New strategy for suppressing decoherence in quantum computation

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Abstract: Controlable strong interaction of the qubit's bath with an external system (i.e. with the bath's environment) allows for choosing the conditions under which the decoherence of the qubit's states can be substantially decreased (in a certain limit: completely avoided). By "substantially decreased" we mean that the correlations which involve the bath's states prove negligible, while the correlations between the qubit's and the environment's states can be made ineffective during a comparatively long time interval. So, effectively, one may choose the conditions under which, for sufficiently long time interval, the initial state of "qubit + bath" remains unchanged, thus removing any kind of the errors. The method has been successfully employed in the (simplified) model of the solid-state-nuclear quantum computer (proposed by Kane).

1. Introduction

The issue of decoherence in quantum computation is one of the central subjects in both fundamentals and practical realizability of the quantum computers.

Here we propose a new strategy (method) for substantial suppression of decoherence in quantum computers for arbitrary type of the errors. As opposite to the existing approaches/methods [1], our strategy is simple both conceptualy and mathematicaly.

Physicaly, the *central idea* of our approach relies on the following assumptions: **(i)** extension of the composite system "qubit + bath (Q+B)" with the bath's environment, so obtaining the new composite system "qubit + bath + (bath's) environment (Q+B+E)", and **(ii)** assumption of existence of *robust states* of the bath which can be selected (and kept (approximately) unchanged) by *controlable strong* (quantum-measurement-like) interaction of the bath with its environment.

Given the points (assumptions) (i) and (ii), the standard *stationary perturbation theory* points out the next possibiity: with a proper choice of the initial state of the bath, one may obtain *substantial suppression of decoherence*: the correlations involving the bath's states are "(virtually) arbitrarily small", while for comparatively long time interval the initial state of Q+B remains unchanged, thus removing any kind of the errors!

We give a brief account of the issues naturally appearing in this context, appointing (without details) that the method works in the (simplified) model of the solid-state-nuclear quantum computer.

2. General background

We employ the standard assumptions of the general decoherence theory [2,3]: (a) initially, the Q+B+E system is decoupled, and (b) the dominant terms are the interaction-Hamiltonians, $\hat{H}_{QB} + \hat{H}_{BE}$. Then we apply the standard algebra [2,3], based on the direct use of the unitary time-evolution of the composite system Q+B+E.

One should remind that the standard models of the theory of decoherence (including the considerations in quantum computation) deal with the composite system Q+B, where the interaction $\hat{H}_{QB}$ usually is of the type (of the separable kind [3]):

$$\hat{H}_{QB} = c \sum_{p,q} \gamma_{pq} \hat{P}_{Qp} \otimes \hat{\Pi}_{Bq}, \tag{1}$$

where $\hat{P}_{Qp}$ and $\hat{\Pi}_{Bq}$ represent the projectors onto the corresponding subspaces of the Hilbert state-spaces of Q, and of B, respectively; c represents the coupling constant.

Then the decoherence is loosely (but sufficiently) presented by the (time) decrease of the off-diagonal elements of the Q's "density matrix":

$$\rho_{Qpp'} = C_p C^*_{p'} z_{pp'}(t), \tag{2a}$$

where the "correlation amplitude" $z_{pp'}(t)$" [2]:

$$z_{pp'}(t) = \sum_q p_q \exp(-\imath ct(\gamma_{pq} - \gamma_{p'q})/\hbar), \quad \sum_q p_q = 1; \tag{2b}$$

notice the dependence on the eigenvalues of $\hat{H}_{QB}$.

It is important to note that in the context of the, so-called, *macroscopic considerations* [4], the states enumerated by the indeces p, p' - which are the elements of the "pointer basis" [2] - are considered to be *robust under the influence of the environment.* I.e., most of the basic assumptions necessary for the "transition from quantum to classical" [5, 6] presuppose invariance of the "pointer basis" under the transformations generated by the interaction Hamiltonian. In idealized form it reads:

$$\hat{H}_{QB} |\Psi_p\rangle_Q \otimes |\chi\rangle_B = |\Psi_p\rangle_Q \otimes |\chi'\rangle_B, \tag{3}$$

where $|\Psi_p\rangle_Q$ is an element of the "pointer basis" - i.e., of the set of states bearing the (semi)classical character [2-4] of an open quantum system.

3. The strategy

We extend the system Q+B by the bath's environment (E): thus dealing with the new composite system Q+B+E.

