An analytic model for tool trajectory error in 5-axis machining

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Analysis and modelling

ABSTRACT

Purpose: This paper proposes an analytical method of evaluating the maximum error by modeling the exact tool path when the tool traverses singular region in five-axis machining.

Design/methodology/approach: It is known that the Numerical Control (NC) data obtained from the inverse kinematic transformation can generate singular positions, which have incoherent movements on the rotary axes. Such movements cause unexpected errors and abrupt operations, resulting in scoring on the machined surface. To resolve this problem, previous methods have calculated several tool positions during a singular operation, using inverse kinematic equations to predict tool trajectory and approximate the maximum error. This type of numerical approach, configuring the tool trajectory, requires a lot of computational time to obtain a sufficient number of tool positions in the singular region. We have derived an analytical equation for the tool trajectory in the singular area by modeling the tool operation, by considering linear and nonlinear parts that are a general form of the tool trajectory in the singular area and that are suitable for all types of five-axis machine tools. In addition, evaluation of the maximum tool-path error shows high accuracy, using our analytical model.

Findings: : In this study, we have separated the linear components of the tool trajectory from the nonlinear ones, to propose a tool trajectory model that is applicable to any kind of 5-axis machine. We have also proposed a method to calculate the maximum deviation error based on the proposed tool trajectory model.

Practical implications: The algorithms proposed in this work can be used for evaluating NC data and for linearization of NC data with singularity.

Originality/value: Our algorithm can be used to modify NC data, making the operation smoother and reducing any errors within tolerance.

Keywords: CAD/CAM, 5-Axis machining, Tool trajectory error, Singular area, NC programming

1. Introduction

Five-axis milling is an efficient machining technology since its tool orientation is more flexible, with two additional rotational axes than three-axis milling, but it can has greater tool trajectory error than that with three-axis[8]. The reason for causing bigger error is that the relationship between the workpiece coordinate system for tool position and orientation, and the machine coordinate system for operating commands is nonlinear. The actual tool trajectory is not the same as the programmed trajectory, and when the tool orientation is nearly vertical to the workpiece table, the nonlinearity can increase severely and cause a fatal machining error[1]. Therefore, the effort to evaluate the tool trajectory and to minimize error is essential for good machining quality.

Many researchers have previously studied the tool trajectory error for 5-axis machining. Tool path error is an important factor for determining the step length of NC data. Hwang[5] presented

the theory that the tool path of a machine with rotational axes has more deviation than that with the circular arc suggested by Zeid[10]. Wei and Lin[9] suggested the equation for tool path error with step length of a 4-axis machine, and calculated the maximum error assuming that the maximum deviation occurs at mid value of the parametric range. Ho and coworkers[3] and Ho, M.C. and Hwang[4] suggested an algorithm for calculating the nonlinear tool path error of a 5-axis machine of the table-tilting type. However, their methods cannot be applied in the case of large rotation. Bohez and Makhanov et al.[2] proposed a method to construct parameters in a workpiece coordinate system in uniform grid, by transforming the parameters into those of a linear machine coordinate system. Their method can be applied only when the transformed grids do not overlay.

In this paper, we propose an analytical equation for tool trajectory when the tool traverses a singular area, where the unexpected deviation error increases. We also propose a method to calculate the maximum deviation error of the trajectory based on an analytical trajectory model. In the next section, we describe the singular position, which is one of the important problems in 5-axis machining. Section 3 discusses in detail the method we propose for modeling the tool trajectory with a singularity. The proposed equation for tool trajectory can be applied to any kind of 5-axis machine with a serial kinematic chain. Section 4 describes the method to calculate the maximum trajectory deviation error in the singular area, and Section 5 illustrates the proposed method.

