Effectiveness of acoustic power dissipation in lossy layers

Daniel Razansky[a)]
Department of Biomedical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel

Pinchas D. Einziger
Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel

Dan R. Adam
Department of Biomedical Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel

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The effect of losses in the dissipative object becomes crucial when maximal power absorption of the incident wave is of top priority. In order to assess the phenomenon of acoustic power absorption in finite size dissipative medium, a prototype model of linear pressure waves absorption in dissipative layer is considered. The conditions, parameters and bounds for the optimal (maximal) incident power absorption within the layer have been found analytically and explicitly versus its normalized thickness. These conditions are presented in terms of the basic wave propagation parameters, namely sound velocity and attenuation constant. It is shown that, for thin layers (in terms of acoustic wavelength), the upper bound on the absorptivity tends to the value of 50%, when prescribed resonant dispersion/absorption conditions, characterized by the so-called Kramers–Kronig relations, are met within the layer. For sufficiently thick layers absorption of close to 100% of the incident wave power can be achieved, when specific optimal values are selected for the corresponding real and imaginary parts of dissipative layer wave number. The model may serve as a canonical prototype problem for engineered dissipative materials design and optimization of the sound/ultrasound absorption in lossy targets, e.g., biological tissues. © *2004 Acoustical Society of America.* [DOI: 10.1121/1.1756671]

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I. INTRODUCTION

Transmission and reflection of plane longitudinal pressure waves in a stratified nondissipative fluid or gaseous media is a well-known phenomenon, e.g., Refs. 1–4. The principle of resonant layered structures is also widely used in electromagnetics and optics, e.g., Fabry–Perot interferometers and filters,[5] though the losses in the layers have usually much less consideration. The effect of losses in the dissipative object becomes crucial when maximal absorption of the incident wave power is of top priority, e.g., when designing optimal acoustic or elastic wave absorbers,[6,7] noise insulators,[8] or optimizing hyperthermia-based ultrasonic treatments.[9,10]

Various effects of acoustic wave transmission through finite-thickness layers were considered by many authors. Some of the related studies are discussed below. Various configurations of acoustic waves interaction with layered media are described by Brekhovskikh,[1,2] where no special attention is drawn to lossy media. Gramotnev and coworkers[11–13] investigated an anomalous absorption of acoustic and electromagnetic waves by ultra-thin layers of complex media (viscous fluids in the case of acoustic waves). Their studies have an impact on the interaction of longitudinal and shear elastic waves with dissipative fluid surface. Prosperetti[14] and others[15,16] studied extensively linear and nonlinear effects of bubbly liquid layers on the propagation of acoustic waves. It was found that, when gas bubbles are added to the liquid (even lossless liquid), the mixture may give a substantial rise to absorptivity and/or reflectivity of the medium by changing its basic wave propagation parameters, namely, acoustic velocity and dissipation constant. Recently, the optimization of ultrasonic power absorption has become of increased interest due to an extensive research in the area of therapeutic ultrasound, where several studies propose the use of free microbubbles[10] or ultrasound contrast agents[17] for the improvement of biological media absorptivity, again, by actually synthesizing the medium inside the region of interest in terms of its acoustic velocity and dissipation constant.

The main intention here was to obtain a closed-form results and analytical bounds for a general optimal dissipative layer in terms of the mentioned above basic linear wave propagation parameters, independently of the specific nature of the dissipative medium. Herein, the focus is on the basic prototype model of linear plane acoustic wave absorption in a dissipative finite thickness layer. The optimal dissipative layer parameters (maximal incident power absorption, optimal acoustic velocity and dissipation constant) are found analytically via closed-form explicit expressions and expressed versus layer thickness (normalized to the wavelength of the incident acoustic wave). Finally, the asymptotic bounds on layer absorptivity are derived in the limits of thick and thin layers.

