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An investigation on thermoelastic behaviour of functionally graded thick spherical vessels under combined thermal and mechanical loads

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Analysis and modelling

ABSTRACT

Purpose: The subject of this paper is to study the thermoelastic behavior of thick functionally graded hollow sphere under thermal and mechanical loads. The mechanical and thermal properties of FG sphere are assumed to be functions of radial position.

Design/methodology/approach: In present study, two methods are used to estimate the effective mechanical properties of FG sphere. One of the simplest methods in estimation of the effective mechanical and thermal properties of a mixture of two constituent materials is the Rule of Mixture (R-M) scheme. Another scheme for estimating the mechanical properties is due to the work of Mori-Tanaka. When the mechanical properties of FG sphere are estimated by using the Mori-Tanaka scheme, thermal material properties of FG body may be determined utilizing the R-M or the other methods which will be discussed as follows.

Findings: Results for the temperature, radial displacement, radial stress and hoop stress fields through the geometry of the sphere are give. The figures reveal that some minor difference may be obtained for two schemes and the difference between the results for displacement distribution is larger than difference of temperature and stress distributions.

Practical implications: The thermal material properties are obtained utilizing the Hatta-Taya and Rosen-Hashin relations. Also, the mechanical properties are estimated using the Mori-Tanaka scheme. In addition to the methods of approximation of material properties cited above, the rule of mixture scheme for determination of thermal and mechanical properties is also considered and results of these two schemes are compared for two cases of material composition through the geometry of FG sphere. The FG sphere is assumed to be symmetrically loaded and one-dimensional steady-state analysis of isotropic linear thermoelastic FG sphere under combined thermal and mechanical loads is investigated. Solution of the heat conduction equation and the Navier equation are obtained by using the Galerkin finite element method and by generating 100 elements along the radial direction of FG sphere.

Originality/value: This paper presents the FEM analysis of a functionally graded thick hollow sphere which its thermal and mechanical material properties only depend on the radial position.

Keywords: Functionally graded material; Hollow sphere; Thermal stress

1. Introduction

Functionally graded materials (FGMs) are new of branch of materials which can be used for various conditions such as thermal and mechanical load applications. The FGMs are microscopically nonhomogeneous materials where the composition of the constituents of materials is changed continuously. The mechanical benefits obtained by a material gradient may be significant, as can be seen by the excellent structure performance of some of these materials. Hence, there has been considerable interest in recent years in the application of such materials in areas such as lightweight armors, high temperature applications and industrial fields such as electronics, biomaterials and so on [1].

Among some articles which dealt with the subject of stress analysis of functionally graded structures the following papers may be referenced. Kwon et al. [2] studied the case of a graded sphere under non-uniform temperature variations by using a numerical integration procedure. Obata and Noda [3] used a perturbation approach to study the thermal stresses in functionally graded hollow sphere that was uniformly heated. Lutz and Zimmerman [4] solved the problem of uniform heating of spherical body whose elastic modulus and thermal expansion coefficients each vary linearly with radial position. Eslami et al. [5] analytically solved the governing equation of a functionally graded spherical vessel and investigated the temperature, displacement and relevant thermal stresses due to the thermal and mechanical loads

Wang and Mai [6] considered the finite element method to analyze one-dimensional transient heat conduction problems. Durodola and Adlington[7] presented the use of numerical methods to assess the effect of various forms of gradation of material properties to control deformation and stresses in rotating axisymmetric components such as disks and rotors. Nadeau and Ferrari [8] presented a one-dimensional thermal stress analysis of a transversely isotropic layer that was inhomogeneous along its thickness. Using the infinitesimal theory of elasticity, Naki and Murat [9] obtained close-form solution for stresses and displacements in functionally graded cylindrical and spherical vessels subjected to internal pressure. Fukui et al. [10] studied the problem of uniform heating of a radial inhomogeneous thick-walled cylinder.

In this paper, a thick hollow sphere made of functionally graded materials under one-dimensional steady-state temperature distribution with general type of boundary conditions (thermal and mechanical) on inside and outside of the sphere is considered. The thermal properties only depend on the radial position and are obtain utilizing the Hatta-Taya [11] and Rosen-Hashin[12] relations. Also, the mechanical properties of sphere are functions of position and are assumed to obey the Mori-Tanaka [13] relation. The results of using the power law equation for the material distribution and the methods used in present paper in determining of the materials properties are compared. Solution of the heat conduction equation and the Navier equation are obtained by using the Galerkin finite element method. Results for the temperature, radial displacement, radial stress and hoop stress fields through the geometry of the sphere are give.