2. Singular position in 5-axis machining

Five-axis milling has nonlinear characteristics between the workpiece and the machine coordinate systems due to the rotation axes.
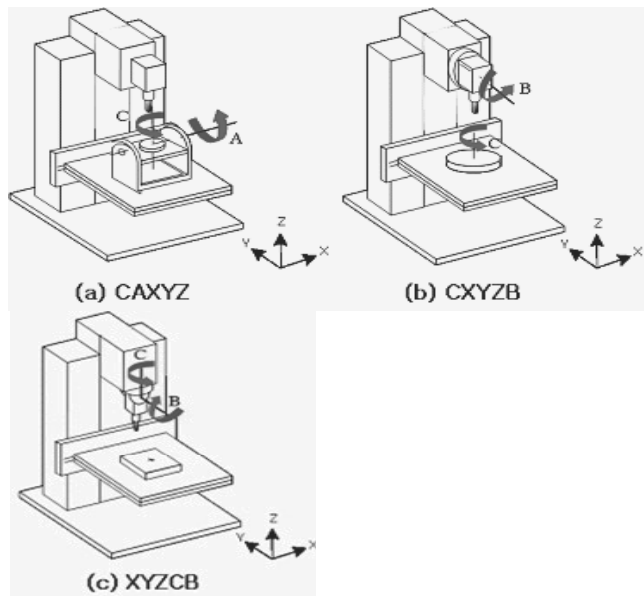

Fig. 1. 5-Axis Machine Types Including the Singular Area

This nonlinearity can increase severely when a specific type of 5-axis machines move in a specific tool orientation, especially when the tool orientation approaches the vertical to the workpiece table, which causes fatal machining error. This area of tool orientation is called 'singular area'. We represented in Fig. 1 the types of 5-axis machines that can have the singular problem. All of those types have a rotational axis about the Z-axis of the machine coordinate system. In the singular area, the prior rotational axis in the mechanical chain of a 5-axis machine makes more excessive movement than the other rotation axis. Afford[1] adopted the concept of a singular cone for the detection of singularity in 5-axis machining.

3. Tool trajectory near a singular position

In a singular area, a tool along the machine Z-axis shows excessive rotational motion than that with the other rotation axis, while the translational motion along this axis is insignificantly small. When the tool orientation approaches the vertical to the workpiece table, we represent the rotation axis about the machine Z-axis as U-axis, and the other as V-axis. The machine coordinates in an NC block are represented as eq. (1) with parameter u.

$$\begin{bmatrix} X \\ Y \\ Z \\ U \\ V \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \\ U_0 \\ V_0 \end{bmatrix} + u \begin{bmatrix} X_1 - X_0 \\ Y_1 - Y_0 \\ Z_1 - Z_0 \\ U_1 - U_0 \\ V_1 - V_0 \end{bmatrix}$$

where u : parameter $(0 \leq u \leq 1)$ (1)

Since the rotation about the U-axis is dominant to that about the V-axis when the tool traverses the singular area, the tool trajectory moves from p_i to p_{i+1} and can be represented as a curve, as shown in Fig. 2.
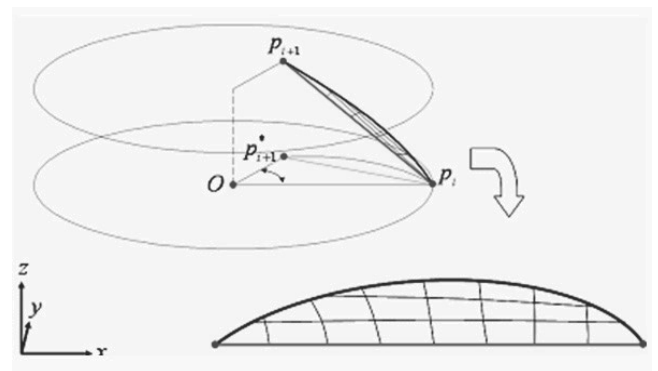

Fig. 2. Tool Trajectory Model in Singular Region

We can assume that the curve is on a plane, since the excessive rotation motion within singular region is about the Z-axis and the tool deviation from the line of p_i to p_{i+1} is mostly in the X and Y direction. In order to test the assumption, we examined the actual tool trajectory when the tool traverses the singular area, even when tool displacement in the Z direction is not small, as in Fig. 3(a). Then, we present the trajectory as viewed from p_i to p_{i+1} in Fig. 3(b). With Fig. 3(b), we can ascertain that the assumption is reasonable, since the height of deviation is small compared with the width. Since the tool displacement in the Z direction is generally small in a singular area, the height of the deviation curve would be much smaller.