II. FORMULATION

A three layers simplified model is considered. This model is used herein for the absorptivity optimization proce-

[a)]Electronic mail: danir@tx.technion.ac.il

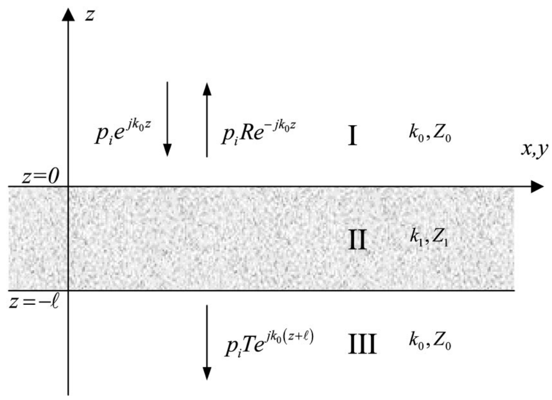

FIG. 1. Physical configuration of linear pressure wave impingement upon the dissipative layer.

dure outlined below. It consists of a spatially-infinite dissipative acoustic layer (Fig. 1) of thickness l characterized by complex specific acoustic impedance $Z_1=\sqrt{\rho_1/K_1}$ and the corresponding complex acoustic wave number $k_1=\omega\sqrt{\rho_1 K_1}$, where ρ_1 and K_1 are the medium density and its complex compressibility, respectively, while it is assumed that $\Im\{k_1\}\leqslant 0$ and $\Im\{Z_1\}\geqslant 0$. The dissipative layer is surrounded by nondissipative media with appropriate constants $Z_0=\sqrt{\rho_0/K_0}$ and $k_0=\omega\sqrt{\rho_0 K_0}$. An incident plane longitudinal acoustic wave, having a pressure amplitude of p_i and harmonic time dependence $e^{j\omega t}$, propagates in the $-z$ direction through layer I and impinges normally upon a boundary between layer I and layer II.

For the above stated problem, the fraction of incident power absorbed in the dissipative layer, namely, the absorption efficiency, is given as

$$\eta_{\mathrm{abs}}=1-|R|^2-|T|^2, \tag{1}$$

where R and T denote global layer reflection and transmission coefficients, respectively, as depicted in Fig. 1. The solution procedures for obtaining these coefficients are well known[1–3,5] and the resulting expressions, for the present purposes, are

$$R=\frac{\frac{1-Z}{1+Z}\left(1-e^{-j2\delta Z}\right)}{1-\left(\frac{1-Z}{1+Z}\right)^2 e^{-j2\delta Z}} \tag{2}$$

and

$$T=\frac{\left[1-\left(\frac{1-Z}{1+Z}\right)^2\right]e^{-j\delta Z}}{1-\left(\frac{1-Z}{1+Z}\right)^2 e^{-j2\delta Z}}, \tag{3}$$

where $Z=u-jv$ and δ, denoting the normalized complex impedance ratio and the normalized layer thickness, are given via

