Coherent states, displaced number states and Laguerre polynomial states for su(1,1) Lie algebra

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Abstract

The ladder operator formalism of a general quantum state for su(1,1) Lie algebra is obtained. The state bears the generally deformed oscillator algebraic structure. It is found that the Perelomov's coherent state is a su(1,1) nonlinear coherent state. The expansion and the exponential form of the nonlinear coherent state are given. We obtain the matrix elements of the su(1,1) displacement operator in terms of the hypergeometric functions and the expansions of the displaced number states and Laguerre polynomial states are followed. Finally some interesting su(1,1) optical systems are discussed.

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I. INTRODUCTION

The harmonic oscillator is a fundamental exactly solvable physical system and the coherent state(CS)[1] defined in this system is well studied. The generalization of the CS to multi-photon case[2] and the extention to various systems have been made.

As for su(1,1) Lie algebra, the Perelomov's coherent state(PCS) is well known[3] . The su(1,1) Lie algebra is of great interest in quantum optics because it can characterize many kinds of quantum optical systems. In particular, the bosonic realization of su(1,1) describes the degenerate and non-degenerate parametric amplifiers.

The generators of su(1,1) Lie algebra, K_0 and $K_\pm$, satisfy the commutation relations

$$[K_+, K_-] = -2K_0,\ [K_0, K_\pm] = \pm K_\pm. \tag{1}$$

Its discrete representation is

$$\begin{aligned} K_+|n,k\rangle &= \sqrt{(n+1)(2k+n)}|n+1,k\rangle, \\ K_-|n,k\rangle &= \sqrt{n(2k+n-1)}|n-1,k\rangle, \\ K_0|n,k\rangle &= (n+k)|n,k\rangle. \end{aligned} \tag{2}$$

Here $|n,k\rangle (n = 0, 1, 2, ...)$ is the complete orthonormal basis and $k = 1/2, 1, 3/2, 2, ...$ is the Bargmann index labeling the irreducible representation[$k(k-1)$ is the value of Casimir operator]. We introduce the number operator $\mathcal{N}$ by

$$\mathcal{N} = K_0 - k, \mathcal{N}|n,k\rangle = n|n,k\rangle. \tag{3}$$

The PCS is defined as

$$\begin{aligned} |\alpha,k\rangle_P &= S(\xi)|0,k\rangle \\ &= (1-|\alpha|^2)^k \sum_{n=0}^{\infty} \sqrt{\frac{\Gamma(2k+n)}{\Gamma(2k)n!}}\alpha^n |n,k\rangle, \end{aligned} \tag{4}$$

where $\xi = r\exp(i\theta)$,$\alpha = \exp(i\theta)\tanh r$, $\Gamma(x)$ is the gamma function, $S(\xi) = \exp(\xi K_+ - \xi^* K_-)$ is the su(1,1) displacement operator. There is another coherent state of su(1,1) which is known as the Barut-Girardello(BG) coherent state(BGCS)[4]. The BGCS is defined as the eigenstate of the lowering operator K_-

$$K_-|\alpha,k\rangle_{BG} = \alpha|\alpha,k\rangle_{BG}, \tag{5}$$

and it can be expressed as[4]

$$|\alpha,k\rangle_{BG} = \sqrt{\frac{|\alpha|^{2k-1}}{I_{2k-1}(2|\alpha|)}}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!\Gamma(n+2k)}}|n,k\rangle, \tag{6}$$

where $I_\nu(x)$ is the first kind modified Bessel function.

The PCS is defined as the displacement operator formalism, while the BGCS as the ladder operator formalism. We ask if the PCS admits the ladder operator formalism? The answer is affirmative. We will discuss it in the next section. We also give the ladder operator formalism of a general su(1,1) quantum state and find that the PCS is a su(1,1) nonlinear coherent state(NLCS). The complete expansion and exponential form of the su(1,1) NLCS are obtained.