2. Derivation of equations

2.1. Heat conduction problem

Consider a ceramic-metal FG sphere where its material composition is graded along the radial direction. The inner surface of the sphere which is ceramic rich side is exposed to symmetric thermal and mechanical loads. The heat conduction equation for one-dimensional steady-state problem in spherical coordinates is given as

$$k\frac{d^2T}{dr^2}+\frac{2k}{r}\frac{dT}{dr}+\frac{dk}{dr}\frac{dT}{dr}=0 \qquad (1)$$

where $T=T(r)$ is the temperature change, $k=k(r)$ is the thermal conductivity, and r is the radial position variable. In general form, the applied thermal boundary conditions due to the heat convection, heat flux and or prescribed temperature at the inner and outer surfaces of the sphere are given as

$$\begin{aligned}&A_{11}\frac{dT(a)}{dr}+A_{12}T(a)=C_1\\&A_{21}\frac{dT(b)}{dr}+A_{22}T(b)=C_2\end{aligned} \qquad (2)$$

In which A_{ij} are thermal conduction coefficient k or convection coefficient h, depends on the type of boundary conditions which may be considered. The terms C_1 and C_2 are known constants on the inside and outside radii of the sphere.

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2.2. Navier equation

The equilibrium equation of a symmetrically loaded functionally graded sphere in radial direction, ignoring the body force, is

$$\frac{d\sigma_{rr}}{dr}+\frac{2}{r}(\sigma_{rr}-\sigma_{\theta\theta})=0 \qquad (3)$$

The stress-strain relations for isotropic, linear thermoelastic functionally graded materials are given as follows

$$\begin{aligned}&\sigma_{rr}=\lambda e+2\mu\varepsilon_{rr}-\beta T\\&\sigma_{\theta\theta}=\lambda e+2\mu\varepsilon_{\theta\theta}-\beta T\end{aligned} \qquad (4)$$

where σ_{ij} and $\varepsilon_{ij}(i,j=r,\theta)$ are the components of stress and strain tensors, e is the first invariant of the strain tensor defined as $e=\varepsilon_{rr}+2\varepsilon_{\theta\theta}$, $\beta=(3\lambda+2\mu)\alpha$ is the effective stress-temperature modulus of FG sphere, and T is the temperature change which should be obtained from heat conduction equation. Also, terms λ and μ are the effective Lam`e constants and α is the effective linear thermal expansion coefficient. The kinematical strain-displacement relations are

$$\varepsilon_{rr} = \frac{du}{dr} \quad , \quad \varepsilon_{\theta\theta} = \frac{u}{r} \tag{5}$$

Substitution of strain components into the stress-strain equations of (4) and inserting the resulting relations in equilibrium equation of (3) lead to the Navier equations as

$$A_1 \frac{d^2u}{dr^2} + A_2 \frac{du}{dr} + A_3 u - \frac{d}{dr}(\beta T) = 0 \tag{6}$$

Where

$$\begin{aligned} A_1 &= \lambda + 2\mu \\ A_2 &= \frac{d(\lambda + 2\mu)}{dr} + \frac{2}{r}(\lambda + 2\mu) \\ A_3 &= \frac{2}{r}\frac{d\lambda}{dr} - \frac{2(\lambda + 2\mu)}{r^2} \end{aligned} \tag{7}$$

3. Estimating the effective thermal and mechanical properties of fg sphere

In present study, two methods are used to estimate the effective mechanical properties of FG sphere. One of the simplest methods in estimation of the effective mechanical and thermal properties of a mixture of two constituent materials is the Rule of Mixture (R-M) scheme. Another scheme for estimating the mechanical properties is due to the work of Mori-Tanaka [13]. When the mechanical properties of FG sphere are estimated by using the Mori-Tanaka scheme, thermal material properties of FG body may be determined utilizing the R-M or the other methods which will be discussed as follows.

