

The analysis of change with configural frequency analysis using different base models

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Summary

Types or antitypes are based on significant deviations from the null hypothesis. Therefore, the meaning of types or antitypes depends on how the null hypothesis is specified. Here, three different base models for the analysis of change using configural frequency analysis (CFA) are presented, two traditional models and one new model: (1) directed configural frequency analysis (DCFA), (2) prediction configural frequency analysis (PCFA), and configural frequency analysis of change (Change-CFA). The new model is the Change-CFA, which is based on the concept of marginal homogeneity in contingency tables; the method uses a probability model that considers equal pre-treatment and post-treatment marginals. Types or antitypes are interpreted as shifts from pre- to posttest. All methods are applied to a dataset from psychopharmacological treatment; the paper supports the notion of flexibility of CFA in testing different hypotheses.

Key words: Configural Frequency Analysis, pre-post treatment designs, contingency table analysis, prediction table, null hypothesis, null models, analysis of change

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1. Introduction

Configural frequency analysis (CFA) is a non-parametric tool for the analysis of d -dimensional contingency tables (von Eye, 2002). The method was proposed in 1968 by G. A. Lienert, and introduced to the English audience seven years later (Lienert and Krauth, 1975). In CFA, the cells of a contingency table, called configurations, are analyzed by comparing expected frequencies to observed frequencies. The binomial test or Pearson's chi-square are the commonly used test statistics to compare the expected with the observed frequencies (von Eye, 1990). Expected frequencies may be based on many hypothetical models (Mellenbergh, 1996) and are usually estimated using log-linear models, often a main effect model (von Eye and Nesselroade, 1992). Whereas log-linear models look for a match between model and the data, i.e., a match between the expected and the observed frequencies, CFA searches for differences. Significant differences between observed (f_o) and expected frequencies (f_e) are termed "types", if there are more observed than expected cases (i.e., $f_o > f_e$) and "antitypes", if there are fewer observed than expected cases (i.e., $f_o < f_e$). Since its introduction to the scientific community, CFA developed to be a universally useful statistical tool for the analysis of contingency tables. This paper introduces a new version of CFA for the analysis of pre-post treatment effects.

A data example is used in which in a pre-post treatment design, a sample of 65 volunteers were given the drug LSD (i.e., lysergic acid diethylamide; LSD-25); assessed were two variables, speed and accuracy of solving simple mental arithmetic tasks. The data were derived from the well-known Lienert-LSD experiment (see Lienert, 1970; von Eye, 1990, Lautsch & von Weber, 1994). In this experiment, a sample of $N = 65$ students was tested twice: before and after administration of LSD; the medication and second testing were administered one week after the first. Among other psychometric tests the students were given a concentration performance test (the "Konzentrationsleistungstest" KLT; translated: concentration performance test) of Düker and Lienert (1959). The KLT consists of a number of simple arithmetic tasks; the subject has to solve as many arithmetic tasks as possible within a given time. Two variables were assessed; X represents "speed" and was measured as the total number of given answers; Y represents "number of incorrect answers". The post-treatment variables are called U and Z , that is, speed " U " and number of incorrect answers " Z ". Speed and number of incorrect answers at Time 1 (X and Y) are used as predictors of speed and number of incorrect answers at Time 2 (U and Z).

The new version of CFA is compared to two well established versions of CFAs, the directed configural frequency analysis (DCFA) based on von Eye (1985) and the predictive configural frequency analysis (PCFA) proposed by Heilmann, Lienert, and Maly (1979; see also Netter, 1996):

1) DCFA calculates expected frequencies according to the following log-linear model:

$$\log(y) = u_0 + u_{\text{predictors}} \quad (1.0)$$

or, in the present example,

$$\log(y) = u_0 + u_x + u_y + u_{xy} \quad (1.1)$$

In the case of a 4 by 4 table, the expected frequencies of DCFA can also be calculated as $e_{ij} = f_{i.} / 4$. The base model of the directed CFA proposes that:

- there are two groups of variables, predictors and criteria. These groups are independent of each other;
- in the group of independent variables, predictors, interactions of any order may prevail;
- in the group of dependent variables, criteria, main effects and interactions do not exist.

2) PCFA calculates expected frequencies according to the following log-linear model:

$$\log (y) = u_0 + u_{\text{predictors}} + u_{\text{criteria}} \tag{2.0}$$

or, in the present example

$$\log (y) = u_0 + u_x + u_y + u_u + u_z + u_{xy} + u_{uz} \tag{2.1}$$

If all predictor patterns constitute the rows of a 2-dimensional table, and all criterion patterns constitute the columns, the expected frequencies of PCFA can also be calculated as $e_{ij} = f_{i.} f_{.j} / N$. The base model of prediction CFA proposes that:

- predictor variables are independent of criterion variables;
- within the predictors and within the criteria, interactions of any order may prevail.