$$Z=Z_0/Z_1=k_1\rho_0/(k_0\rho_1)=u-jv \tag{4}$$

and

$$\delta=k_0 l\rho_1/\rho_0, \tag{5}$$

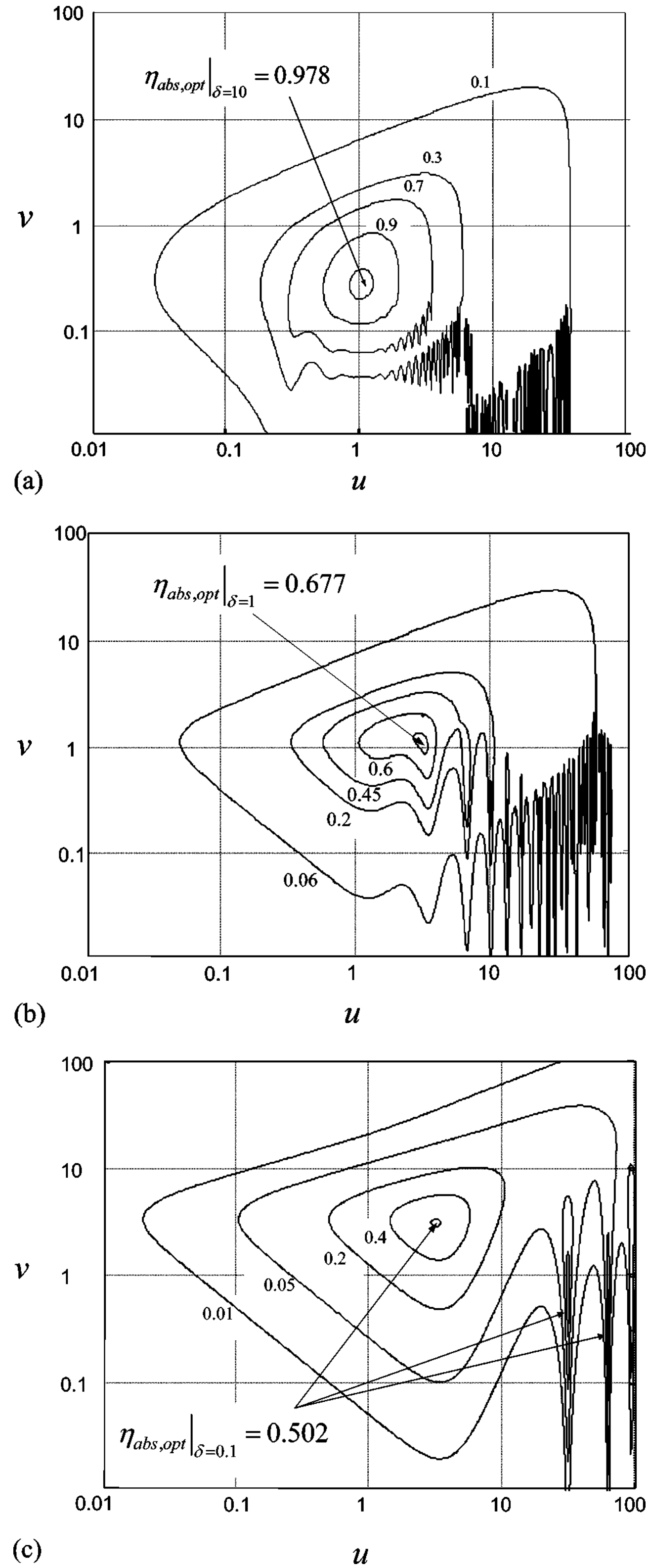

FIG. 2. Equaefficiency contours of η_{abs} in the $u-v$ plane, calculated via (1)–(5), for three different values of δ: (a) $\delta=10$; (b) $\delta=1$; (c) $\delta=0.1$.

respectively.

Note that the speed of sound c_1 in the dissipative layer is now determined (for low losses), via (4), by $\Re\{k_1\}=\omega/c_1=k_0 u\rho_1/\rho_0$, whereas $\Im\{k_1\}=-k_0 v\rho_1/\rho_0$ is the attenuation coefficient of the layer, directly related to the penetration depth $\Delta_1=-1/\Im\{k_1\}$. Thus, when the wave penetration ratio l/Δ_1, given via

$$l/\Delta_1 = -\Im\{k_1 l\} = v\,\delta, \qquad (6)$$

is sufficiently large, the dissipative layer behavior approaches that of a semi-infinite layer.

III. OPTIMIZATION PROCEDURE

It should be noted that when Z is pure real (i.e., $v=0$), for any u and δ, the power of the incident wave will not be absorbed in the intermediate layer (layer II), since it becomes nondissipative, namely $\eta_{\rm abs}=0$. On the other hand, for $v\to\infty$ the incident acoustic wave will be totally reflected at $z=0$ boundary, since $|Z_1|\to\infty$, thus, again, $\eta_{\rm abs}=0$. Between these two extreme cases one may expect that, for any given u and δ, there exists at least one optimal value $v_{\max}$, which maximizes the power absorption efficiency $\eta_{\rm abs}$. In a more general way, two-dimensional optimization must be performed on $\eta_{\rm abs}$ in terms of both u and v, i.e., finding the maximal values of $\eta_{\rm abs}$ for any given δ. In other words, one seeks for $\eta_{\rm abs,opt}$, $u_{\rm opt}$, and $v_{\rm opt}$, depending on δ. The existence of the predicted maximal efficiencies can be readily verified via Fig. 2, where the two-dimensional equaefficiency contours of $\eta_{\rm abs}$ in the $u-v$ plane are calculated via (1)–(5) for three different values of δ.

While precise optima values could be obtained via the contour maps, as depicted in Fig. 2, an efficient analytical optimization scheme can be facilitated by expressing $\eta_{\rm abs}$ in terms of two independent complex variables, namely, $Z=u-jv$ and its complex conjugate $Z^*=u+jv$, leading to

$$\eta_{\rm abs}=1-\frac{\sin(\delta Z)\sin(\delta Z^*)+\dfrac{4ZZ^*}{(1-Z^2)(1-Z^{*2})}}{\left[\dfrac{1+Z^2}{1-Z^2}\sin(\delta Z)-j\,\dfrac{2Z}{1-Z^2}\cos(\delta Z)\right]\left[\dfrac{1+Z^{*2}}{1-Z^{*2}}\sin(\delta Z^*)-j\,\dfrac{2Z^*}{1-Z^{*2}}\cos(\delta Z^*)\right]}. \qquad (7)$$