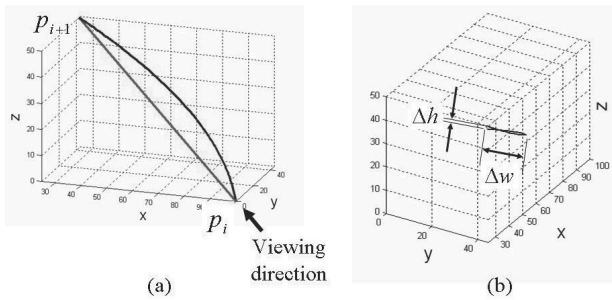(a)   (b)

Fig. 3. Projection of Tool Trajectory Curve

As a result, we can assume that the curve is on a plane, and the tool trajectory in a singular area can be modeled if we can calculate the X and Y components of the curve. The abrupt excessive rotation in the singular area is mainly due to rotation about the U-axis. Therefore, the X and Y components of the actual tool trajectory can be represented as components that are independent of the rotation about U-axis and others that are dependent on U, as in eq. (2),

$$\begin{bmatrix} x(u) \\ y(u) \end{bmatrix} = \begin{bmatrix} G_x(u) \\ G_y(u) \end{bmatrix} + Rot(U) \begin{bmatrix} M_x(u) \\ M_y(u) \end{bmatrix}$$

$$where \quad Rot(U) = \begin{bmatrix} \cos(U) & -\sin(U) \\ \sin(U) & \cos(U) \end{bmatrix}$$

(2)

where $G(u)$ are the components that are independent of rotation about the U-axis and $M(u)$ are components that are dependent on rotation about this axis. In eq. (2), $G(u)$ and $M(u)$ are functions of the rotation about the V-axis and translation in X, Y, and Z directions. Since rotation about V-axis in a singular area is insignificantly small and the tool angle in V-axis is near zero, $G(u)$ and $M(u)$ can be approximated as linear and be represented as lines in the X-Y plane. Therefore, the tool trajectory in the singular area can be considered as a curve that is decomposed into the line $\overline{G_0 G_1}$ and rotation of the line

$\overline{M_0 M_1}$, as in Fig. 4. The solid curve in Figure 4 represents the composite curve of tool trajectory in the singular area. The curve is represented in eq. (3).
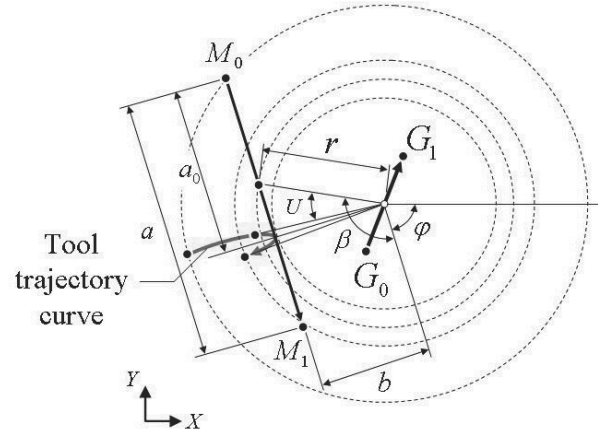

Fig. 4. Tool Trajectory Curve on X-Y Plane

$$p(u) = \begin{bmatrix} X(u) \\ Y(u) \\ Z(u) \end{bmatrix}$$

$$= \begin{bmatrix} \{G_{0x} + u(G_{1x} - G_{0x})\} + r(U)\cos(\theta) \\ \{G_{0y} + u(G_{1y} - G_{0y})\} + r(U)\sin(\theta) \\ Z_0 + u(Z_1 - Z_0) \end{bmatrix}$$

(3)

$$where \quad \theta = U + \varphi + \beta(u), \ U = U_0 + u(U_1 - U_0)$$

$$r(u) = \sqrt{(a_0 + ua)^2 + d^2}$$

$$a = \|M_1 - M_0\|, \quad a_0 = -k a$$

$$\varphi = \tan^{-1}(\frac{\Delta M_y}{\Delta M_x})$$

$$\beta(u) = -s \cdot \tan^{-1}(\frac{b}{a_0 + ua})$$

$$s = sign(\frac{M_{1y}}{M_{1x}} - \frac{M_{0y}}{M_{0x}})$$

In eq. (3), φ is the slope of line $\overline{M_0 M_1}$ with respect to the X-Y coordinate system, and β represents the angle of the line from the origin to a point on line $\overline{M_0 M_1}$ in the cylindrical coordinates system, which is in the range from zero to π. The a_0 is the distance to M_0 from the point on the line

$\overline{M_0 M_1}$ with minimum distance from the origin. The s in eq. (3) represents the directional sign of the line from M_0 to M_1 with respect to the origin of the coordinate system, which is positive when the line direction is counter-clockwise, and negative otherwise.