3) These two models are compared to what is here called change-CFA based on marginal homogeneity. This base model assumes marginal homogeneity between pre- and posttest. The model cannot be expressed in terms of a log-linear model because it assumes that the pre- and the post-treatment marginals are equal (Wickens, 1989). The expected frequencies can be calculated as $e_{ij} = f_{i.} f_{.i} / N$. The term $f_{.i}$ postulates the same frequencies for the columns or post-treatment marginals as for the rows or pre-treatment marginals. Therefore, deviations from this expected model can be interpreted as shifts from pre- to post-treatment; those shifts describe the change caused by the treatment.

2. Bivariate Pre-Post-Treatment Experiment

The four original continuous variables were dichotomized at the medians of the pre-treatment variables. “+” denotes a score above the median and “-” denotes a score at or below the median. The resulting 4 x 4 table can be taken from Table 1.

Table 1:
Frequencies for total number of answers and incorrect answers above and below the respective medians

	U + Z +	U + Z -	U - Z +	U - Z -	Σ
X + Y +	4	5	8	4	21
X + Y -	2	4	1	4	11
X - Y +	0	0	4	5	9
X - Y -	0	0	7	17	24
Σ	6	9	20	30	N = 65

For the null hypothesis of $H_0: f_{ij} = f_{ji}$, the Bowker Test (1948) may be applied ($i = \text{rows}$, $j = \text{columns}$, $k = \text{number of rows or columns}$):

$$\chi^2 = \sum \sum \frac{(f_{ij} - f_{ji})^2}{f_{ij} + f_{ji}}, i > j \quad df = \binom{k}{2}$$

resulting in the following chi-square:

$$\chi^2 = \frac{(2-5)^2}{(2+5)} + \frac{(0-8)^2}{(0+8)} + \frac{(0-4)^2}{(0+4)} + \frac{(0-1)^2}{(0+1)} + \frac{(0-4)^2}{(0+4)} + \frac{(7-5)^2}{(7+5)} = 18.5$$

which is significant for $df = 6$ ($\chi^2_{\alpha=0.05; df=6} = 12.59$) indicating that the pre-treatment results were better than the post-treatment results. Unfortunately, the Bowker Test does not indicate which of the patterns changed significantly.

3. Directed CFA

If we specify the base model according to DCFA (von Eye, 1985) the expected frequencies are calculated through $e_{ij} = f_i/4$ (see Table 2). The null hypothesis according to the DCFA model assumes that the post-treatment scores were uncorrelated and independent of the pre-treatment scores (i.e., no interaction effects under the H_0).

Table 2:
Directed CFA: Listed are the expected frequencies based on $e_{ij} = f_i/4$ and the observed frequencies from Table 1

	U + Z +	U + Z -	U - Z +	U - Z -	Σ
X + Y +	5.25 (4)	5.25 (5)	5.25 (8)	5.25 (4)	21
X + Y -	2.75 (2)	2.75 (4)	2.75 (1)	2.75 (4)	11
X - Y +	2.25 (0)	2.25 (0)	2.25 (4)	2.25 (5)	9
X - Y -	6.00 (0)	6.00 (0)	6.00 (7)	6.00 (17)*	24
	6	9	20	30	N = 65

Note. The observed frequencies are in parentheses; * = significant type or antitype.

This prediction model assumes that there are only pre-treatment effects. The pattern $f_{x-y/u-z} = 17$ is tested for significance through

$$\chi^2 = \frac{(f - e)^2}{e} = \frac{(17 - 6.00)^2}{6.0} = 20.17$$

This configuration says that there are 17 students with a stable pattern of slow but accurate work who are unaffected by the treatment. Another pattern $f_{x-y+/u-z} = 5$ seems to be also significant:

$$\chi^2 = \frac{(f - e)^2}{e} = \frac{(5 - 2.25)^2}{2.25} = 3.36$$

Because we perform $R = 16$ possible simultaneous tests, the α -level of 5% must be adjusted; applying the Bonferroni-adjustment ($\alpha^* = \alpha/16 = 0.003125$) results in a critical chi-square limit of $\chi^2_{(0.05/16)} = 8.76$ with $df = 1$ (see also Dunkl, Ludwig, and Lotz, 1990). The value of 3.36 does not exceed the adjusted Bonferroni limit. Other seemingly significant patterns such as $f_{x-y-/u+z} = 0$ and $f_{x-y-/u+z} = 0$ also do not reach the level of significance ($\chi^2 = 6.00$).

4. Prediction CFA

The base model is specified such that the expected frequencies are calculated according to $e_{ij} = f_i \cdot f_j / N$ (see Table 3). This model assumes that the predictor variables are totally independent of the criterion variables, that the predictors interact with each other, and that the criteria interact with each other.

