGP approach. This was in the context of ration formulation introduced by Rehman & Romero (1984).

WGP formulation is expressed as a mathematical model with a single objective (achievement) function (the weighted sum of the deviations variables). The optimal compromise solution is found through the philosophy of 'distance measure' that measures the discrepancy between the desired goal and the performance level of a goal. To consider all goals simultaneously normalization techniques should be applied (Tamiz et al., 1998).

Rehman & Romero (1984) introduced PF paradigm into the WGP to keep deviations within desired limits and to distinguish between different levels of deviations. This system is coupled with the achievement function (WGP) through penalty coefficients and with additional constraints defining deviation intervals. Such approach enables one to define allowable positive and negative deviation intervals separately for each goal. Depending on the goal's characteristics (nature and importance of 100% matching), these intervals might be different. Sensitivity is dependent on the number and size of defined intervals and the penalty scale utilized (s_i; for i=1 to n).

Tool for three phase pig ration formulation

Presented optimization tool for organic pig ration formulation was developed in MS Excel as an add-in application. This tool is capable of formulating least-cost, nutritionally balanced, and environmental acceptable rations for 'organically' growing pigs in different production periods. It also gives information about which feed-mix provides the optimal energy content.

The tool is organized as a three phase approach that merges two sub-models based on MP techniques. The first sub-model is an example of a common least-cost ration formulation, based on the LP paradigm. The purpose of including this into the tool is to get an approximate estimate of expected ration cost. In this manner, the tool calculates the target economic goal, which is one of the goals in the second sub-model. The first sub-model is therefore, from the perspective of constraints, as simple as possible and is intended to exclusively measure the 'rough' cost estimation. Through cost function, this is linked to the second sub-model. The latter is based on WGP and is supported by a system of six sided PF. In this approach, the desired nutrition levels and ration costs are modelled as goals instead of as constraints. Besides in the second sub-model, additional constraints with indirect influence on the environment are added. Consequentially, the model is much more complex, and it finally yields a better solution. For more detailed mathematical description of the model one can refer to Žgajnar & Kavčič (2008), where the similar approach has been applied.

Due to the importance of energy concentration of the feed-mix and its influence on the ration structure and cost, the tool also includes a third phase. In this phase, a macro loop is added that runs the first and the second sub-models for n-times, and consequently it yields n-formulated rations. The number of iterations in the third phase depends on the starting/ending energy content of the feed-mix and on the energy rise in each iteration step (e.g. 0,1 MJ kg^{-1}). From the n-obtained solutions, the tool selects the cheapest option and marks it as the 'optimal' feed-mix structure for this given example.

Analyzed example

The tool has been applied for hypothetic organic pork production, with an average genotype for less intensive fattening. In this paper we present just the fattening period between 50 and 100 kg with an average daily gain of 700 g. We considered that the tool should formulate the complete ration/feed-mix in relation to the nutritional requirements. It is presumed that most of the fodder is produced at the farm under organic conditions and is evaluated with the full cost approach. The rest feed (less than 20%) that cannot be produced at the farm is accounted for at market price. However no synthetic substances (e.g. amino acids supplement) could be added, since they are banned by law.

The nutrition requirements (Metabolizable energy (35.2 MJ day^{-1}), Crude protein (399 g day^{-1}), Amino acids (Lys–19.7 g day^{-1}; Met+Cys–11.3 g day^{-1}; Thr–13 g day^{-1}; Trp–3.6 g day^{-1}) and Minerals (Ca–12.88 g day^{-1}; P–11.59 g day^{-1}; $P_{available}$–4.89 g day^{-1}; Na–2.58 g day^{-1})) are taken from Blair (2007). In order to prevent unrealistic solution that has too much of one feed in the diet, we considered recommendations for maximal feed inclusion (Blair, 2007) and (Futtermittelspecifische …, 2006), namely through additional upper-bound constraints (Table 1). In the process of ration formulation the tool could choose between twelve different feeds (Table 1) that might be produced at the farm (except: alfalfa-dehydrated, yeast-brewer's dried, potato protein concentrate that might be purchased at market price), and four mineral components (limestone, salt, monocalcium phosphate and dicalcium phosphate) that could be purchased at market price.

Table 1. Prices and nutritive values of available feed and their suggested maximal share of the ration.

