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The Human Femoral Head as a Tilted Solid of Revolution

The opposed surfaces of the human hip joint have earlier been reported to be non-spherical.[1] The data available in the literature provide us with quantitative information on the curvatures measured at different sites and directions of the human femoral head. Clarke and Amstutz[2] have reported changes in curvatures between meridians of the femoral head, as referred to the femoral neck axis. They found that the radius of curvature increased from the posterior meridian to the superior meridian and decreased again towards the anterior meridian. In the equatorial plane, on the other hand, the measured curvatures varied within very close limits, and this plane could therefore be considered as virtually circular. Cathcart[3] noted that planes parallel to the equatorial plane were "more convex" just anterior to the superior meridian, causing a decrease of the radius for this region. The geometrical interpretation of these data was, however, primarily confined to the confirmation of the "out of roundness" of the human femoral head. It is proposed here that the human femoral head is a solid of revolution with an axis of symmetry which does not coincide with the femoral neck axis. An analytical description supported by the existing experimental data is given and the orientation of the axis of symmetry is determined.

We consider a solid of revolution. Let an axis which passes through the centre *O* of its equatorial circle, other than the axis of symmetry, intersect the surface of this solid of revolution at an apex (also called pole) *P*, as shown in Fig. 1. The angle between the tilted axis and the axis of symmetry is denoted by α. A plane perpendicular to the tilted axis will generate on the surface of the body of revolution a non-circular and inclined closed latitude curve *ABCD*. Planes including this axis will generate on the surface of the body of revolution, symmetrically about plane *POQ*, non-identical longitudes. These deviated longitudes intersect the meridians of the body by angles varying from zero at point *A* to α at point *B* and back to zero at point *C*. The curvatures of the longitudes vary from point to point along the curve *ABCD*, due to both the inclination of the latitudes and the directional deviation of the longitudes. Using the methods of Geometry on a Surface (cf. for example Brand[4]), these curvatures were calculated and compared with the curvatures of the human femoral head, as found in the literature (and measured with reference to the femoral neck axis). For this purpose three different groups of parameters were investigated separately for the average male and the average female:

1) The shape of the solid of revolution. Two solids of revolution were considered:

$$(x^2/a^2) + (y^2/b^2) + (z^2/b^2) = 1 \qquad (1)$$

$$x^2/b^2\,(1 + m^2x^2) + (y^2/b^2) + (z^2/b^2) = 1 \qquad (2)$$

With both having x as an axis of symmetry, the first is the ordinary ellipsoid of revolution while the second includes a parameter $m^2 = k/b^2$ with a constant k indicating departure from sphericity. At small values of x the second solid of revolution resembles a sphere, which keeps a tilted equator practically circular.

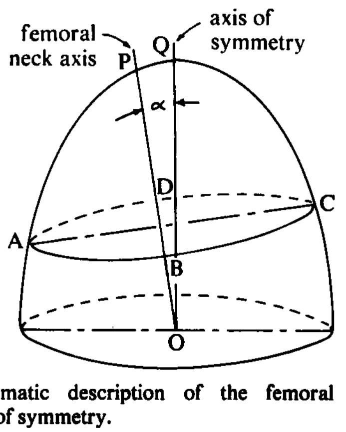

Fig. 1. Schematic description of the femoral neck axis and the axis of symmetry.

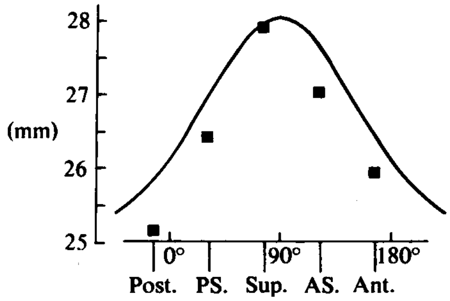

Fig. 2. Comparative variation of the theoretical (solid line) and experimental (■) radii of curvatures of the human femoral head of an average male. The experimental values are taken from Clarke and Amstutz.[2] (Post = posterior, PS = posterior-superior, Ant = anterior.) The solid of revolution is described by Equation 2, with $b = 25.2$ mm, $k = 0.09$ and $\alpha = 16°$. The shift between Sup and the 90° point corresponds to 13°, which is the angle of tilt of plane *POQ* to the vertical line towards the anterior point.

2) The size of the solid of revolution, that is, the parameters *a*, *b* and *k*.

3) The orientation of the axis of symmetry with respect to the femoral neck axis, giving the best fit between the calculated and experimental data.

A computer program was developed and used throughout the analysis.

From the two solids of revolution described above, the second (Equation 2) was found more suitable. Departure from the axis of symmetry to the femoral neck axis, that served as axis of reference in the above-mentioned measurements, caused negligible changes in the calculated curvatures near the original equatorial circle, as compared to changes of the longitudinal curvatures. In fact there was a slight decrease in the radius of curvature of the tilted equator at the superior point. The size parameters found were: $b = 25.2$ mm, $k = 0.09$ for an average male, and $b = 22.8$ mm, $k = 0.05$ for an average female. The comparative variations of the radii of curvature for the average male are displayed in Fig. 2. It was found that the axis of symmetry was elevated to the femoral neck axis towards the superior point of the femoral head by the angle α defined in Fig. 1. This angle was 16° for the average male and 14° for the average female. Both axes together formed a plane *POQ* tilted to the vertical line towards the anterior point by an angle of 13° for the average male and 10° for the average female.

An axi-symmetrical geometry can be easily and accurately reproduced. The formulation presented here of the femoral head geometry therefore opens the possibility of a more correct fit of femoral head replacement.

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