The extremum values of $\eta_{\rm abs}=\eta_{\rm abs,opt}$ are obtained at the set of points $Z=Z_{\rm opt}$ and $Z^*=Z^*_{\rm opt}$, which are solutions of

$$\partial\eta_{\rm abs}/\partial Z=0, \quad \partial\eta_{abs}/\partial Z^*=0. \qquad (8)$$

Due to symmetry of (7) with respect to Z and Z^*, it can be readily verified that $(Z_{\rm opt})^*=Z^*_{\rm opt}$ and $(\partial\eta_{\rm abs}/\partial Z)^*=\partial\eta_{\rm abs}/\partial Z^*$. Hence, Eq. (8) can equivalently be represented, at $Z=Z_{\rm opt}$, as

$$\Re\{d\eta_{\rm abs}/dZ\}=0, \quad \Im\{d\eta_{\rm abs}/dZ\}=0, \qquad (9)$$

where $Z^*_{\rm opt}=(Z_{\rm opt})^*$ is taken as a parameter. Taking the derivative $d\eta_{\rm abs}/dZ=0$ at $Z=Z_{\rm opt}$, via (7), i.e.,

$$\left[j2\sin(\delta Z_{\rm opt})\sin(\delta Z^*_{\rm opt})-\frac{4\delta Z_{\rm opt}Z^*_{\rm opt}}{1-Z^{*2}_{\rm opt}}\right]\times\left[\frac{1+Z^2_{\rm opt}}{1-Z^2_{\rm opt}}\cos(\delta Z_{\rm opt})+j\,\frac{2Z_{\rm opt}}{1-Z^2_{\rm opt}}\sin(\delta Z_{\rm opt})\right]+\frac{4Z^*_{opt}}{1-Z^{*2}_{opt}}\sin(\delta Z_{\rm opt})-j\,\delta 2Z_{\rm opt}\sin(\delta Z^*_{\rm opt})=0, \qquad (10)$$

leads to an implicit representation of $Z_{\rm opt}$ dependence on the normalized layer thickness δ.

IV. EXTREME CASES

A. Thick layer approximation: $\delta\gg 1$

For obtaining maximal power absorption efficiency in case of a thick dissipative layer it would be reasonable to set the real part of Z close to unity, i.e., $Z\to 1-jv$, in order to obtain minimal wave reflection at $z=0$. Then, the imaginary part v must be set to the values that will cause most of the wave to be attenuated while passing through the layer. Thus, v must meet a compromise—it should be large enough in order to absorb the great majority of incident wave in the dissipative layer, however, it must be also much less than unity otherwise the absolute value of the total reflection coefficient at $z=0$ will be affected. One would expect that, given these conditions, the maximal absorption efficiency would be close to unity, i.e., $\eta_{\rm abs,opt}\approx 1$ ($\delta\gg 1$). Indeed, this intuition is justified via Fig. 2(a), where the contours of power absorption efficiency are depicted for a relatively thick layer ($\delta=10$). The numerical evaluation of Eq. (7) readily shows that maximal efficiency of $\eta_{\rm abs,opt}|_{\delta=10}=0.978$ is obtained for $u_{\rm opt}=1.026$ and $v_{\rm opt}=0.256$.

For large values of u and small values of v an oscillatory behavior of $\eta_{\rm abs}$ can be observed. This is due to the fact that, in this region, the exponential terms in Eq. (7), which are proportional to v, approach the unit value, while the arguments of the sinusoidal functions are large (proportionally to u). Thus, it may be expected that the whole expression would be very sensitive to changes in u.

Motivated by the above discussion, one should seek for solution of the form $Z_{\rm opt}=1+\epsilon_{\rm opt}$, where $|\epsilon_{\rm opt}|\ll 1$ is a solution of (10). Accounting for the leading terms of the highest exponential and algebraic growth [i.e., the terms in the first square brackets in (10)], one obtains $\epsilon_{\rm opt}=-jv_{\rm opt}$ and an implicit thick layer approximation for (10), i.e.,

$$v_{\rm opt}e^{2v_{\rm opt}\delta}\sim 4\delta. \qquad (11)$$

Upon converting (11) into the following iterative expression:

$$v^{(p)}_{\rm opt}\sim\frac{\ln(4\delta/v^{(p-1)}_{\rm opt})}{2\delta}, \qquad (12)$$

the solution of (11) can be expressed, after two iterations, as

$$v_{\rm opt}^{(0)}=1,\quad v_{\rm opt}^{(1)}=\frac{\ln(4\delta)}{2\delta},\quad v_{\rm opt}^{(2)}=\frac{\ln[8\delta^2/\ln(4\delta)]}{2\delta}. \quad (13)$$