Feed on disposal	Price* (Cent kg^{-1})	ME MJ kg^{-1}	DM	CP	Lys	Met+ Cys	Thr	Trp	Max** %
			g kg^{-1}						
Maize	18	14.1	880	85	2.5	3.5	3.0	0.8	0.6
Wheat	21	13.8	880	120	3.4	4.5	3.5	1.5	0.7
Barley	21	12.6	880	106	3.8	3.7	3.7	1.4	-
Oats	26	11.2	880	108	4.3	4.1	3.7	1.4	0.25
Wheat flour	17	12.5	880	167	7.3	5.6	6.5	2.0	0.15
Wheat bran	14	8.3	880	141	6.2	5.0	5.5	2.5	0.25
Alfalfa, dehydrated	33	6.1	910	180	8.7	4.5	7.8	2.9	-
Yeast, brewer's dried	71	13.2	900	452	32.1	11.7	21.8	5.1	0.05
Potato protein concentrate	132	15.7	930	780	56.9	20.1	45.3	10.6	0.15
Lupinseed meal	58	14.1	890	349	15.4	7.8	12.0	2.6	0.15
Faba beans	42	12.7	870	254	16.2	5.2	8.9	2.2	0.2
Pea - field	38	13.4	890	228	15.0	5.2	7.8	1.9	0.3

**Prices are estimated with model calculations – own source*

*** Suggested maximum inclusion of feedstuffs in pig diets*

Table 2. Importance of goals with corresponding penalty function intervals.

Goal	Unit (day^{-1})	Weight (w_i)	Penalty function intervals p_{i1}^{+} %	p_{i1}^{-}	p_{i2}^{+}	p_{i2}^{-}	Together p_i^{+} %	p_i^{-}
ME	MJ	75	1	0	2	0	3	0
CP	g	60	1	0	2	0	3	0
Lys	g	80	5	1	5	3	10	4
Met + Cys	g	60	5	1	5	3	10	4
Thr and Trp	g	60	5	1	10	3	15	4
$P_{available}$	g	40	3	1	5	3	8	4
Ca and Na	g	30	3	1	5	3	8	4
Cost	cent	5/90	10	∞	20	∞	30	∞

p_{i1}^{+}, p_{i1}^{-}, p_{i2}^{+}, p_{i2}^{-} *penalty intervals at the first and the second stage*

The tool offers the option to switch between goals and constraints, depending on the needs and preferences of the decision maker. In the analyzed case, we chose ten goals (Table 2) that should be met as accurately as possible.

The importance of each goal is defined by weights (w_i) ranging between 0 and 100. Relatively high values are set for amino acids, since reduction of unbalanced protein fraction by increased protein quality (fulfilling the amino acids ratios in relation to the energy) reduces nitrogen excretion and pollution. For each goal, deviation intervals are defined separately (Table 2). They are measured in percentage deviation from the desired level. The cost goal is the only one that is not penalized for negative deviation and simultaneously the negative interval is unlimited.

RESULTS AND DISCUSSION

The main objective of the tool presented in this paper is to assist organic producers in formulating diets that are balanced and at the same time as cheap as possible. On a simple example we present how the tool could be applied and what might be the benefits. Namely, for organic producers this task is due to numerous limitations and constraints very complex. We have presumed that the decision maker prepares a feed-mix for growing pigs, looking from two different viewpoints (scenarios). In the first scenario, the most important element is quality of the ration ($W_{cost}=5$), while in the second one, cost is more important ($W_{cost}=90$).

The results obtained are presented in Figs 1and 2. Fig. 1 illustrates the structure of the diet for the situation when economics is preferred to the quality (Scenario II). Fig. 2 illustrates the level of ration costs dependent on the energy concentration of the diet. The range of the energy content of the ration was set between 12.3 and 13.7 MJ of metabolizable energy (ME).