There are three definitions of coherent states, that is, (1) the displacement operator acting on the vacuum states, (2) the eigenstates of the annihilation operator, (3) the minimum uncertainty states. These three definitions are identical only for the simplest harmonic oscillator. For su(1,1) system, the PCS is defined according to the first and the BGCS to the second. The minimum uncertainty states(MUSs) for su(1,1) are defined as[5]

$$(\mu K_+ + \nu K_-)|\alpha,k\rangle_{MUS} = \alpha|\alpha,k\rangle_{MUS}, \tag{7}$$

where μ and ν are complex constants satisfying $|\mu/\nu| < 1$. One type of the MUS is the Laguerre polynomial state(LPS)[6]. The LPS is only given formally in the literature. We will give the expansion of the LPS in terms of states $|n,k\rangle$ in section III. The PCS, BGCS and MUS cover the three definations of the coherent states. In section IV, we consider several interesting su(1,1) optical systems, namely, the density-dependent

Holstein-Promakoff system[7], amplitude-squared system[8], and two-mode system[9]. A conclusion is given in Sec.V.

II. SU(1,1) COHERENT STATES AND NONLINEAR COHERENT STATES

We consider a general state

$$|x, k\rangle_G = \sum_{n=0}^{\infty} C(n, x, k)|n, k\rangle, \tag{8}$$

where x denote parameter and all the coefficients $C(n, x, k)$ are non-zero. Now we try to give the ladder operator formalism of the above general state. The key point is to let the number operator $\mathcal{N}$ and the operator $f(\mathcal{N})K_+$ act on Eq.(8), respectively. Here $f(\mathcal{N})$ is a real function of $\mathcal{N}$. The operations lead to

$$\begin{aligned} \mathcal{N}|x, k\rangle_G &= \sum_{n=1}^{\infty} C(n, x, k)n|n, k\rangle, \\ f(\mathcal{N})K_+|x, k\rangle_G &= \sum_{n=1}^{\infty} f(n)C(n-1, x, k)\sqrt{n(n+2k-1)}|n, k\rangle. \end{aligned} \tag{9}$$

If we choose

$$f(\mathcal{N}) = \frac{C(\mathcal{N}, k, x)\sqrt{\mathcal{N}}}{C(\mathcal{N}-1, k, x)\sqrt{\mathcal{N}+2k-1}}, \tag{10}$$

the following equation is obtained

$$[\mathcal{N} - f(\mathcal{N})K_+]|x, k\rangle_G = 0. \tag{11}$$

This is the ladder operator formalism of the general state $|x, k\rangle_G$.

Let us examine the algebraic structure involved in the general state. Define $\mathcal{A}$ as an associate algebra with generators

$$\mathcal{N}, A_+ = \frac{C(\mathcal{N}, k, x)\sqrt{\mathcal{N}}}{C(\mathcal{N}-1, k, x)\sqrt{\mathcal{N}+2k-1}}K_+, A_- = (A_+)^\dagger. \tag{12}$$

Then it is easy to verify that these operators satisfy the following relations

$$[\mathcal{N}, A_\pm] = \pm A_\pm,\ A_+A_- = S(\mathcal{N}),\ A_-A_+ = S(\mathcal{N}+1), \tag{13}$$

where the function

$$S(\mathcal{N}) = \frac{\mathcal{N}^2 C^2(\mathcal{N}, k, x)}{C^2(\mathcal{N}-1, k, x)} \tag{14}$$

This algebra $\mathcal{A}$ is nothing but the generally deformed oscillator(GDO)[10] algebra with the structure function $S(\mathcal{N})$. So we see that the general state $|x, k\rangle_G$ bears generally deformed oscillator algebraic structure.