3.1. Rule of mixture (R-M) scheme

In rule of mixture scheme, the fractions of mixtures are only considered and the forces between the phases are not included in consideration. The R-M scheme states that the material properties of a matrix phase and particulate phase in a mixture can be determined by application of only the effect of volume fraction of each phase on material properties. Thus

$$P = P_1 V_1 + P_2 V_2 \tag{8}$$

$$V_2 = \left(\frac{r-a}{b-a}\right)^n \tag{9}$$

where P is the effective properties of FG body, the subscript 1 denotes the matrix phase properties and the subscript 2 indicates the materials properties of particulate phase which is added to the matrix phase. Also, terms *a* and *b* are the inner and outer radii of FG sphere, V_1 and V_2 are volume fraction of material constituents of FG sphere which obey the relation $V_1 + V_2 = 1$, and *n* is the power index which by proper selection of its value a desired structure with suitable distribution of stresses or thermal field may be obtained. It should be noted that Lam`e constants λ and μ, thermal conductivity, *k*, and linear thermal expansion may be determined by using the R-M scheme while the other material properties of isotropic linear elastic FG sphere can be calculated by using the proper relations between the material properties. In this case, the Poisson's ration is considered to be constant.

3.2. Mori-Tanaka (M-T) scheme

The Mori–Tanaka scheme is applicable for estimation of effective mechanical properties of a graded microstructure which has a continuous matrix and a randomly distributed particulate phase. This scheme considers the forces between the matrix and particulate phases and takes the interaction of the elastic fields among neighboring inclusions into the account [13, 14]. It is assumed that the matrix phase, denoted by the subscript 1, is reinforced by spherical particulates, denoted by the subscript 2. In M-T scheme, the Bulk modulus and shear modulus are estimated by

$$\begin{aligned} \frac{K - K_1}{K_2 - K_1} &= \frac{V_2}{1 + 3(1 - V_2)(K_2 - K_1)/(3K_1 + 4\mu_1))} \\ \frac{\mu - \mu_1}{\mu_2 - \mu_1} &= \frac{V_2}{1 + (1 - V_2)(\mu_2 - \mu_1)/(\mu_1 + f_1)} \\ \lambda &= K - \frac{2\mu}{3} \end{aligned} \tag{10}$$

where K_1, K_2, are the bulk modulus of the constituents of FG body, μ_1, μ_2 are the shear modulus of constituent materials of FG body and $f_1 = \mu_1(9K_1 + 8\mu_1)/6(K_1 + 2\mu_1)$.

3.3. Estimation of thermal properties of FG sphere

As noted above, the thermal properties of FG body may be determined by using the R-M scheme. The other alternative method to estimate these material properties is given in the following. The thermal conductivity of the functionally graded sphere can be estimated utilizing the Hatta-Taya relation as [6]

$$\frac{k - k_1}{k_2 - k_1} = \frac{V_2}{1 + (1 - V_2)(k_2 - k_1)/3k_1} \tag{11}$$

where k_1 and k_2 are thermal conduction coefficient of the sphere material constituents. Also, the linear thermal expansion, α of a two phase domain may be determined by Rosen-Hashin equation as

$$\frac{\alpha-\alpha_1}{\alpha_2-\alpha_1}=\frac{1/K-1/K_1}{1/K_2-1/K_1} \tag{12}$$

in which the material properties given by subscript 1 and 2 are the material properties of matrix and particulate phases and the bulk modulus appeared in foregoing relation should be obtained by using Eq. (10).

4. Finite element implementation

In order to solve and analyze the functionally graded spherical pressure vessels, the governing equations may be solved analytically or numerically. Since the analytical solution of the difference equations of FG sphere is complicated to achieve, the numerical procedures are most considered by investigators. Among the numerical methods, the finite difference, finite element, and meshless methods are most interested and utilized for the analysis of structures. In this paper, the finite element method for its applicability and convergence is used to obtain the distribution of temperature, displacement and stress through the geometry of the FG sphere. To this end, the geometry of the sphere is divided into some discretized elements along the radial direction. The temperature and displacement in a base element can be approximated by

$$u^{(e)}=\sum_{i=1}^{l}N_i U_i \quad ; \quad T^{(e)}=\sum_{i=1}^{l}N_i T_i \tag{13}$$

