Photon-added one-photon and two-photon nonlinear coherent states

Xiaoguang Wang*

Laboratory of Optical Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100080, P.R.China

and CCAST(World Laboratory), P.O.Box 8730, Beijing 100080, P. R. China

(February 1, 2008)

Abstract

From the photon-added one-photon nonlinear coherent states $a^{\dagger m}|\alpha, f\rangle$, we introduce a new type of nonlinear coherent states with negative values of m. The nonlinear coherent states corresponding to the positive and negative values of m are shown to be the result of nonunitarily deforming the number states $|m\rangle$ and $|0\rangle$, respectively. As an example, we study the sub-Poissonian statistics and squeezing effects of the photon-added geometric states with negative values of m in detail. Finally we investigate the photon-added two-photon nonlinear coherent states and find they are still the two-photon nonlinear coherent states with certain nonlinear functions.

PACS: 42.50.Dv, 03.65.Fd

*email:xyw@aphy.iphy.ac.cn

1. Introduction

Recently there has much interest in the study of nonlinear coherent states(NLCSs) [1,2], which are right-hand eigenstates of the product of the boson annihilation operator a and a nonlinear function $f(N)$ of the number operator N,

$$f(N)a|\alpha, f\rangle = \alpha|\alpha, f\rangle. \tag{1}$$

Here α is a complex eigenvalue. It has been shown that a class of NLCSs may appear as stationary states of the centre-of-mass motion of a trapped ion [1]. These nonlinear coherent states exhibit nonclassical features like squeezing and self-splitting.

Another type of interesting nonclassical states consists of the photon-added states [3,4]

$$|m, \psi\rangle = \frac{a^{\dagger m}|\psi\rangle}{\langle\psi|a^m a^{\dagger m}|\psi\rangle}, \tag{2}$$

where $|\psi\rangle$ may be an arbitrary quantum state, $a^\dagger$ is the boson creation operator, m is a non-negative integer-the number of added quanta. For the first time these states were introduced by Agarwal and Tara [3] as photon-added coherent states. The photon-added squeezed states [5] ,even(odd) photon-added states [6] and photon-added thermal state [7] were also introduced and studied. The photon-added states can be produced in the interaction of a two-level atom with a cavity field initially prepared in the state $|\psi\rangle$ [3].

Sivakumar showed that the photon-added coherent states are nonlinear coherent states [8]. As a generalization we showed a general result that photon-added NLCSs(PANLCSs) are still NLCSs with different nonlinear functions [9]. The PANLCSs are defined as

$$|m, \alpha, f\rangle = \frac{a^{\dagger m}|\alpha, f\rangle}{\langle\alpha, f|a^m a^{\dagger m}|\alpha, f\rangle}. \tag{3}$$

They satisfy [9]

$$f(N-m)[1-m/(N+1)]a|m, \alpha, f\rangle = \alpha|m, \alpha, f\rangle. \tag{4}$$

As seen from Eq.(4), the PANLCS is an NLCS with the nonlinear function $f(N-m)[1-m/(N+1)]$. Naturally Eq.(4) reduces to Eq.(1) when $m=0$. The well-known geometric states(GSs) [10] and negative binomial states(NBSs) [11] are NLCSs [9]. Therefore, the photon-added GSs [12,13] and photon-added NBSs [14] are still NLCSs and are special cases of the PANLCSs.

In the present paper we show that the PANLCSs are the result of nonunitarily deforming the number state $|m\rangle$. We introduce the PANLCS with negative values of m , which are the result of nonunitairily deforming the vacuum state $|0\rangle$. As an example, we study the sub-Poissonian statistics and squeezing effects of the photon-added geometric states with negative values of m in detail. We also investigate the photon-added two-photon nonlinear coherent states.

2.The PANLCS as deformed number state $|m\rangle$

In this section we show that the PANLCS can be written as a nonunitarily deformed number state. This is achieved by the method given by Shanta et al [15]. Here we give a brief review of the method.