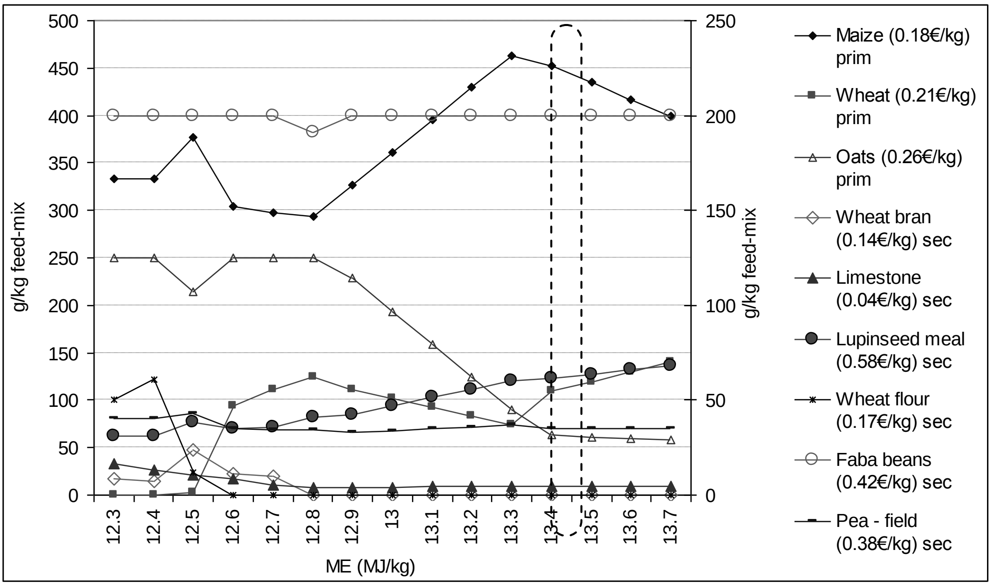

Fig. 1. Formulated feed-mix under scenario of high (W=90) ration cost importance (prim = primary axis; sec = secondary axis).

One of the factors that define how much a pig is going to eat is the energy content of the feed-mix. If the feed-mix is more concentrated, an animal is going to eat less, and vice versa (Blair, 2007). Fig. 1 presents formulated rations for the analysed fattening period. It is obvious that the energy content of the ration strongly influences selection of the feed. With increasing energy content, the quantity of maize increases and the quantity of oats decreases. From Fig. 1, it is apparent that in spite of expensive faba-beans, it enters into the solution, which is due to its favourable amino acids structure. The same holds for pea. Both are important substitutes for banned synthetic amino acid supplements.

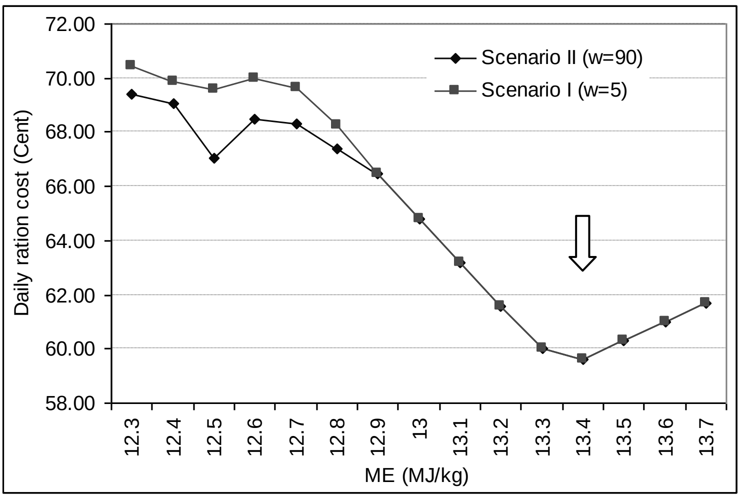

Fig. 2. Daily ration costs dependent on the feed-mix energy content.

The difference in daily ration costs between different energy concentrations is obvious. It ranges from 59.61 cents up to 70.43 / 69.42 cents per day per pig (scenario I/II). In any case, it should be an important issue to find the 'optimal' energy concentration of feed-mix in the daily management of organic pork production. In the Fig. 2 daily ration costs are presented for both scenarios. It is apparent that for analysed case importance of diet cost (Scenario II) has major influence only in the range of lower energy concentrations of feed-mixes (12.8 MJ kg^{-1} backwards), while from 12.9 MJ kg^{-1} onwards the trend of cost is the same. This is due to the fact that a feed-mix with lower energy content is harder to formulate especially more balanced one, which highly increases the costs. Consequently the minimal cost is achieved at relatively high energy concentration of feed-mix (Fig. 2), which is not usual in organic practise that is general less intensive. One could have legitimate scruples about the discrepancy between these results and practice, which is mainly due to poor quality of organically produced cereals in the sense of high nutritive value variability.

CONCLUSIONS

The results of this study show that the three phase optimization approach, supported by mathematical programming (LP and WGP with PF), can be efficiently applied to the diet formulation for organic pork production. The tool enables formulation of efficient diets, since it supports the farmer to find the optimal ration' energy content under various economic circumstances. With application of this tool problems like unbalanced protein composition, increased feed cost, increased burdening of the environment etc. might be mitigated. In this way the discrepancy between the aim of organic farming and practice could be reduced.

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