in which terms N_i are the shape functions and U_i, T_i are, respectively, the nodal value of displacement and temperature, and l is the number of nodes per element. Now, the formal Galerkin finite element method may be applied to the governing equations (1) and (6) to find the nodal unknown temperature and displacement values. Thus

$$\int_{V(e)}\left(k\frac{d^2T}{dr^2}+\frac{2k}{r}\frac{dT}{dr}+\frac{dk}{dr}\frac{dT}{dr}\right)N_i dV=0 \tag{14}$$

$$\int_{V(e)}\left\{(\lambda+2\mu)\frac{d^2u}{dr^2}+\left[\frac{d(\lambda+2\mu)}{dr}+\frac{2}{r}(\lambda+2\mu)\right]\frac{du}{dr}+\left[\frac{2}{r}\frac{d\lambda}{dr}-\frac{2(\lambda+2\mu)}{r^2}\right]u-\frac{d}{dr}(\beta T)\right\}N_i dV=0 \tag{15}$$

Here, $V^{(e)}$ is the volume of base element. Application of the weak form to the first term of Eq. (14) and the first term, first term of second bracket and the last term of Eq. (15) leads to the following form of governing equations

$$\int_{r_f}^{r_l} k\,r^2\frac{dN_i}{dr}\frac{dT}{dr}dr=-N_i\,r^2\,q_r\Big|_{r_f}^{r_l} \tag{16}$$

$$\int_{r_f}^{r_l}\left\{\left(\left[(\lambda+2\mu)r^2\frac{dN_i}{dr}+2\lambda rN_i\right]\frac{d}{dr}+\left[4(\lambda+\mu)N_i+2\lambda r\frac{dN_i}{dr}\right]\right)u-\beta T\frac{d}{dr}(N_i r^2)\right\}dr=N_i r^2 t_r\Big|_{r_f}^{r_l} \tag{17}$$

where r_f and r_l are, respectively, the radial position of the first and last nodes of base element, respectively. Also, terms $q_r=-k\frac{dT}{dr}$ and $t_r=\sigma_{rr}\hat{e}_r$ are the heat flux and traction on the boundaries of the base element, respectively, and $\hat{e}_r$ is the outward unit normal to the boundary of the base element. To obtain the final form of the finite element formulation, the displacement and temperature should be replaced by the approximating Eqs. (13). Upon substitution of Eqs. (13) into Eqs. (14) and (15), the matrix form of the finite element formulation is obtained as

$$\begin{bmatrix}[K_{11}] & [K_{12}]\\ [K_{21}] & [K_{22}]\end{bmatrix}\begin{Bmatrix}\{T_i\}\\ \{U_i\}\end{Bmatrix}=\begin{Bmatrix}\{Q\}\\ \{F\}\end{Bmatrix} \tag{18}$$

where the submatrices [K_{11}], [K_{12}], [K_{21}], [K_{22}], { F}, and {Q} are as follows

$$\left[K_{11}^{ij}\right]=\int_{r_f}^{r_l}k\,r^2\frac{dN_i}{dr}\frac{dN_j}{dr}dr \tag{19}$$

$$\left[K_{12}^{ij}\right]=0 \tag{20}$$

$$\left[K_{21}^{ij}\right]=\int_{r_f}^{r_l}-\beta\frac{d(N_i r^2)}{dr}N_j dr \tag{21}$$

$$\left[K_{22}^{ij}\right]=\int_{r_f}^{r_l}\left(\left[(\lambda+2\mu)r^2\frac{dN_i}{dr}+2\lambda rN_i\right]\frac{dN_j}{dr}+\left[4(\lambda+\mu)N_i+2\lambda r\frac{dN_i}{dr}\right]N_j\right)dr \tag{22}$$

$$\left\{Q^i\right\}=-N_i r^2 q_r\Big|_{r_f}^{r_l} \tag{23}$$

$$\left\{F^i\right\}=N_i r^2 t_r\Big|_{r_f}^{r_l} \tag{24}$$

5. Numerical results

In this section, the finite element formulation is implemented for the analysis of FG sphere under combined thermal and mechanical loads. To investigate the accuracy of the results, a comparison between the results of FEM code developed in present work and those of analytical solution given in the literature is accomplished. To this end, 100 elements are generated along the radial direction of the FG sphere and the results for the case that the material properties obey a simple power law form function given in Ref. [5] are compared. The material properties of FG sphere in Ref. [5], where an analytical solution of the governing equations is presented, are assumed to be as follows