Hence, the optimal impedance ratio $Z_{\rm opt}$ and the optimal power absorption efficiency $\eta_{\rm abs,opt}$ take the asymptotic forms

$$Z_{\rm opt}\sim 1-j\ln[8\delta^2/\ln(4\delta)]/2\delta \quad (14)$$

and

$$\eta_{\rm abs,opt}\sim 1-v_{\rm opt}^2/4-e^{-2\delta v_{\rm opt}}=1-\{\ln^2[8\delta^2/\ln(4\delta)]+\ln(16\delta^2)\}/16\delta^2\sim 1, \quad (15)$$

respectively.

B. Thin layer approximation: $\delta\ll 1$

The behavior of power absorption efficiency for the cases of thin dissipative layers is much more intriguing. In this limit, one may expect that the attenuation coefficient must be large enough to produce high absorptivity, however, not too large otherwise large reflectivity will arise at $z=0$. Obviously, the limit $\delta\ll 1$ in conjunction with finite $|Z|$ (i.e., $\delta|Z|\rightarrow 0$) is of no interest since in this limit $R\rightarrow 0$ [Eq. (2)] and $T\rightarrow 1$ [Eq. (3)], leading to $\eta_{\rm abs}\rightarrow 0$ [Eq. (1)]. Hence, to obtain higher efficiency, the normalized impedance has to be large (i.e., $|Z|\gg 1$), so as to provide finite $\delta|Z|$. In the subsequent analysis it will be shown that the efficiency in this case can be enhanced up to $\eta_{\rm abs,opt}=1/2$.

An analytic optimization procedure can be repeated, similar to that performed for the thick layer's approximation. The thin layer limit renders, via (10), $\sin(\delta Z)\rightarrow 0$ and $\cos(\delta Z)\rightarrow 1$. Maintaining terms up to $O(|\sin(\delta Z)|^2)$ and $O(\sin(\delta Z)/Z)$ results in the thin layer approximation for Eq. (10), i.e.,

$$\delta Z_{\rm opt}[\sin(\delta Z_{\rm opt}^*)-j2\cos(\delta Z_{\rm opt})/Z_{\rm opt}^*]+\sin(\delta Z_{\rm opt})\times[\sin(\delta Z_{\rm opt}^*)\cos(\delta Z_{\rm opt})-j2/Z_{\rm opt}^*]\sim 0. \quad (16)$$

The solution of (16) for both square brackets, associated with the leading orders, discussed above, is readily given by the following quadratic equation:

$$\delta Z_{{\rm opt},m}-\pi m\sim -j2/Z_{{\rm opt},m}, \quad (17)$$

leading to

$$Z_{{\rm opt},m}\sim\frac{\pi m}{2\delta}[1+\sqrt{1-j8\delta/(\pi m)^2}]. \quad (18)$$

Note that the second root of (17) was ignored since the associated $Z_{\rm opt}$ corresponds to an active media, i.e., $\Im\{Z_{\rm opt}\}=v_{\rm opt}>0$. The asymptotic limit $Z\gg 1$ in (18) can be satisfied if and only if $\delta\ll 1$, revealing that optimal light medium ($Z\gg 1$) solution in (18) is restricted to thin layers. The substitution of $m=0$ provides the following approximation for the zeroth-order mode of the optimal impedance ratio:

$$Z_{{\rm opt},m}|_{m=0}\sim(1-j)/\sqrt{\delta}. \quad (19)$$

For either $\delta\ll 1$ or $m\gg 1$ one obtains

$$Z_{{\rm opt},m}|_{m\neq 0}\sim\pi m/\delta-j2/\pi m. \quad (20)$$

For thin layers, all modes (all values of m) provide the same asymptotic value of the optimal power absorption efficiency, namely,

$$\eta_{\rm abs,opt}\sim 1/2. \quad (21)$$

C. Intermediate range

The optimization procedure for the normalized impedance ratio $Z_{\rm opt}=u_{\rm opt}-jv_{\rm opt}$ and the resultant optimal power absorption efficiency $\eta_{\rm opt}$ as well as the wave penetration ratio $v_{\rm opt}\delta$ in (7) and (6), respectively, can be carried out numerically via (10), recovering various limiting cases, as discussed above and depicted in Fig. 3. The exact solutions of (10), i.e., $u_{\rm opt}$ and $v_{\rm opt}$, are shown in Fig. 3(a), whereas its substitutions into (7) and (6), namely $\eta_{\rm abs,opt}$ and $l/\Delta_{1,\rm opt}$, are depicted in Figs. 3(b) and 3(c), respectively.