Consider an annihilation operator A which annihilates a set of number states $|n_i\rangle, i = 1, 2, ...k$. Then we can construct a sector S_i by repeatedly applying $A^\dagger$, the adjoint of A, on the number state $|n_i\rangle$. Thus we have k sectors corresponding to the states that are annihilated by A. A given sector may turn out to be either of finite or infinite dimension. If a sector, say S_j, is of infinite dimension then we can construct an operator $G_j^\dagger$ such that $[A, G_j^\dagger] = 1$ holds in that sector. Then the eigenstates of A can be written as $\exp(\alpha G_j^\dagger)|n_j\rangle$. If an operator A is of the form $f(N)a^p$, where p is non-negative integer, such that it annihilates the number state $|j\rangle$ then $G_j^\dagger$ is constructed as [15]

$$G_j^\dagger = \frac{1}{p} A^\dagger \frac{1}{AA^\dagger}(a^\dagger a + p - j). \tag{5}$$

It is interesting that the operator $f(N-m)[1-m/(N+1)]a$ in Eq.(4) annihilates both the vacuum state $|0\rangle$ and Fock state $|m\rangle$. The states between the vacuum state and Fock state $|m\rangle$ are not annihilated. To discuss the case of the PANLCS $|m, \alpha, f\rangle$ let

$$A = f(N-m)[1-m/(N+1)]a, A^\dagger = a^\dagger f(N-m)[1-m/(N+1)] \tag{6}$$

We construct sector S_0 by repeated applying $A^\dagger$ on the vacuum state. S_0 is the set $|i\rangle, i = 0, 1, 2, ..., m-1$ and it is of finite dimension. The sector S_m, built by the repeated application of $A^\dagger$ on $|m\rangle$, is the set $|i\rangle, i = m, m+1, ...$ and it is of infinite dimension. Hence we can construct an operator $G^\dagger$ such that $[A, G^\dagger] = 1$ holds in S_m. To construct $G^\dagger$, we set $p = 1$ and $j = m$ in Eq.(5) and this yields

$$G^\dagger = a^\dagger \frac{1}{f(N-m)} \tag{7}$$

In fact, by direct verification, we have

$$[f(N-m)[1 - m/(N+1)]a, a^\dagger \frac{1}{f(N-m)}] = 1. \tag{8}$$

Therefore the PANLCS can be written as

$$|m, \alpha, f\rangle = \exp(G^\dagger)|m\rangle = \exp[a^\dagger \frac{1}{f(N-m)}]|m\rangle \tag{9}$$

up to a normalization constant. From the above equation it is shown that the PANLCS can be viewed as nonunitarily deformed Fock(number) state $|m\rangle$.

3.The PANLCS with negative m

The form of A, given by Eq.(6), suggests that it is a well-defined operator-valued function also for negative values of m on the Fock space. In this section the PANLCS with negative m is constructed. Denoting the the PANLCS with negtive m by $|-m, \alpha, f\rangle$, the equation to determine them are

$$f(N+m)[1 + m/(N+1)]a|-m, \alpha, f\rangle = \alpha|-m, \alpha, f\rangle. \tag{10}$$

The operator $A = f(N+m)[1 + m/(N+1)]a$ only annihilates the vacuum state. When $f(N) \equiv 1$, the state $|-m, \alpha, f\rangle$ reduces to that studied in Ref. [8]. The sector S_0, built by the repeated application of $A^\dagger = a^\dagger f(N+m)[1+m/(N+1)]$ on $|0\rangle$, is the set $|i\rangle, i = 0, 1, ...$

and it is just the infinite dimensional Fock space. To construct $G^\dagger$, corresponding to the operator $A = f(N+m)[1+m/(N+1)]a$, we set $p = 1$ and $j = 0$ in Eq.(5) and this yields

$$G^\dagger = a^\dagger \frac{N+1}{f(N+m)(N+m+1)} \tag{11}$$

Thus the PANLCS with negative m can be written as

$$|-m,\alpha,f\rangle = \exp(G^\dagger)|0\rangle = \exp[a^\dagger \frac{N+1}{f(N+m)(N+m+1)}]|0\rangle \tag{12}$$

up to a normalization constant. The state $|-m,\alpha,f\rangle$ is obtained by nonunitarily deforming the vacuum state $|0\rangle$ while the state $|m,\alpha,f\rangle$ is obtained by nonunitarily deforming the Fock state $|m\rangle$.