By acting the annihilation operator K_- on Eq.(11) from left, we get

$$[f(\mathcal{N}+1)(\mathcal{N}+2k) - K_-]|x, k\rangle_G = 0. \tag{15}$$

In the derivation of the above equation, we have used the fact that the operator $\mathcal{N}+1$ is non-zero in the whole space. The function $f(\mathcal{N})$ is completely determined by the coefficients of the state $|x, k\rangle_G$. From the coefficients of the BGCS(Eq.(6)), the operator-valued function $f(\mathcal{N}) = \alpha/(\mathcal{N}+2k-1)$. It is easily seen that Eq.(15) reduces to Eq.(5) as we expected. From the coefficients of the PCS(Eq.(4)), we obtain the corresponding operator-valued function $f(\mathcal{N}) = \alpha$. This simple result leads to the ladder operator formalism of the PCS

$$\frac{1}{\mathcal{N}+2k} K_- |\alpha, k\rangle_P = \alpha |\alpha, k\rangle_P \tag{16}$$

In fact, by direct verification, we have

$$[\frac{1}{\mathcal{N}+2k} K_-, K_+] = 1. \tag{17}$$

Therefore, the exponential formalism of the PCS can be given by

$$|\alpha, k\rangle_P = \exp(\alpha K_+)|0, k\rangle \tag{18}$$

up to a normalization constant.

Reminding the definition of nonlinear coherent states in Fock space[11], we can call the state $|\alpha, k\rangle_P$ as a su(1,1) NLCS. The NLCS is defined as

$$G(\mathcal{N}) K_- |\alpha, k\rangle_{NL} = \alpha |\alpha, k\rangle_{NL}, \tag{19}$$

where $G(\mathcal{N})$ is a real function of $\mathcal{N}$. The PCS and the BGCS are recovered for the special choices of $G(\mathcal{N}) = 1/(\mathcal{N}+2k)$ and $G(\mathcal{N}) = 1$, respectively. Thus the two coherent states, PCS and BGCS, are unified within the framework of the su(1,1) NLCS.

Assuming the expansion of the NLCS is

$$|\alpha, k\rangle_{NL} = \sum_{n=0}^{\infty} D(n, \alpha, k)|n, k\rangle \tag{20}$$

and substituting it into Eq.(19), we get the following recursion relation

$$\frac{D(n+1, \alpha, k)}{D(n, \alpha, k)} = \frac{\alpha}{G(n)\sqrt{(n+1)(2k+n)}}. \tag{21}$$

Eq.(21) leads to

$$D(n, \alpha, k) = \frac{\alpha^n D(0, \alpha, k)}{G(n-1)G(n-2)...G(0)\sqrt{n!\Gamma(2k+n)/\Gamma(2k)}}. \tag{22}$$

The combination of Eq.(20) and (22) gives the expansion of the su(1,1) NLCS,

$$\begin{aligned} |\alpha, k\rangle_{NL} &= D(0, \alpha, k) \sum_{n=0}^{\infty} \frac{\alpha^n D(0, \alpha, k)\sqrt{\Gamma(2k)}}{G(n-1)G(n-2)...G(0)\sqrt{n!\Gamma(2k+n)}}|n, k\rangle, \\ &= D(0, \alpha, k) \sum_{n=0}^{\infty} \frac{\alpha^n \Gamma(2k) K_+^n}{G(n-1)G(n-2)...G(0)n!\Gamma(2k+n)}|0, k\rangle. \end{aligned} \tag{23}$$

The coefficient $D(0, \alpha, k)$ can be determined by normalization. Let $G(\mathcal{N}) = 1/(\mathcal{N}+2k)$, we naturally reduce Eq.(23) to Eq.(4) up to a normalization constant.