The basic classification of power absorption mechanism is readily obtained via Fig. 3(c). While the thin layer limit, supporting the lossy modes of index $m=0,1,2,...$, extends approximately over $0\leqslant v\delta\leqslant 1$, the thick layer limit, in the complementary range $1<v\delta<\infty$, supports a consecutive continuation of the $m=1$ mode only. It should be noted, though, that the pressure distribution within the layer for the $m=1$ mode in the range $0\leqslant v\delta\leqslant 1$ is basically that of a standing wave (half period) field, whereas its continuation in the range $1<v\delta<\infty$ decays exponentially. In applications like ultrasonic hyperthermia of living tissue (e.g., ablation of cancerous tissue) it is important to know the actual distribution of wave intensity as it passes through the dissipative object since it directly affects the level of local energy absorption of the wave and, consequently, the local temperature rise within the layer. Considering the $1<v\delta<\infty$ range for $m=1$ mode in Fig. 3(c): the power distribution uniformity within the layer is determined in terms of wave penetration depth defined by Eq. (6). It can be readily evaluated numerically using Eqs. (4)–(7) that the penetration depth $\Delta_{1,\rm opt}$ of the optimal absorbing layer becomes less than its actual thickness l for approx. $\delta>1$. The ratio $l/\Delta_{1,\rm opt}$, however, grows relatively slowly with δ, following the logarithmic dependence of (13), thus, allowing for the optimal penetration depth to be of the order of the layer thickness, over a wide range of thicknesses. It should also be noted that thicker layers $\delta\gg 1$ support higher order modes $m\gg 1$, which, however, extend over finite δ range and eventually terminate.

For a zeroth-order mode ($m=0$), the real part of the normalized impedance ratio becomes almost equal to its imaginary part (in absolute values), i.e., $u_{\rm opt}\approx v_{\rm opt}$. This is the case when the phase between the applied pressure and the velocity of the particles in the medium reaches the 90° value. For linear pressure waves, this kind of anomalous behavior is usually related with an anomalous dispersion/absorption, well described by the so-called Kramers–Kronig relations.[18,19] It usually occurs when real part of the medium complex compressibility becomes zero or very small comparatively to its imaginary part (equivalently to the negligible real part of the complex permittivity in electrodynamics[18]). In this case, a composition of an optimal thin dissipative layer is set by the dispersive relations, consequently, having resonant properties with resonance fre-

