COMMENT ON "QUANTUM PHASE SHIFT CAUSED BY SPATIAL CONFINEMENT"

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Received 3 September 1999; revised 22 November 1999

The analysis of phase shifts in executed and proposed interferometry experiments on photons and neutrons neglected forces exerted at the boundaries of spatial constrictions. When those forces are included it is seen that the observed phenomena are not in fact geometric in nature. A new proposal in the reply to this comment avoids that pitfall.

Key Words: interferometry, phase shifts, force-free effects.

1. INTRODUCTION

Allman *et al.* [1] have proposed a neutron interference experiment wherein the neutrons in one arm of an interferometer pass through a channel in an otherwise reflecting barrier and the resulting phase shift is to be measured. Following an earlier discussion [2], they calculate that the phase shift expected to be induced by the neutrons' passage through the channel in the barrier will be given for an appropriate range of the parameters by

$$\Delta\Phi \approx \frac{\pi}{4}\,\frac{\lambda\ell}{a^2} = \frac{\pi^2\hbar\ell}{2a^2\sqrt{2mE}}\ , \tag{1}$$

where ℓ and a are the length and width of the channel and λ, m, and E are the wavelength, mass, and energy of the neutrons. They assert that no force is exerted on the neutrons and from that they conclude that the proposed experiment will demonstrate a new, purely geometrical, force-free effect of the Aharonov-Bohm type.

That conclusion is not correct. The phase shifts induced by force-free interactions are necessarily independent of the energy of the neutrons [3,4,5], contrary to Eq. (1). The neutrons in the proposed experiment are in fact acted on by forces having non-vanishing components in the direction of the beam.

2. FORCES ON THE NEUTRON

In reality, any forces on the neutrons are exerted in the neutrons' exchange of momentum with atoms in the barrier. Allman *et al.* substitute a boundary condition for the interaction of the neutrons with the atoms in the barrier. That approximation gives an adequate wave function and leads to the correct phase shift. No potential gradient appears in the Schroedinger equation, but Allman *et al.*, in saying that no force is exerted, neglect the force exerted on the neutrons at the boundary.

To focus on the principle involved, consider an idealized situation in which a single neutron, initially in a state represented by some wave packet $\psi(t)$, is aimed at a long channel so that there is a time interval when the neutron's wave packet is for practical purposes entirely within the channel. In the best case, the wave packet enters the channel with only minimal reflection, proceeds through the channel at reduced speed as described by Ref. 1, then exits the channel, again with minimal reflection, and continues on with its initial speed. Although the reflection is minimal, the wave function unavoidably spreads into some diffraction region around the ends of the channel. There the boundary exerts a force F_b in the beam direction whose expectation is given by

$$\begin{aligned} \left\langle F_b \right\rangle_t &= \frac{d}{dt}\left\langle p_b \right\rangle = i\hbar \int d^3x \left(\frac{\partial \psi^*}{\partial t} \nabla_b \psi - \frac{\partial \psi}{\partial t} \nabla_b \psi^* \right) \\ &= \mp \frac{\hbar^2}{2m} \int d^2x \left| \nabla_b \psi \right|^2 , \end{aligned} \tag{2}$$

where the two-dimensional integral is carried over the boundary segments normal to the beam direction, *i.e.* over the surfaces of the barrier outside of the channel. In Eq. (2), the minus sign applies to the first face of the barrier encountered by the neutron and the plus sign to the second face, corresponding to the neutron's losing momentum when it enters the channel and regaining that momentum when it leaves. Ehrenfest's theorem guarantees that the force given in Eq. (2) accounts correctly for the reduced momentum p' in the channel, correctly given by Allman *et al.* as

$$p' = p - \Delta p \approx p - \frac{\pi^2 \hbar^2}{2a^2 p} , \tag{3}$$

where p is the free-space momentum before and after the passage through the channel. In the case of a neutron that misses the channel and is reflected from the barrier, the time integral of the force given in Eq. (2) agrees with a net momentum transfer equal to $-2p$, as it must.

That the momentum shift is brought about by the force exerted on the neutron at the boundary is especially clear when the wave packet exits the channel, because there the negligible momentum carried in the small reflected wave is directed oppositely to the momentum transferred to the neutron by the force at the boundary, so the reflected wave cannot carry off the transferred momentum.

The same physics appears in a more physical model where the barrier is represented by a finite repulsive potential instead of by a boundary condition. If the potential is taken to be rounded at the edge of the barrier, a finite force appears in the Schroedinger equation in the form of the gradient of the potential. If the potential ends with a step, the force becomes a delta function. Although the details of the force vary with those of the model, the momentum the force imparts to the neutron is in all cases the same as that imparted by the boundary in the boundary-condition model.

3. CONCLUSIONS

This phenomenon stands in contrast to genuinely force-free interference phenomena [3,4,5], in which the phase shift is independent of the energy. It is also very different from the Aharonov-Bohm magnetic scattering effect, in which electrons are scattered from a solenoid containing a magnetic flux. There too the interaction of the beam particles with the atoms of the solenoid can adequately be represented by a boundary condition and the force at the boundary accounts correctly for the momentum change when the electrons are scattered [6]. The interest in that version of the Aharonov-Bohm effect arises from the fact that the scattering, and with it the momentum transfer, depends upon the magnitude of the magnetic flux in the solenoid even though there is no force and no momentum transfer between the electrons and the local, present magnetic field. As expected, the magnetic-flux-dependent part of the phase shift is independent of the electron's energy. No analogous consideration arises when neutrons pass through a channel in a barrier.

These considerations apply equally to the optical interference experiments in Ref. [1] as to the proposed neutron experiment, but they do not apply to the temporally modulated constriction in a new experiment proposed by Allman *et al.* [7], which involves no force in the direction of motion of the wave packet. That proposed experiment falls into the force-free class that includes the Aharonov-Casher effect, the Scalar Aharonov-Bohm effect, and the force-free nuclear phase shifter but not the electric and magnetic Aharonov-Bohm effects [4,8].

Acknowledgements. This work is supported by the U.S. Department of Energy, Nuclear Physics Division, under contract W-31-109-ENG-38. I thank Brendan Allman for informing me about the new proposed experiment.

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