Equation (3) can be applied to any type of 5-axis machine for modeling tool trajectory in a singular area. The curve represented by this equation approaches the real tool trajectory curve when the tool approaches the singular position and the tool displacement in the X and Y directions is larger than that in Z direction.

The $G(u)$ plays a role in eq. (3) only when the translational axes are prior to the U-axis in the kinematic chain of the 5-axis machine. Equation (3) for the type of 5-axis machine that has no translational axes prior to the U-axis is simplified to the following eq. (4).

$$p(u) = \begin{bmatrix} X(u) \\ Y(u) \\ Z(u) \end{bmatrix} = \begin{bmatrix} r(u)\cos(\theta) \\ r(u)\sin(\theta) \\ Z_0 + u(Z_1 - Z_0) \end{bmatrix}$$

(4)

4. Calculation of maximum deviation with singularity

When an NC data commands to move the tool from position $\overrightarrow{p_0}$ to $\overrightarrow{p_1}$, the actual tool path can be a deviated curve as in Figure 5. When we define the maximum deviation error as the maximum distance that is perpendicular to $\overrightarrow{D}$ from $\overrightarrow{p_0}$ to $\overrightarrow{p_1}$, the parameter k of the point that corresponds to the maximum deviation can be obtained by eq. (5), and the maximum distance by eq. (6).

$$k = \frac{\overrightarrow{D}}{\left|\overrightarrow{D}\right|^2} \bullet \overrightarrow{P}$$

$$where \ \overrightarrow{D} = (\Delta X, \Delta Y, \Delta Z)$$

(5)

$$d = \sqrt{(\overrightarrow{P})^T (I - L)^2 \overrightarrow{P}}$$

$$where \ L = \frac{1}{\left|\overrightarrow{D}\right|^2} \begin{bmatrix} \Delta X^2 & \Delta X \Delta Y & \Delta X \Delta Z \\ \Delta Y \Delta X & \Delta Y^2 & \Delta Y \Delta Z \\ \Delta Z \Delta X & \Delta Z \Delta Y & \Delta Z^2 \end{bmatrix}$$

(6)

The parameter u_{max} of the maximum deviation point of the real tool trajectory has the same value as the parameter u_{max}^* of

the projected curve on the plane perpendicular to U-axis. The projected curve is obtained by $X(u)$ and $Y(u)$ of eq. (3), as in Fig. 5.

Therefore, if we can calculate the parameter u_{max}^* of the projected curve, we can calculate the maximum deviation error of the real tool trajectory with a singularity. The point of maximum deviation error is obtained when the slope of the projected curve is the same as the slope h of the line from $\overrightarrow{p_0}$ to $\overrightarrow{p_1^*}$.
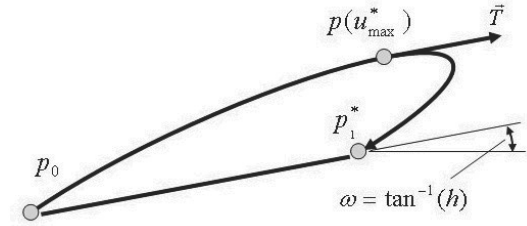

Fig. 5. The Position of Maximum Tool Trajectory Error

The slope of the tangent vector of the projected curve is calculated by partial differentiation as in eq. (7). The parameter u_{max}^* of the maximum deviation point is calculated by eq. (8).

$$p'(u) = \frac{G_y + n(u)\sin\{U + \varphi + \beta(u) + \lambda(u)\}}{G_x + n(u)\cos\{U + \varphi + \beta(u) + \lambda(u)\}}$$

$$where \ \lambda(u) = \tan^{-1}\left\{\frac{r(u)(U_1 - U_0 + \beta'(u))}{r'(u)}\right\}$$

$$n(u) = \sqrt{r'(u)^2 + r(u)^2(U_1 - U_0 + \beta'(u))^2}$$

(7)

$$f(u) = \sin(U + \varphi - \omega + \beta(u) + \lambda(u)) + \frac{G_y - h G_x}{n(u)\sqrt{h^2 + 1}}$$

$$= 0$$

(8)

However, since eq. (8) is nonlinear, the solution is difficult to calculate, but we approximated by following polynomial eq. (9).