The PANLCS is obtained by the action of $a^{\dagger m}$ on the NLCS $|\alpha,f\rangle$. The state $|-m,\alpha,f\rangle$ can be written in a similar form using the inverse operators a^{-1} and $a^{\dagger -1}$. [16] These operators are defined in terms of their actions on the number state $|n\rangle$ as follows

$$\begin{aligned} a^{-1}|n\rangle &= \frac{1}{\sqrt{n+1}}|n+1\rangle, \\ a^{\dagger -1}|n\rangle &= \frac{1}{\sqrt{n}}|n-1\rangle, \\ a^{\dagger -1}|0\rangle &= 0. \end{aligned} \tag{13}$$

Using these inverse operators the state $|-m,\alpha,f\rangle$ can be rewritten as $|-m,\alpha,f\rangle = a^{\dagger -m}a^{-m}|\alpha,f'\rangle$ up to a normalization constant. Here $|\alpha,f'\rangle$ is the NLCS with the nonlinear function $f'(N) = f(N+m)$. The state $|-m,\alpha,f\rangle$ is obtained by the action of the operator $a^{\dagger -m}a^{-m}$ on the NLCS $|\alpha,f'\rangle$ while the state $|m,\alpha,f\rangle$ is obtained by the action of the operator $a^{\dagger m}$ on the NLCS $|\alpha,f\rangle$.

From Eq.(12) the number state expansion of the PANLCS with negative m can be easily obtained as

$$|-m,\alpha,f\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n \sqrt{n!}}{f(n+m-1)...f(0)(n+m)!}|n\rangle \tag{14}$$

up to a normalization constant. The expansion is useful in the following discussions.

4.Photon-added geometric state with negative m

In this section we consider a special example of the PANLCS with negative m, the photon-added geometric state with negative m.

The geometric state is defined as [10]

$$|\eta\rangle = \eta^{1/2}\sum_{n=0}^{\infty}(1-\eta)^{n/2}|n\rangle,\ 0<\eta<1, \tag{15}$$

It satisfies

$$\frac{1}{\sqrt{N+1}}a|\eta\rangle = \sqrt{1-\eta}|\eta\rangle. \tag{16}$$

In comparison with Eq.(1), we see that the geometric state is an NLCS with the nonlinear function $1/\sqrt{N+1}$. The photon-added geometric state is defined as

$$|m,\eta\rangle = \frac{a^{\dagger m}|\eta\rangle}{\langle\eta|a^m a^{\dagger m}|\eta\rangle} \tag{17}$$

$$= \eta^{(m+1)/2}\sum_{n=0}^{\infty}\binom{m+n}{n}^{n/2}(1-\eta)^{n/2}|n\rangle, \tag{18}$$

which is just the negative binomial state introduced by Barnett [12]. We have studied the statistical properties and algebraic characteristics of the photon-added geometric state in detail [13]. From Eqs.(4) and (16) we get

$$\frac{\sqrt{N-m+1}}{N+1}a|m,\eta\rangle = \sqrt{1-\eta}|m,\eta\rangle. \tag{19}$$

The state $|m,\eta\rangle$ is an NLCS with the nonlinear function $f(N)=\sqrt{N-m+1}/(N+1)$. When $m=0$, Eq.(19) reduces to Eq.(16) as we expected.

We would like to study the state $|-m,\eta\rangle$, the photon-added geometric state with negative values of m, which satisfies

$$\frac{\sqrt{N+m+1}}{N+1}a|-m,\eta\rangle = \sqrt{1-\eta}|-m,\eta\rangle. \tag{20}$$

From Eq.(14), the number state expansion of the state $|-m,\eta\rangle$ is given by

$$|-m,\eta\rangle = \sqrt{\frac{m!}{{}_2F_1(1,1;m+1;1-\eta)}}\sum_{n=0}^{\infty}(1-\eta)^{n/2}\sqrt{\frac{n!}{(n+m)!}}|n\rangle, \tag{21}$$

where ${}_2F_1(1,1;m+1;1-\eta)$ is the hypergeometric function.