One can show that

$$\begin{aligned} \mathcal{N}K_+ &= K_+(\mathcal{N}+1), f(\mathcal{N})K_+ = K_+ f(\mathcal{N}+1), \\ [f(\mathcal{N})K_+]^n &= (K_+)^n f(\mathcal{N}+1)f(\mathcal{N}+2)...f(\mathcal{N}+n). \end{aligned} \tag{24}$$

Then as a key step, by using Eq.(24) with

$$f(\mathcal{N}) = \frac{\alpha}{G(\mathcal{N}-1)(\mathcal{N}+2k-1)}, \tag{25}$$

the NLCS is finally written in the exponential form

$$
\begin{aligned}
|\alpha,k\rangle_{NL} &= D(0,\alpha,k)\sum_{n=0}^{\infty}\frac{1}{n!}[\frac{\alpha}{G(\mathcal{N}-1)(\mathcal{N}+2k-1)}K_+]^n|0,k\rangle, \\
&= D(0,\alpha,k)\exp[\frac{\alpha}{G(\mathcal{N}-1)(\mathcal{N}+2k-1)}K_+]|0,k\rangle. \qquad (26)
\end{aligned}
$$

From the above equation, the exponential form of the BGCS is easily obtained by setting $G(\mathcal{N})=1$,

$$
|\alpha,k\rangle_{BG} = [\exp\frac{\alpha}{(\mathcal{N}+2k-1)}K_+]|0,k\rangle \qquad (27)
$$

up to a normalization constant. Let $G(\mathcal{N}) = 1/(\mathcal{N}+2k)$ in Eq.(26), Eq.(18) is recovered as we expected.

Actually we have

$$
[G(\mathcal{N})K_-,\frac{1}{G(\mathcal{N}-1)(\mathcal{N}+2k-1)}K_+] = 1. \qquad (28)
$$

By this observation, Eq.(26) is naturally obtained.

III. DISPLACED NUMBER STATES AND LAGUERRE POLYNOMIAL STATES

As a generalization of the PCS, we define the displaced number state(DNS) for su(1,1) Lie algebra in analogous with the definition of the displaced number state in Fock space,

$$
|\xi,m,k\rangle_{DN} = S(\xi)|m,k\rangle = \sum_{n=0}^{\infty}\langle n,k|S(\xi)|m,k\rangle|n,k\rangle,\ \xi = r\exp(i\theta). \qquad (29)
$$

All the work left is to calculate the matrix elements $S^k_{nm}(\xi) = \langle n,k|S(\xi)|m,k\rangle$. Using the decomposed form of the displacement operator

$$
S(\xi) = \exp(\alpha K_+)(1-|\alpha|^2)^{K_0}\exp(-\alpha^* K_-)\ ,\alpha = \exp(i\theta)\tanh r \qquad (30)
$$

and the relation

$$\exp(-\eta^* K_-)|m,k\rangle = \sum_{q=0}^{m} \frac{(-\eta^*)^{m-q}}{(m-q)!}\sqrt{\frac{m!\Gamma(2k+m)}{q!\Gamma(2k+q)}}|q,k\rangle \tag{31}$$

we obtain the matrix elements as

$$\begin{aligned} S^k_{nm}(\xi) &= (1-|\alpha|^2)^k \alpha^n (-\alpha^*)^m \sqrt{m!n!\Gamma(2k+m)\Gamma(2k+n)} \\ &\sum_{q=0}^{\min(m,n)} \frac{(1-1/|\alpha|^2)^q}{q!(n-q)!(m-q)!\Gamma(2k+q)}. \end{aligned} \tag{32}$$

Using the relations

$$(-m)_q = (-1)^q \frac{m!}{(m-q)!}, (-n)^q = (-1)^q \frac{n!}{(n-q)!}, (2k)_q = \frac{\Gamma(2k+q)}{\Gamma(2k)}, \tag{33}$$

we can write the matrix elements in terms of hypergeometric function as

$$\begin{aligned} S^k_{nm}(\xi) &= (1-|\alpha|^2)^k \alpha^n (-\alpha^*)^m \\ &\sqrt{\frac{\Gamma(2k+m)\Gamma(2k+n)}{\Gamma(2k)\Gamma(2k)m!n!}}\ {}_2F_1(-m,-n;2k;1-\frac{1}{|\alpha|^2}). \end{aligned} \tag{34}$$

Here the hypergeometric function

$${}_2F_1(\alpha,\beta;\gamma;z) = \sum_{n=0}^{\infty} \frac{(\alpha)_n(\beta)_n}{n!(\gamma)_n} z^n, \tag{35}$$

and

$$(x)_n = x(x+1)...(x+n-1), (x)_0 \equiv 1. \tag{36}$$

The combination of Eqs.(29) and (34) gives the expansion of the DNS in terms of the basis state $|n,k\rangle$. It is easily checked that Eq.(29) reduces to Eq.(4) when $m=0$. The matrix elements abtained here are useful in the study of su(1,1) quantum states.