$$f(u) = C_a u^4 + C_b u^3 + C_c u^2 + C_d u + C_e$$

$$where \ \begin{bmatrix} C_a \\ C_b \\ C_c \\ C_d \\ C_e \end{bmatrix} = \begin{bmatrix} u_0^4 & u_0^3 & u_0^2 & u_0 & 1 \\ u_1^4 & u_1^3 & u_1^2 & u_1 & 1 \\ u_2^4 & u_2^3 & u_2^2 & u_2 & 1 \\ u_3^4 & u_3^3 & u_3^2 & u_3 & 1 \\ u_4^4 & u_4^3 & u_4^2 & u_4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} f(u_0) \\ f(u_1) \\ f(u_2) \\ f(u_3) \\ f(u_4) \end{bmatrix}$$

(9)

Since the exactness of the approximated eq. (9) can be affected by the positions of the sampling parametric points

$u_0, u_1, u_2, u_3,$ and u_4, the sampling parameters are determined considering the characteristics of the actual tool trajectory curve as follows. Fig. 6 shows three cases of tool trajectories, indicating that the parameter u^*_{max} of the maximum deviation point is about 0.5 and is on the opposite side of parameter k from 0.5. k is the apex parameter of the actual tool trajectory, as calculated by eq. (5). For example, when the value k is bigger than 0.5, as in Fig. 6(b), u^*_{max} has a value smaller than 0.5.
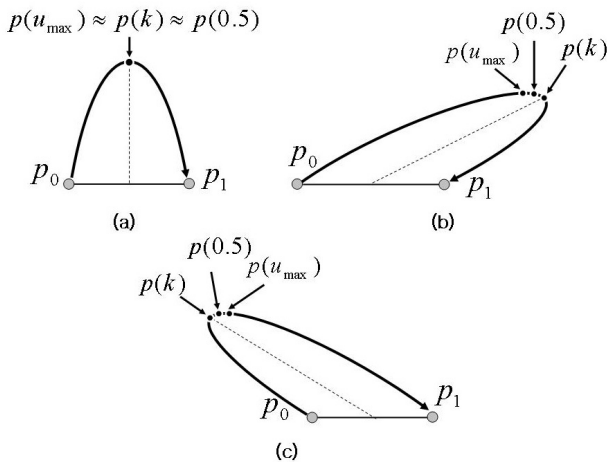

Fig. 6. Three Cases of the Maximum Trajectory Error Position

Considering these characteristics, the sampling parametric points described in Table 1 are used for the polynomial equation (9). Therefore, the parameter u^*_{max} of the maximum deviation position can be calculated by the formulas of roots for the eq. (9), which is the real solution in the valid range. Consequently, the maximum deviation error d can be calculated by eq. (6) with the parameter u^*_{max} that is the same as u_{max}.

Table 1.
Sampling parameters

CASE	$k \approx 0.5$	$k \prec 0.5$	$0.5 \prec k$
para-meters	$u^*_{max} = 0.5$	$u_0 = 0.0$	$u_0 = 0.0$
		$u_1 = 0.5 - 2h$	$u_1 = 0.5$
		$u_2 = 0.5 - h$	$u_2 = 0.5 + h$
		$u_3 = 0.5$	$u_3 = 0.5 + 2h$
		$u_4 = 1.0$	$u_4 = 1.0$
		where $h = |0.5 - k|$	