The photon statistics of a quantum state can be conveniently studied by Mandel's Q-parameter [17]

$$Q = \frac{\langle N^2\rangle - \langle N\rangle^2 - \langle N\rangle}{\langle N\rangle} \tag{22}$$

A negative Q indicates that the photon number distribution is sub-Poissonian and it is a nonclassical feature. A positive Q indicates the super-Poissonian distribution and Q=0 indicates Poissonian distribution. The photon-added geometric state $|m,\eta\rangle$ can be sub-Poissoian depending on the parameter η [13]. For the state $|-m,\eta\rangle$(Eq.(21)), the mean value of N^k is easily obtained as

$$\langle N^k\rangle = \frac{m!}{{}_2F_1(1,1;m+1;1-\eta)}\sum_{n=0}^{\infty}n^k(1-\eta)^n\frac{n!}{(n+m)!}. \tag{23}$$

In Fig.1 the Q-parameter, calculated using Eqs.(22) and (23), for the state $|-m,\eta\rangle$ is shown as a function of η. The Q-parameter is always greater than zero indicating that they are super-Poissonian. For larger values of η, the Q-parameter is close to zero since the state $|-m,\eta\rangle$ reduces to $|0\rangle$ in the limit $\eta\rightarrow 1$.

5.Squeezing in $|-m,\eta\rangle$

Define the quadrature operators X(coordinate) and Y (momentum) by

$$X = \frac{1}{2}(a+a^\dagger), Y = \frac{1}{2i}(a-a^\dagger). \tag{24}$$

Then their variances

$$Var(X) = \langle X^2\rangle - \langle X\rangle^2, Var(Y) = \langle Y^2\rangle - \langle Y\rangle^2 \tag{25}$$

obey the Heisenberg's uncertainty relation

$$Var(X)Var(Y) \geq \frac{1}{16}. \tag{26}$$

If one of the variances is less than 1/4, the squeezing occurs. In the present case, $\langle a \rangle$ and $\langle a^2 \rangle$ are real. Thus, the variances of X and Y can be written as

$$Var(X) = \frac{1}{4} + \frac{1}{2}(\langle a^\dagger a \rangle + \langle a^2 \rangle - 2\langle a \rangle^2), \tag{27}$$

$$Var(Y) = \frac{1}{4} + \frac{1}{2}(\langle a^\dagger a \rangle - \langle a^2 \rangle). \tag{28}$$

From Eq.(21), the expectation value $\langle -m, \eta | a^k | - m, \eta \rangle$ is directly obtained as

$$\langle -m, \eta | a^k | - m, \eta \rangle = \frac{m!}{{}_2F_1(1, 1; m+1; 1-\eta)} \sum_{n=0}^{\infty} (1-\eta)^{n+k/2} \frac{(n+k)!}{\sqrt{(n+m)!(n+m+k)!}} \tag{29}$$

The variances of X and Y can be calculated from Eqs.(27), (28) and (29). In Fig.2 we show the variances of the quadrature operators X and Y as a function of η for different values of m. The squeezing exists in the quadrature Y. For the quadrature Y, the degree of the squeezing becomes deep with the increase of the parameter m. In the limit $\eta \to 1$, the variances are all equal to 1/4. This is because the state $| - m, \eta \rangle$ reduces the vacuum state $|0\rangle$ in this limit.

5. Photon-added two-photon nonlinear coherent states

In this section, we investigate the photon-added two-photon nonlinear coherent states. The two-photon nonlinear coherent state is defined as [18]

$$F(N)a^2|\alpha, F\rangle = \alpha|\alpha, F\rangle, \tag{30}$$

and the corresponding photon-added two-photon nonlinear coherent state is

$$|\alpha, F, m\rangle = a^{\dagger m}|\alpha, F\rangle \tag{31}$$

up to a normailization constant.