The methodology distingusihed in Section 2 points to the following assumptions: (i) initially, the system is in uncorrelated state $|\Psi\rangle_Q \otimes |0\rangle_B \otimes |\chi\rangle_E$, and (ii) the unitary-evolution operator $\hat{U}_{QBE}$ can be written as:

$$\hat{U}_{QBE} \cong \exp(-\imath t(\hat{H}_{QB} + \hat{H}_{BE})/\hbar). \tag{4}$$

These *standard assumptions* are extended by the following *crucial assumptions*: **(A)** The interaction Hamiltonian $\hat{H}_{BE}$:

$$\hat{H}_{BE} = C \sum_{i,j} \kappa_{ij} \hat{P}_{Bi} \otimes \hat{\Pi}_{Ej}, \tag{5}$$

(compare to eq.(1)) is the *dominant term*, thus making $\hat{H}_{QB}$ *the perturbation* (C is the coupling constant), and **(B)** The initial state $|0\rangle_B$ can be chosen such that one may state (cf. eq. (3)):

$$\hat{H}_{BE}|0\rangle_B \otimes |\chi\rangle_E = |0\rangle_B \otimes |\chi'\rangle_E. \tag{6}$$

Then one may employ the standard stationary perturbation theory [7].

3.1 The perturbation theory employed

The basic idea of the perturbation theory [7] is presented by the following expressions::

$$\hat{H}_{BE}|\Phi_n\rangle_{QBE} = E_n^{(0)}|\Phi_n\rangle_{QBE}, \tag{7}$$

$$(\hat{H}_{BE} + \hat{H}_{QB})|\Psi_n\rangle_{QBE} = E_n|\Psi_n\rangle_{QBE}, \tag{8}$$

where in the limit $c \to 0$ one has: $|\Psi_n\rangle_{QBE} \to |\Phi_n\rangle_{QBE}$, and $E_n \to E_n^{(0)}$. (Remind that: the coupling constant c is given and the above limit should not be literaly understood (it is here for the formal completeness of the considerations); what we shall further need is the ratio of the coupling constants c/C, where the limit $c/C \to 0$ is legitimate.)

As it directly follows from eq.(5), the states $|\Phi\rangle_{QBE}$ can be chosen as $|pij\rangle \equiv |p\rangle_Q \otimes |i\rangle_B \otimes |j\rangle_B$. Then, in accordance with (7) and (8) one may write for the normalized eigenstates:

$$|\Psi_n\rangle_{QBE} \equiv |\Psi_{pij}\rangle_{QBE} = (1 - \epsilon_{pij}^2)^{1/2}|pij\rangle + \epsilon_{pij}|\chi_{pij}\rangle, \tag{9}$$

where $\langle pij|\chi_{pij}\rangle = 0$ and $\langle \chi_{pij}|\chi_{pij}\rangle = 1$, and

$$E_n \equiv E_{pij} = E_{pij}^{(0)} + \lambda_{pij} \equiv C\kappa_{ij} + \lambda_{pij}. \tag{10}$$

Notice: the corrections of the $\hat{H}_{BE}$'s eigenstates and eigenvalues are $\epsilon_{pij}|\chi_{pij}\rangle$ and λ_{pij}, respectively, while eqs.(9, 10) are *exact*!

Then from eq. (4), (9), (10), one directly obtains:

$$\hat{U}_{QBE} \cong \hat{U}_1 + \hat{U}_2, \tag{11a}$$

where

$$\hat{U}_1 = \sum_{(pij)} \exp(-\imath t E_{pij}/\hbar)(1 - \epsilon_{pij}^2)|pij\rangle\langle pij|, \tag{11b}$$

where the sum runs over the different combinations of the indeces "p, i, j". Bearing in mind that $\|\hat{U}_1 + \hat{U}_2\| = 1$, it is a matter of straightforward algebra to prove that:

$$1 = n_1 + n_2, \quad n_1 = \langle\Psi|\hat{U}_1^\dagger \hat{U}_1|\Psi\rangle, \tag{12}$$

and

$$n_1 \geq (1 - \epsilon_{max}^2)^2, \tag{12b}$$

where ϵ_{max} is the maximal value of ϵs defined by eq.(9).

So, applying $\hat{U}_{QBE}$ onto the initial state $|\Psi\rangle_Q \otimes |0\rangle_B \otimes |\chi\rangle_E$, one obtains:

$$\hat{U}_{QBE}|\Psi\rangle_Q \otimes |0\rangle_B \otimes |\chi\rangle_E \cong \sum_{(pij)} C_p \alpha_i \beta_j |pij\rangle + O(\epsilon_{max}), \tag{13}$$

where $C_p = \langle p|\Psi\rangle$, $\alpha_i = \langle i|0\rangle$, $\beta_j = \langle j|\chi\rangle$, and the bases $\{|i\rangle_B\}$ and $\{|j\rangle_E\}$ diagonalize [3] $\hat{H}_{BE}$.