5. Application and results

As mentioned, 5-axis machining with singularity causes incoherent movements of the rotary axes, which lead to unexpected errors and abrupt operations, resulting in scoring on the machined surface. Therefore, even though the NC data for machining an impeller look smooth as in Fig. 7(a), the actual machining can generate unexpected poor results, as in Fig. 7(b). We have represented the tool trajectory with the singularity as real lines in Fig. 8 by applying the forward kinematic equation, whereas the analytic tool trajectory, indicated as dotted lines in Fig. 8, was obtained by applying the proposed eq. (3) of this study. The result shows that the trajectory generated by our analytical equation is almost the same as the real tool trajectory.
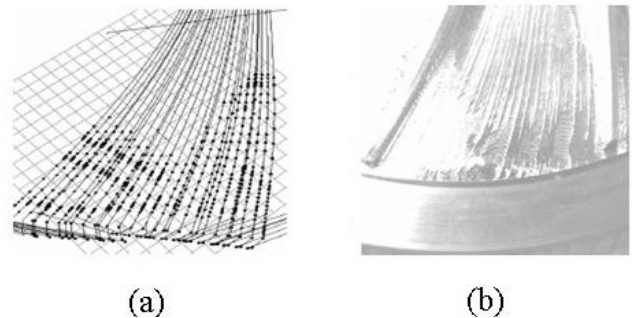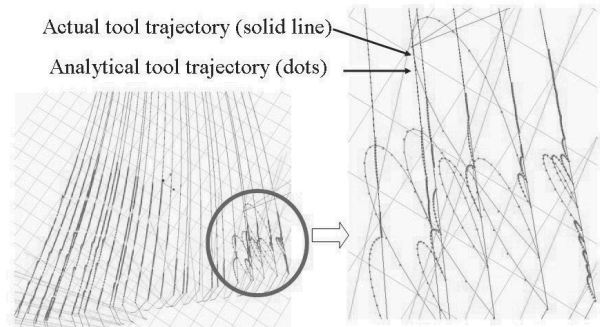

Fig. 7. NC Data and Machining Result with Scoring

Fig. 8. Actual and Analytic Tool Trajectory in Singular Area

Table 2.
One step of Tool Trajectory

	CL data in workpiece coordinate system	NC data in machine Coordinate system
p_0	$x_0 = 75.2110$	$X_0 = -90.9595$
	$y_0 = -51.4295$	$Y_0 = 7.3103$
	$z_0 = 228.6808$	$Z_0 = 227.6341$
	$i_0 = 0.0095944$	$A_0 = 1.0664$
	$j_0 = 0.0159479$	$C_0 = 148.9685$
	$k_0 = 0.9998268$	
p_1	$x_0 = 74.5130$	$X_0 = -82.0656$
	$y_0 = -50.5510$	$Y_0 = -34.8175$
	$z_0 = 228.6857$	$Z_0 = 228.3975$
	$i_0 = 0.0175393$	$A_0 = 1.1793$
	$j_0 = 0.0107682$	$C_0 = 121.5477$
	$k_0 = 0.9997882$	

In order to verify the proposed method for calculating the maximum deviation error with the singularity, we show one step of the tool trajectory for the impeller (Table 2).

Table 2 shows cutter location data for the step in the workpiece coordinate system and the NC data for the Swiss made MIKRON UCP-710 machine in machine coordinate system. For the step, we represent in Fig. 9 the actual trajectory with solid lines and the analytical trajectory from eq. (3) with dots. The maximum deviation error in the actual tool trajectory is 0.9383 mm when the parameter u_{max} is 0.4101, while the calculated maximum error is 0.9382 mm, which means 99.99% accuracy.
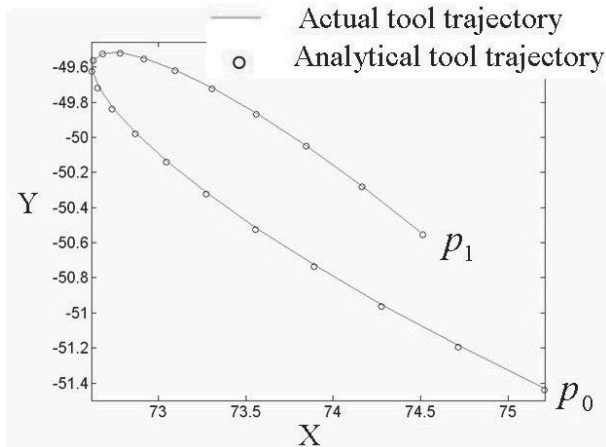

Fig. 9. Maximum Deviation Error of Tool Trajectory

6. Conclusions

Five-axis machining has nonlinear characteristics between the workpiece and the machine coordinate systems. Even when NC data give a command of a linear operation, the actual tool trajectory can be a curved path. Since this nonlinearity can increase severely when the tool traverses a singular region, the actual tool trajectory in the singular area has to be checked before machining. In this study, we have separated the linear components of the tool trajectory from the nonlinear ones, to propose a tool trajectory model that is applicable to any kind of 5-axis machine. We have also proposed a method to calculate the maximum deviation error based on the proposed tool trajectory model. The algorithms proposed in this work can be used for evaluating NC data and for linearization of NC data with singularity.

Acknowledgements

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