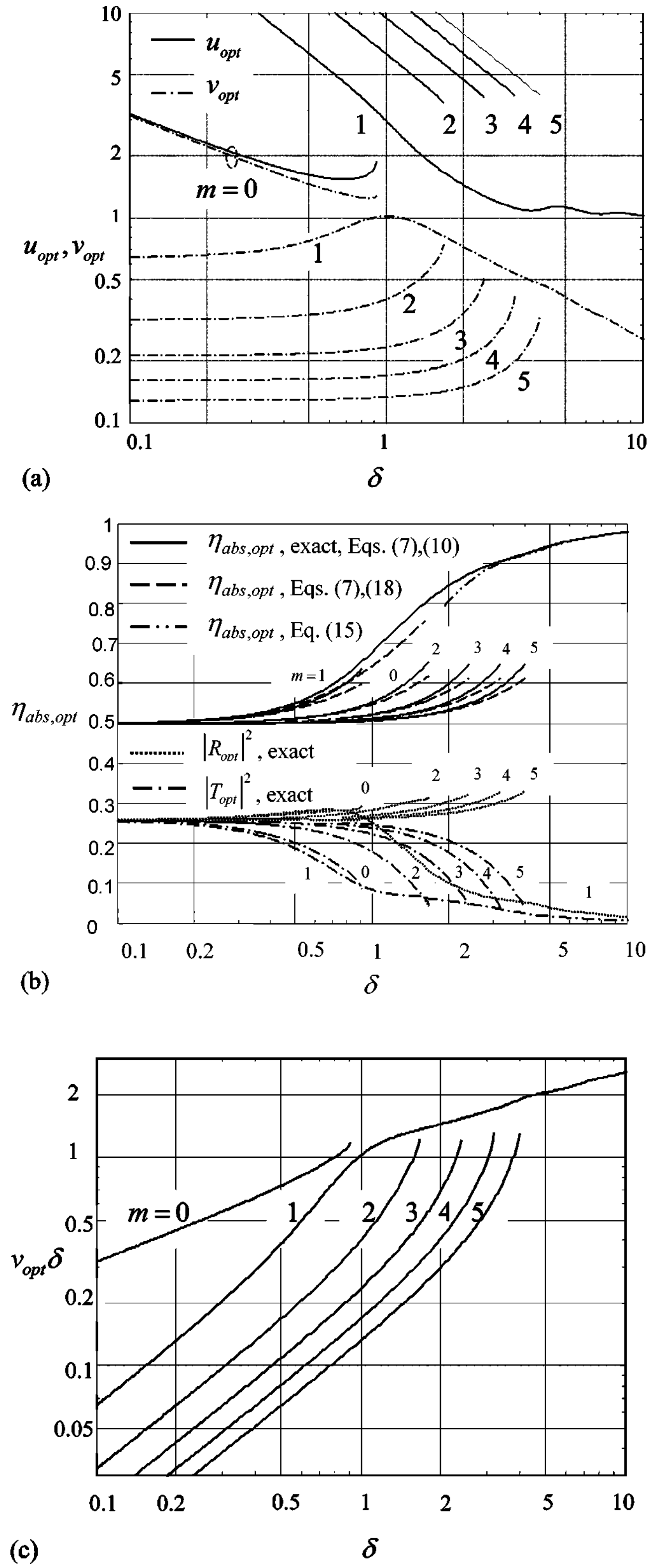

FIG. 3. Optimal power absorption efficiency $\eta_{\text{abs,opt}}$, normalized impedance ratio $Z_{\text{opt}} = u_{\text{opt}} - jv_{\text{opt}}$, and wave penetration ratio $v_{\text{opt}}\delta$, depicted in (a), (b), and (c), respectively, vs normalized thickness δ for first five lossy modes $(m = 0,1,...,5)$.

quency (frequencies). Thus, in order to reach the upper bound on power absorption efficiency by applying the optimal absorption conditions in the thin dissipative layer (resulting in $u_{\text{opt}} \approx v_{\text{opt}}$), the ensonifying frequency must be close to this/these resonance frequency/frequencies. To obtain this kind of resonant behavior in a relatively low frequency range (KHz or MHZ range) resonant elements can be added to the layer. Good examples are porous media, bubbly liquid layers[15] or media containing ultrasound contrast agents.[17]

The substitution of the approximations for u_{opt} and v_{opt} in (18)–(20) into (7) results in approximated values of power absorption efficiency, which agree very well with the exact numerically evaluated results, as depicted in Fig. 3(b). Both $|R_{\text{opt}}|^2$ and $|T_{\text{opt}}|^2$, the constituents of $\eta_{\text{abs,opt}}$, are also depicted in Fig. 3(b) for reference. One notes that the $m = 1$ mode establishes an upper bound (global maxima) for all the other modes. It coincides, however, with the other modes as the values of $\eta_{\text{abs,opt}} \rightarrow 1/2$ and $|R_{\text{opt}}|^2 \rightarrow |T_{\text{opt}}|^2 \rightarrow 1/4$ are reached as $\delta \rightarrow 0$.