As one type of MUS for su(1,1) Lie algebra, the LPS is given by[6],

$$|\alpha,k\rangle_{LP} = C_0 S(\beta) L_M(\xi \frac{K_0-k}{K_0+k-1} K_+)|0,k\rangle. \tag{37}$$

Here $\beta = r\exp(i\theta)$ is determined by the equation $\exp(2i\theta)\tanh^2 r = -\nu/\mu$. $\xi = -\exp(i\theta)\tanh(2r)$, C_0 can be determined by normalization, and

$$L_M(x) = \sum_{n=0}^{M} \frac{1}{n!} \binom{M}{M-n} (-1)^n x^n \tag{38}$$

is the Laguerre polynomial. Using Eq.(38), we obtain the expansion of the LPS as

$$\begin{aligned}
|\alpha, k\rangle_{LP} &= C_0 S(\beta) \sum_{m=0}^{M} (-\xi)^n \frac{M!}{(M-m)!\sqrt{m!\Gamma(2k+m)/\Gamma(2k)}} |m, k\rangle, \qquad (39)\\
&= C_0 \sum_{n=0}^{\infty} \left[\sum_{m=0}^{M} (-\xi)^n \frac{M! S_{nm}^{k}(\beta)}{(M-m)!\sqrt{m!\Gamma(2k+m)/\Gamma(2k)}} \right] |n, k\rangle.
\end{aligned}$$

The combination of Eq.(34) and (39) gives the complete expansion of the LPS.

IV. SOME SU(1,1) OPTICAL SYSTEMS

In the previous two sections, we obtain the general results of several quantum states for su(1,1) Lie algebra. Now we want to investigate some interesting su(1,1) optical systems.

A. Density-dependent HP realization

The HP realization of the su(1,1) Lie algebra is[7]

$$K_+ = a^+\sqrt{N+2k}, K_- = \sqrt{N+2k}a, K_0 = N+k. \tag{40}$$

where a^+, a, and $N = a^+a$ are the creation, annihilation, and number operator of a single-mode electromagnetic field satisfying $[a, a^+] = 1$. On the Fock space $|n\rangle = [a^{+n}/\sqrt{n!}]|0\rangle$, we have

$$\begin{aligned}
K_+|n\rangle &= \sqrt{(n+1)(2k+n)}|n+1\rangle, \qquad (41)\\
K_-|n\rangle &= \sqrt{n(2k+n-1)}|n-1\rangle,\\
K_0|n\rangle &= (n+k)|n\rangle.
\end{aligned}$$

In comparison with Eq.(2), we see that the HP realization gives rise to the discrete representation of su(1,1) Lie algebra on the usual Fock space. Therefore, by replacing the state $|n,k\rangle$ by $|n\rangle$, we recover all the results in Sec.II and III.

By the replacement procedure described above, we obtain the PCS via HP realization as

$$|\alpha, M\rangle_{NB} = (1-|\alpha|^2)^{M/2} \sum_{n=0}^{\infty} \binom{M+n-1}{n}^{1/2} \alpha^n |n\rangle \tag{42}$$

This is just the well-known negative binomial state(NBS)[12]. Here $M = 2k$. Since the PCS admits displacement operator formalism, we naturally obtain the displacement operator formalism of the NBS from Eq.(4)[13]

$$|\alpha, M\rangle_{NB} = \exp[\eta a^+ \sqrt{N+2k} - \eta^* \sqrt{N+2k} a]|0\rangle. \tag{43}$$

The parameter η is determined by the equation $\eta/|\eta| \tanh |\eta| = \alpha$.