Acting the operator $a^2 a^{\dagger m}$ on Eq.(30) from the left, we obtain

$$F(N - m + 2)a^2 a^{\dagger m} a^2 |\alpha, F\rangle = \alpha(N+1)(N+2)a^{\dagger(m-2)}|\alpha, F\rangle. \tag{32}$$

Since

$$a^2 a^{\dagger m} a^2 = (N+4-m)(N+3-m)a^2 a^{\dagger(m-2)}, \tag{33}$$

we obtain

$$F(N-m+2)(N+4-m)(N+3-m)a^2 a^{\dagger(m-2)}|\alpha, F\rangle = \alpha(N+1)(N+2)a^{\dagger(m-2)}|\alpha, F\rangle. \tag{34}$$

Let $m-2 \rightarrow m$ in the above equation and note that the operator $(N+1)(N+2)$ is positive in the whole Fock space, we get

$$F(N-m)(1-\frac{m}{N+2})(1-\frac{m}{N+1})a^2|\alpha, F, m\rangle = \alpha|\alpha, F, m\rangle. \tag{35}$$

This shows that the photon-added nonlinear coherent states $|\alpha, F, m\rangle$ are still nonlinear coherent states with the nonlinear function

$$F(N-m)(1-\frac{m}{N+2})(1-\frac{m}{N+1}). \tag{36}$$

Since the squeezed vaccum state and squeezed first Fock state are two-photon nonlinear coherent state [18], we conclude that the photon-added squeezed vaccum state and photon-added squeezed first Fock state are also two-photon nonlinear coherent states as discussed in Ref. [19]. We can also introduce the photon-added two-photon nonlinear coherent states with negative m and make a similar discussion as one-photon case. We will not explicitly present them here.

6.Conclusions

In conclusion, we have studied a special NLCSs, the PANLCSs. From the PANLCS we introduce a new type of quantum state, the PANLCS with negative values of m. The states corresponding to the positive and negative values of m are shown to be the result of nonunitarily deforming the number states $|m\rangle$ and $|0\rangle$, respectively. As a example, we study the sub-Poissonian statistics and squeezing effects in the photon-added geometric state with negative values of m in detail. The results shows that photon-added geometric state with negative values of m are always super-Poissonian and the state can be squeezed in the quadrature Y. We also consider the photon-added two-photon nonlinear coherent states and find a similar concusion as one-photon case, i.e, the photon-added two-photon nonlinear coherent states are still two-photon nonlinear coherent states with certain nonlinear functions.

Acknowledgment: The author thanks for the discussions with Prof. Hong-Chen Fu and help of Prof.Chang-Pu Sun, Shao-Hua Pan, Guo-Zhen Yang. The work is partially supported by the National Science Foundation of China with grant number:19875008.

Figure Captions:

Figure1, Mandel's Q parameter as a function of η for different values of m.

Figure2, Variances of the quadrature operators X and Y as a function of η for different values of m.

REFERENCES

[1] Matos Filho R L de and Vogel W, 1996 *Phys.Rev.*A54 4560.

[2] Man,ko V I,Marmo G ,Sudarshan E C G,
and Zaccaria F, 1997 *Physica Scripta*, **55** 528;
Man,ko O V,1997 *Phys.Lett.*A228 29;
Mancini S,1997 *Phys.Lett.*A233 291;
Roy B,1998 *Phys.Lett.*A249 25.

[3] Agarwal G S and Tara K, 1991 *Phys.Rev.*A43 492.

[4] Dodonov V V, Marchiolli M A, Korennoy Ya A, Man'ko V I and Moukhin Y A 1998 *Phys.Rev.*A58 4087.

[5] Zhang Z and Fan H, 1992 *Phys.Lett.* **A165** 14;
Welsch D.-G., Dakna M, Knöll L, and Opatrny, Los Alamos e-print quant-ph/9708018.

[6] Dodonov V V, Korennoy Y A, Man'ko V I, and Moukhin Y A, 1996 *Quantum Semiclassic. Opt.* **8** 413.

[7] Agarwal G S and Tara K, 1992*, Phys.Rev.*A46 485;
Jones G N, Haight J and Lee C T 1997 *Quantum Semiclassic. Opt.* **9** 411.

[8] Sivakumar S 1999 *J.Phys.A:Math.Gen.* **32** 3441.