Finally, to provide some assessment of the applicability of the derived estimates, one may consider a simple case of the linear $f = 10$ KHz pressure wave, propagating in water with $c_0 = 1500$ m/s and $\rho_0 = 1000\,\text{kg/m}^3$, which impinges normally upon the dissipative layer of air at STP, having $c_1 = 330$ m/s and $\rho_1 = 1.2\,\text{kg/m}^3$. For instance, one may look for the thickness l of this layer, providing the optimal power absorption efficiency for the dominant mode $m = 1$. Given these values and utilizing definitions and results of Eqs. (4), (5), and (20) it is readily obtained that $u = \rho_0 c_0 / \rho_1 c_1 = 3787.9$, $\delta = \pi m / u = 8.294 \times 10^{-4}$, and, consequently, $l = 16.5$ mm. It should be emphasized that the obtained optimum value of l is extremely sensitive to changes in u, as expected for a lossy mode with very small δ at the optimum. Furthermore, while the value of u is approximately constant for an air layer in water, the value of v is determined via attenuation coefficient $\alpha_1 = -\Im\{k_1\}$ of air at $f = 10$ KHz, which is taken as approximately[19] $\alpha_1 = 0.01$ [Neper/m], thus, $v = \alpha_1 \rho_0 / k_0 \rho_1 \approx 0.2$. Substitution of the obtained values into (1)–(5) renders $\eta_{\text{abs}} = 0.3638$. Note that the required optimal value of v must be slightly different, i.e., $v_{\text{opt}} = 2/\pi m = 0.64$, for which the optimal value of $\eta_{\text{abs}} = 0.5$ is reached. For comparison, the intensity of the same pressure wave, propagating in air, is attenuated by 0.033% only every 16.5 mm.

V. CONCLUSIONS

The phenomenon of acoustic power absorption in a finite size dissipative medium is analyzed utilizing an elementary prototype model of plane longitudinal acoustic wave impingement upon the dissipative finite-thickness layer, surrounded by semi-infinite nondissipative media. It is found that, for sufficiently thick layers (in terms of its thickness normalized to wavelength), the absorption of close to 100% of the incident wave power can be achieved provided that specific optimal values are selected for the real and the imaginary parts of the wave number of the dissipative layer. These values are found analytically and explicitly depending on the normalized thickness of the layer. It was also found that, as the thickness of the optimal absorbing layer becomes greater, the ratio between layer thickness and wave penetration depth, at the optimum, also grows. For a very thick optimal layer, with absorption close to 100%, most of the power is absorbed in the vicinity of its boundary. Another observation, important for the modeling of realistic media, can be made regarding the optimal results for a thick layer. The acoustic velocity in homogeneous media can be usually considered as approximately constant over wide range of frequencies, i.e., u is also constant. This rule is also true for many macroscopically quasihomogeneous media like living tissues. On the contrary, the normalized attenuation constant υ is usually frequency dependent. For instance, the attenuation in living tissues grows,[19] approximately, with f^n $(1.1 < n < 2)$, where f is the frequency of insonation, thus, υ, since it is normalized to $k_0 \sim f$, is expected to slowly grow with the frequency. Naturally, if u is set to unity, the selection of frequency becomes important since it directly determines the value of υ and, consequently, the value of the power absorption efficiency.

For thin layers, the upper bound (maximum) on power absorption efficiency tends to the value of 50% as layer thickness becomes smaller. This maximum occurs for infinite number of possible optimal layers (lossy modes), whereas all modes, except for the zeroth-order one, are supported by slightly dissipative layers. The important observation acutally means that even very thin contrast layers are able to absorb up to 50% of the incident energy, provided that specific physical parameters of the layer and its surroundings are established. Due to this kind of resonant absorption, it is hypothesized that, in some special cases, the commonly used definitions in ultrasonic safety assessments, based on bulk attenuation constants[19], may fail in their ability to predict an enhanced local power deposition in thin layers (small sites) of biological tissue. The zeroth-order lossy mode occurs for highly losses provided almost equal optimal (absolute) values of real and imaginary parts of the layer wave number. For linear pressure waves, the latter kind of wave propagation usually corresponds to an anomalous absorption or an anomalous dispersion propagation regime, which is well described by the so-called Kramers–Kronig relations. This behavior, resulting in an almost zero real part of complex compressibility of the dissipative layer (equivalently to zero real part of complex permittivity in electrodynamics), may occur when the dissipative medium has resonant properties (and, consequently, resonance frequency/frequencies). In this case, the ensonifying frequency must normally be close to this/these resonant frequency/frequencies in order to cause $\Re\{k_1\} \approx |\Im\{k_1\}|$ within the dissipative layer. Naturally, as the wavelength of pressure waves approaches the dimensions of the molecular structure of matter, the anomalous absorption occurs. However, in order for the wavelength to reach these values, the frequency of insonation must be on the order of GHz for liquids and solids and on the order of MHz for gases. Nevertheless, when the medium is porous or a gaseous bubbles are inserted into it, e.g., ultrasound contrast agents, the resonance frequencies of the matter can be even within the KHz range, providing drastic improvement in absorption efficiency.

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