$$E = E_0 r^{n_1} \;\; ; \;\; \alpha = \alpha_0 r^{n_2} \;\; ; \;\; k = k_0 r^{n_3} \tag{25}$$

where E, E_0 are the elastic modulus of the FG sphere and base constituent, α, α_0 are the linear thermal expansion coefficient, and k, k_0 are the thermal conductivity of the FG sphere and base constituent, respectively, and the Poisson's ratio is assumed to be constant. Also, terms n_1, n_2 and n_3 are the power law indices which determines the material distributions through the geometry of FG sphere.

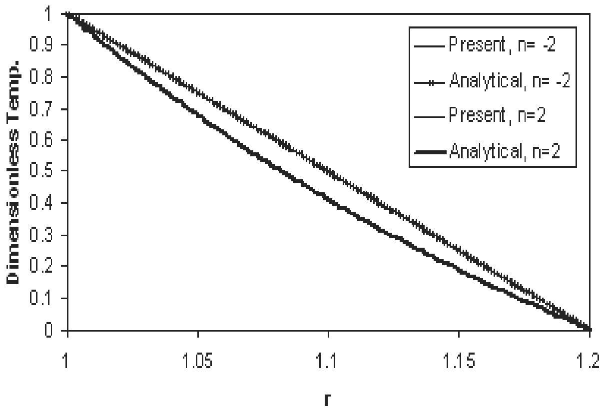

Fig. 1. Dimensionless temperature distribution for two values of power law indices

It should be noted that to obtain the other material properties of isotropic, linear thermoelastic functionally graded materials the following relations may be used

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$
$$\mu = \frac{E}{2(1+\nu)} \tag{26}$$

The thermal boundary conditions are assumed to be prescribed temperature change at the inner and outer surfaces. An FG sphere of inner and outer radii of a=1m and b=1.2m which temperature change at its inner surface is considered to be $T(a)$=10 ^0C, and the outer surface temperature is assumed to be constant is under investigation. Also, the sphere is supposed to be under internal pressure of 50 MPa while is traction free at its outer surface. The material properties at the inner radius of the sphere are assumed to be

$$E_0 = 200\,GPa \;,\; \alpha_0 = 1.2(10^{-6}) \,/\, K$$

and the power law indices are considered to be the same, i.e. $n_1 = n_2 = n_3 = n$. In the following, the temperature, displacement and stresses are normalized by the prescribed temperature at the inner surface, the inner radius of the sphere, and the internal pressure load at the inner surface of the sphere, respectively.

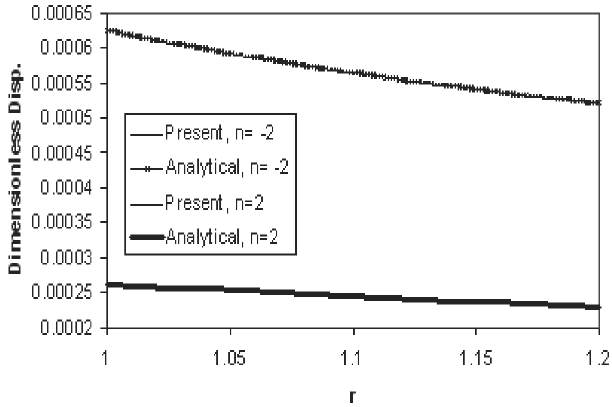

Fig. 2. Dimensionless displacement distribution for two values of power law indices

In Figs. 1 and 2, the temperature and displacement distribution results of analytical and FEM code developed in present work are compared for two values of power law indices, i.e. $n = -2$ and $n = 2$. As may be seen from the figures, the plots are well compared and a close result is obtained.