From Eq.(16), the ladder operator formalism of the NBS is written as

$$\frac{1}{\sqrt{N+M}} a|\alpha, M\rangle_{NB} = \alpha|\alpha, M\rangle_{NB} \tag{44}$$

As seen from the above equation, we conclude that the NBS is a NLCS in Fock space as discussed in our previous paper[14]. In addition, the su(1,1) displaced number states via HP realization are studied in detail by Fu and Wang[15].

It can be seen that some useful results of the NBS are conveniently extracted from the general results for su(1,1) Lie algebra .

B. Amplitude-squared realization

The amplitude-squared su(1,1) is given by

$$K_+ = \frac{1}{2} a^{+2}, K_- = \frac{1}{2} a^2, K_0 = \frac{1}{2}(N + \frac{1}{2}). \tag{45}$$

The representation on the usual Fock space is completely reducible and decomposes into a direct sum of the even Fock space (S_0) and odd Fock space (S_1),

$$S_j = \text{span}\{||n\rangle_j \equiv |2n+j\rangle | n = 0,1,2,...\},\ j = 0,1. \tag{46}$$

Representations on S_j can be written as[6]

$$
\begin{aligned}
K_+||n\rangle_j &= \sqrt{(n+1)(n+j+1/2)}||n+1\rangle_j, \\
K_-||n\rangle_j &= \sqrt{(n)(n+j-1/2)}||n-1\rangle_j, \\
K_0||n\rangle_j &= (n+j/2+1/4)||n\rangle_j.
\end{aligned} \tag{47}
$$

The Bargmann index $k = 1/4(3/4)$ for even(odd) Fock space. From Eq.(4) we see that the PCSs in even/odd Fock space are squeezed vacuum state and squeezed first Fock state

$$
\begin{aligned}
|\xi\rangle_{SV} &= \exp(\frac{\xi}{2}a^{+2} - \frac{\xi^*}{2}a^2)|0\rangle, \\
|\xi\rangle_{SF} &= \exp(\frac{\xi}{2}a^{+2} - \frac{\xi^*}{2}a^2)|1\rangle,
\end{aligned} \tag{48}
$$

respectively. The ladder operator formalisms of the squeezed vacuum state and squeezed first Fock state are easily obtained from Eq.(16)[16]

$$
\begin{aligned}
\frac{1}{N+1}a^2|\xi\rangle_{SV} &= \xi/|\xi|\tanh(|\xi|)|\xi\rangle_{SV,} \\
\frac{1}{N+2}a^2|\xi\rangle_{SF} &= \xi/|\xi|\tanh(|\xi|)|\xi\rangle_{SF.}
\end{aligned} \tag{49}
$$

We see that the the two states are the two-photon nonlinear coherent state $|\alpha\rangle_{TP}$ which is defined as

$$
f(N)a^2|\alpha\rangle_{TP} = \alpha|\alpha\rangle_{TP}. \tag{50}
$$

Here $f(N)$ is a real function of the operator N.

Now we consider the matrix elements $S^k_{nm}(\xi)$ (Eq.(34)) in the representation(Eq.(47)). Reminding that the Bargmann index $k = 1/4(3/4)$ for even(odd) Fock space and substituting $\alpha = \exp(i\theta)\tanh r$ into the Eq.(34), we obtain the matrix elements in the representation as

$$
\begin{aligned}
S_{nm}^{1/4}(\xi) &= \frac{(-1)^m}{m!n!}\sqrt{\frac{(2n)!(2m)!}{\cosh r}}\exp[i(n-m)\theta](\tanh r/2)^{m+n} \\
&\quad {}_2F_1(-m,-n;1/2;-1/\sinh^2 r),
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
S_{nm}^{3/4}(\xi) &= \frac{(-1)^m}{m!n!}\sqrt{\frac{(2n+1)!(2m+1)!}{\cosh^3 r}}\exp[i(n-m)\theta](\tanh r/2)^{m+n} \\
&\quad {}_2F_1(-m,-n;3/2;-1/\sinh^2 r).
\end{aligned} \tag{52}
$$

As special cases of our general result(Eq.(34)), the above two equations with $k = 1/4(3/4)$ have been obtained by Marian[17].