[9] Wang X G and Fu H C 1999 *Mod.Phys.Lett.B* **13** 617.

[10] Shapiro J H and Shephard S R, 1991 *Phys.Rev.Lett.* **62** 2377;
Shapiro J H and Shephard S R, 1991 *Phys.Rev.* A **43** 3795;
Lerner E C, Huang H W and Walters G E, 1970 *J.Math.Phys.* **11** 1679;
Hall M J W, 1993 *J.Mod.Opt.* **40** 809;
Obada A -S F, Hassan S S, Puri R R and Abdalla M S 1993 *Phys.Rev.* **A48** 3174;

Obada A -S F, Yassin O M and Barnett S M, 1997 *J.Mod.Opt.* **44** 149.

[11] Joshi A and Lawande S V , 1989 *Opt.Commun.* **70** 21;

Matsuo K, 1990 *Phys.Rev.* **A41** 519;

Joshi A and Lawande S V, 1991 *J.Mod.Opt.* **38** 2009;

Agarwal G S, 1992 *Phys.Rev.* **A45** 1787;

[12] Barnett S M 1998 *J.Mod.Opt.* **45** 2201.

[13] Wang X G, Pan S H and Yang G Z, *Euro. Phys. J. D*, to appear.

[14] Wang X G, Fu H C *Int.J.Theor.Phys.* to appear.

[15] Shanta P, Chaturvedi S, Srinivasan V, Agarwal G S and Mehta C L 1994 *Phys.Rev.Lett.* **72** 1447.

[16] Mehta C L, Roy A K and Saxena G M 1992 *Phys.Rev.***A 46** 1565.

[17] Mandel L 1979 *Opt.Lett.* **4** 205.

[18] Sivakumar S 1998 *Phys. Lett. A* **250** 257.

[19] Liu N L, Sun Z H and Fan H Y 2000 *J. Phys. A* **33** 1933

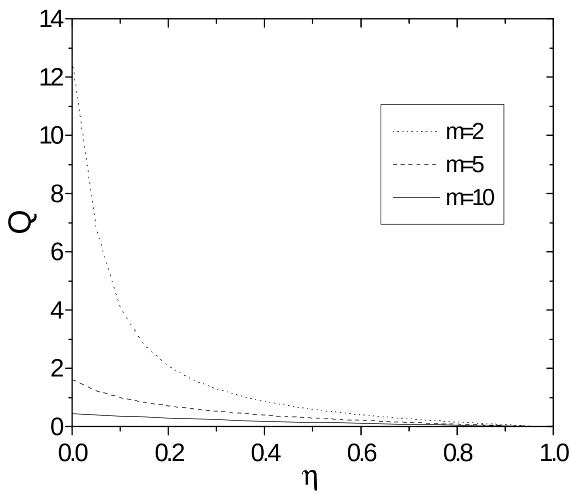
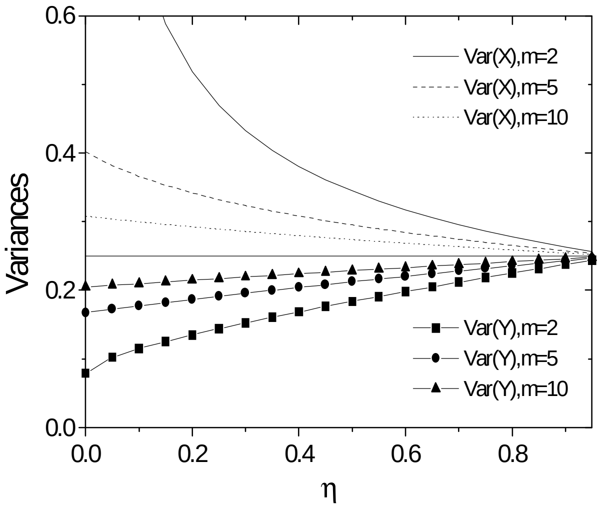


Fig.1



0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
Variances
η
Var(X),m=2
Var(X),m=5
Var(X),m=10
Var(Y),m=2
Var(Y),m=5
Var(Y),m=10

Fig.2