Now, consider the same FG sphere which its material properties are assumed to follow the rule of mixture (R-M) or Mori-Tanaka (M-T) schemes. In this case, the aluminum (Al) and alumina (Al_2O_3) are considered as the material constituents of FG sphere where the inner and outer surfaces of the sphere are supposed to be fully ceramic and fully metal, respectively. The thermal and mechanical properties of the metal and ceramic constituents are given as

$$Al_2O_3: \; E_1 = 380\,GPa \;;\; \alpha_1 = 7.4(10^6)/\,\mathrm{K}$$
$$k_1 = 10.4\,W/mK$$
$$Al: \; E_2 = 70\,GPa \;;\; \alpha_2 = 23(10^{-6})/\,\mathrm{K}$$
$$k_2 = 204\,W/mK \tag{27}$$

As indicated in figures, the effect of material composition on temperature, displacement and stresses is shown and the results for two schemes of estimation of material properties of FG sphere are presented. It may be seen from Fig. 3 that the M-T along with the scheme Hatta-Taya and Rosen-Hashin relations predicts larger values for temperature distribution, especially for the regions near the inner surface of the sphere, i.e. ceramic rich side. Also, it can be observed from Fig. 3 that by increasing the power law index the temperatures through the geometry of the sphere are increased. The reason is that with increase of power law index the amount of ceramic constituent increased and results in larger absorption of thermal energy.

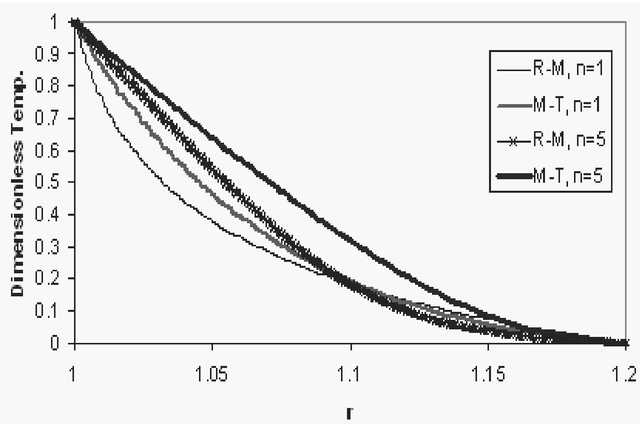

Fig. 3. Dimensionless temperature distribution for two values of power law index and R-M and M-T schemes

Figure 4 depicts that the difference between the results of M-T and R-T schemes for the displacement distribution are considerably large when the combined thermal and mechanical loads are applied to the FG sphere. However increasing the power law index leads to increase of absorption of thermal energy, it can be seen in Fig. 4 that due to increase of amount of ceramic constituent, smaller values for the displacement distribution is obtained. The reason is that the elastic modulus of FG sphere is increased when the value of power law index in increased.

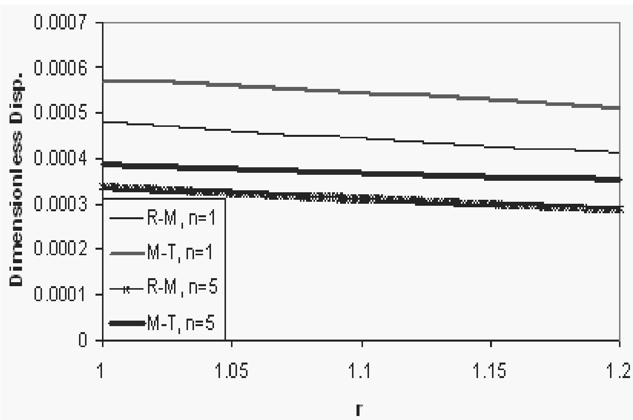

Fig. 4. Dimensionless displacement distribution for two values of power law index and R-M and M-T schemes

It can be inferred from the results shown in Fig. 5 that while some differences are detected for temperature and displacement distributions, there are only minor differences for the radial stress distribution. Also, Fig. 5 shows that with increase of power law index larger values, due to the higher values for the elastic modulus, can be obtained for the radial stress.