C. Two-mode realization

The two-mode photon operators

$$
K_+ = a^+b^+, K_- = ab, K_0 = \frac{1}{2}(N_1 + N_2 + 1) \tag{53}
$$

generate the su(1,1) Lie algebra. Here $N_1 = a^+a$ and $N_2 = b^+b$. The Fock space $\mathcal{F}$ of the two-mode states is decomposed into a direct sum of irreducible invariant subspaces $\mathcal{F}_p^{\pm}$[6]

$$
\begin{aligned}
\mathcal{F} &= \mathcal{F}_0 \oplus \mathcal{F}_1^{\pm} \oplus ... \oplus \mathcal{F}_p^{\pm} \oplus ..., \\
\mathcal{F}_p^{+} &\equiv \text{span}\{||n\rangle_{+p} \equiv |n, n+p\rangle | n = 0, 1, 2, ...\}, \\
\mathcal{F}_p^{-} &\equiv \text{span}\{||n\rangle_{-p} \equiv |n+p, n\rangle | n = 0, 1, 2, ...\}.
\end{aligned} \tag{54}
$$

Representations on $F_p^{\pm}$ are isomorphic and take the form

$$
\begin{aligned}
K_+||n\rangle_{\pm p} &= \sqrt{(n+1)(n+p+1)}||n+1\rangle_{\pm p}, \\
K_-||n\rangle_{\pm p} &= \sqrt{n(n+p)}||n-1\rangle_{\pm p}, \\
K_0||n\rangle_{\pm p} &= [n + (p+1)/2]||n\rangle_{\pm p}.
\end{aligned} \tag{55}
$$

which are representation (2) with $k = (p+1)/2$. Then by replacing $|0, k\rangle$ by $||0\rangle_{\pm p}$ and k by $(p+1)/2$ in Eq.(4), we obtain the two-mode squeezed vacuum state[18]

$$|\xi, p\rangle_{\pm} = \exp(\xi a^{+} b^{+} - \xi^{*} ab)||0\rangle_{\pm p} \tag{56}$$

From Eq.(16), ladder operator formalism of the two-mode squeezed vacuum state is

$$\frac{2}{(N_1 + N_2) + p + 2} ab|\xi, p\rangle_{\pm} = \xi/|\xi| \tanh(|\xi|)|\xi, p\rangle_{\pm} \tag{57}$$

We can define two-mode NLCS as

$$f(N_1, N_2) ab|\alpha\rangle_{TM} = \alpha|\alpha\rangle_{TM}. \tag{58}$$

Therefore, the two-mode squeezed vacuum state can be viewed as the two-mode NLCS. In addtion, the pair coherent state[19] is a special case of two-mode NLCS with $f(N_1, N_2) = 1$.

V. CONCLUSIONS

In this paper we have given the ladder operator formalism of a general quantum state for su(1,1) Lie algebra. The algebra involved in the general state is well-known GDO algebra. The ladder operator formalism of the PCS is found and it is a su(1,1) NLCS. The expansion and exponential form of the NLCS are given. The matrix elements of the su(1,1) squeezing operator is obtained in terms of hypergeometric functions. Using the matrix elements, expansions of the su(1,1) displaced number states and Laguerre polynomial states are obtained. As realizations of su(1,1) Lie algebra, some optical su(1,1) systems are considered. We obtain the ladder operator formalism of the negative binomial state, squeezed vacuum state, squeezed first Fock state, and two-mode squeezed vacuum state. We have generalized the notion of the NLCS in Fock space to the su(1,1) case. It is interesting to study further the su(1,1) NLCS in various quantum optical systems.

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