Table 3:

Prediction CFA: Listed are the expected frequencies based on $e_{ij} = f_i \cdot f_j / N$ and the observed frequencies from Table 1

	U + Z +	U + Z -	U - Z +	U - Z -	Σ
X + Y +	1.94 (4)	2.91 (5)	6.46 (8)	9.69 (4)	21
X + Y -	1.02 (2)	1.52 (4)	3.39 (1)	5.08 (4)	11
X - Y +	0.83 (0)	1.25 (1)	2.77 (4)	4.15 (5)	9
X - Y -	2.22 (0)	3.32 (1)	7.38 (7)	11.09 (17)	24
Σ	6	9	20	30	N = 65

Note. The observed frequencies are in parentheses.

Because of the adjusted Bonferroni limit of $\chi^2_{(0.05/16)} = 8.76$, even the numerically largest observation of $f = 17$ in Table 3 does not deviate significantly from its expected frequency $e = 24 \cdot 30 / 65 = 11.07$; $\chi^2 = (17 - 11.07)^2 / 11.07 = 3.18$. Thus, no prediction type was detected using Prediction-CFA according to Heilmann, Lienert, and Maly (1979). This test is comparable to a test of canonical contingency for pre- and post-treatment patterns.

5. Change CFA based on marginal homogeneity

If we define our base model such that the pre-treatment scores display the same marginals as the post-treatment scores, any deviation from the null hypothesis may be interpreted as shifts or shift-types (see Table 4).

Table 4:

Change CFA based on marginal homogeneity: Listed are the expected frequencies based on $e_{ij} = f_i \cdot f_j / N$ and the observed frequencies from Table 1

	U + Z +	U + Z -	U - Z +	U - Z -	Σ
X + Y +	6.78 (4)	3.55 (5)	2.91 (8)*	7.75 (4)	21
X + Y -	3.55 (2)	1.86 (4)	1.52 (1)	4.06 (4)	11
X - Y +	2.91 (0)	1.52 (0)	1.25 (4)	3.32 (5)	9
X - Y -	7.75 (0)	4.06 (0)	3.32 (7)	8.86 (17)	24
Σ	21	11	9	24	N = 65

Note. The observed frequencies are in parentheses; * = significant type or antitype.

For example, the pattern $f_{X-Y/U-Z-} = 17$ results in the following chi-square = $(17 - 8.86)^2 / 8.86 = 7.48$ with $df = 1$ which misses the Bonferroni adjusted chi-square of 8.74 for $R = 16$ simultaneous tests. This pattern represents a configuration of students who work slowly and accurately and who are not affected by the application of LSD-25. A significant shift-type can be found in $f_{X+Y/U-Z+} = 8$. It suggests that there are significantly more students than expected

who shift from working quickly and inaccurately to working slowly and accurately ($\chi^2 = 8.90$). No student shifted from working slowly and accurately to working quickly and inaccurately as part of the treatment, $f_{x-y/1+2+} = 0$, the chi-square = 7.75 barely missed the significance level.

6. Discussion

Types and antitypes indicate significant deviations from the null hypothesis; therefore, the kind of null hypothesis determines the meaning of the investigated types or antitypes (Mellenbergh, 1996). The base model of the directed configural frequency analysis (DCFA) postulates that the pretest scores have no influence on the post-treatment scores, saying that the treatment has no effect. Significant types or antitypes evolve if the treatment has an effect. The treatment effect may be due to changes that directly dependent on the posttest scores (main effect of U or Z: all subjects change in the same way) or the treatment effect may be due to the interaction between the pre- and posttest measures. DCFA can best be applied in developmental research or as part of within-subjects designs, where one is interested in change due to the pretest measures.

The base model of the prediction CFA (PCFA; von Eye, 2002) postulates that the treatment has no interaction effect. From pretest to posttest assessment, all subjects change in the same way. Significant types or antitypes evolve, if there is an interaction between the pretest variables and the posttest variables; types in PCFA require an interaction treatment effect. PCFA is most useful if such an interaction effect is expected or desired. PCFA can be applied not only in pharmacological research but also in intervention research or clinical psychology in general.

The change-CFA is a special application of the DCFA (von Eye, 1985; von Eye, 1990); it assumes that under the null hypothesis of no treatment effects the resulting $r \times r$ table should be homogeneous with regard to the row frequencies or axially symmetric. The expected frequencies are calculated from the main effects of the pre-treatment variables. Significant configurations are called shift or non-shift types. For Change-CFA and DCFA it is valid to say, that any kind of variation such as individual or spontaneous change, main effect change (all subjects change in the same way) or change due to interaction effects is possible.

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