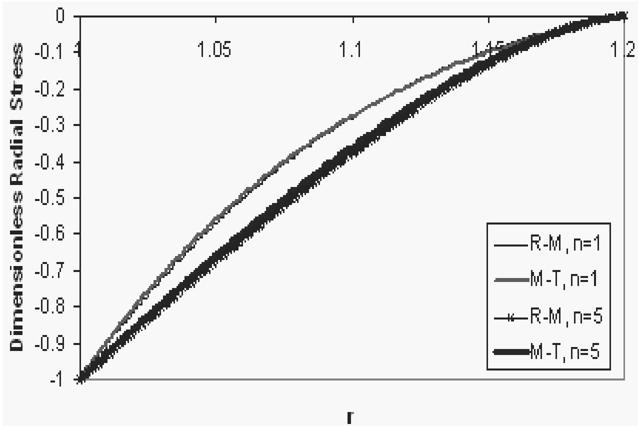

Fig. 5. Dimensionless radial stress distribution for two values of power law index and R-M and M-T schemes

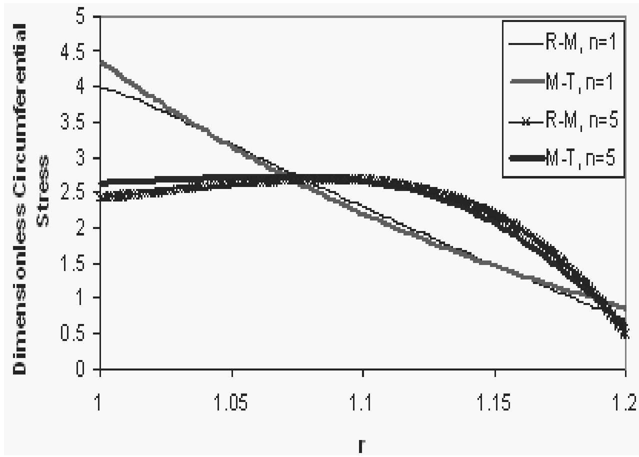

Fig. 6. Dimensionless circumferential stress distribution for two values of power law index and R-M and M-T schemes

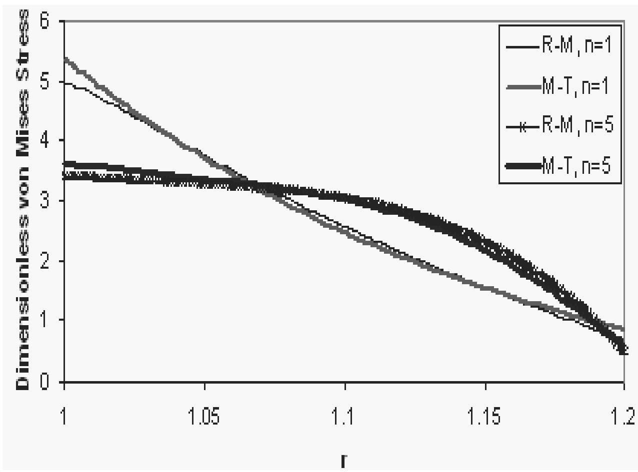

Fig. 7. Dimensionless von Mises stress distribution for two values of power law index and R-M and M-T schemes

Since the thermal field more dominates the circumferential stress than elastic field, it can be seen from Fig. 6 that the difference between the circumferential stresses predicted by two schemes is larger in the vicinity of inner surface of the sphere resulting from the larger difference of temperature distribution at the inner regions. An interesting phenomena which can be observed in Fig. 6 is that, while the larger values of power law index predict smaller values for the circumferential stress, these stresses increases with the larger values of power law index at the vicinity of outer surface of the FG sphere.

This phenomena can also be observed in Fig. 7 where the distribution of von Mises stress is plotted.

6. Conclusions

This paper presents the FEM analysis of a functionally graded thick hollow sphere which its thermal and mechanical material properties only depend on the radial position. The thermal material properties are obtained utilizing the Hatta-Taya and Rosen-Hashin relations. Also, the mechanical properties are estimated using the Mori-Tanaka scheme. In addition to the methods of approximation of material properties cited above, the rule of mixture scheme for determination of thermal and mechanical properties is also considered and results of these two schemes are compared for two cases of material composition through the geometry of FG sphere. The FG sphere is assumed to be symmetrically loaded and one-dimensional steady-state analysis of isotropic linear thermoelastic FG sphere under combined thermal and mechanical loads is investigated. Solution of the heat conduction equation and the Navier equation are obtained by using the Galerkin finite element method and by generating 100 elements along the radial direction of FG sphere. Results for the temperature, radial displacement, radial stress and hoop stress fields through the geometry of the sphere are give. The figures reveal that some minor difference may be obtained for two schemes and the difference between the results for displacement distribution is larger than difference of temperature and stress distributions.

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