With the choice (cf. above point (ii)) presented by eq.(6), $\alpha_i = \delta_{ii_\circ}$, one obtains:

$$\hat{U}_{QBE}|\Psi\rangle_Q \otimes |0\rangle_B \otimes |\chi\rangle_E \cong \sum_{(pj)} C_p \beta_j \exp(-\imath t E_{p0j}/\hbar)|p0j\rangle + O(\epsilon_{max}). \tag{14}$$

Now, since

$$E_{p0j} = C\kappa_{0j} + \lambda_{p0j}, \tag{15}$$

and

$$|\lambda_{p0j}| \leq (1 - \epsilon^2_{p0j})^{-1/2} |\langle p0j|\hat{H}_{QB}|p0j\rangle| +$$
$$+ |\epsilon_{p0j}|(1 - \epsilon^2_{p0j})^{-1} |\langle p0j|\hat{H}_{QB}|\chi_{p0j}\rangle| \tag{16}$$

one obtains the *main result of this paper*:

if one may choose $|0\rangle_B$ so as the maximal value (λ_{max}) of $|\lambda_{poj}|$s can be very small, then one may speak of the *substantial suppression of decoherence*.

Actually, since it can be estimated that $\epsilon_{max} \sim c/C$, one may say that the correlations between Q and B (*and* E) can be considered arbitrarily small, the occurrence of the errors which come *together with the change of B's state* represent substantially rare events: the total probability of these errors not exceding the order of $(c/C)^2$, where C *is virtually arbitrary*. Now, for λ_{max} *very small*, one may write:

$$\hat{U}_{QBE}|\Psi\rangle_Q \otimes |0\rangle_B \otimes |\chi\rangle_E \cong \sum_{(pj)} C_p |p\rangle_Q$$
$$\otimes |0\rangle_B \otimes \sum_j \beta_j \exp(-\imath t C\kappa_{0j}/\hbar)|j\rangle_E$$
$$\equiv |\Psi\rangle_Q \otimes |0\rangle_B \otimes \sum_j \beta_j \exp(-\imath C t \kappa_{0j}/\hbar)|j\rangle_E. \tag{17}$$

for at least the time interval τ:

$$\tau \sim (\lambda_{max}/\hbar)^{-1}. \tag{18}$$

I.e. during this time interval, the correlations between the states of Q and E do not become effective. (Notice: the situation with this regard is even much better, for the correlations are "driven" by the second term on the r.h.s. of eq.(16)!)

3.2 Physical interpretation

The method directly points to the next *protocol for avoiding the errors in quantum computation*:

First, an effective, quantum-measurement-like action of the environment on the bath should be performed, which serves for preparing a robust initial state $|0\rangle_B$. Once such a state is obtained, the interaction between B and E should be strenghtened and prolonged in time, for at least the interval τ. If the interaction is sufficiently strong ($c/C \ll 1$), then, for the proper choice of the initial state of the bath, the above algebra guaranties that for the interval of the order of τ (eq.(18)), the initial state of Q+B would appear unchanged, which is sufficient for preparing the computations in the time intervals much shorter than τ. The same procedure should be repeated for each calculation step.

Being virtually arbitrary, ϵ_{max} and λ_{max} allow for substantial suppression of decoherence of the qubit's states, all the effects of decoherence referring to substantially rare events ($c/C \ll 1$), or falling far beyond the interval τ ($|0\rangle_B$ such that $\lambda_{max} \ll 1$).

Needless to say, in the limits $\epsilon_{max} \to 0$ and $\lambda_{max} \to 0$, the r.h.s. of eq.(17) *is exact*!

4. Discussion

The strategy can be elaborated along the following lines:

(1) Generalizations concerning the interactions of the qubits themselves, likewise the self-Hamiltonians of Q, B, and E.

(2) Existence of unknown part of the bath; i.e., that E interacts with B_1, but not with B_2 (the real bath then would be $B = B_1 + B_2$)

(3) Extension of the (well known) bath B, if it is not sufficiently "macroscopic" as to provide us with the robust states

(4) Existence of the common bath for all the qubits

(5) Avoiding the interaction of Q and E in the realistic situations

(6) Considerations of the "classical environment(E)" [8], inculding the "mixed" initial states of B and E.

As regards the points (1)-(4), the results are encouraging.

The work is in progress as regards the points (5,6).

The *method has been successfully employed in the simplified model of the solid-state-nuclear* quantum computer proposed by Kane[9]. The details will be presented elsewhere.

5. Conclusion

The decoherence in quantum computers can be, at least in principle, suppressed. The idea is to properly, strongly "press" the qubit(s)'s bath, and to produce: the stochastic change of the initial state of Q+B represents an improbable event (in the limit $c/C \to 0$ it is stochastically impossible), and for sufficiently long time interval this state remains unchanged. So, one may say that we use decoherence (on B), to combat decoherence (on Q).

In practical realizations one should try to choose the interaction of B and E which should be very strong ($C \gg c$), and such the initial state of B so as to one may state $|\langle p0j|\hat{H}_{QB}|p0j\rangle| \ll 1$. **These choices are really a matter of the particular model !** The preliminarysuccess with the solid-state-nuclear computer is encouraging.

After the interval τ, the correlations between Q and E become effective, leading to decoherence presented by:

$$\rho_{Qpp'} = C_p C^*_{p'} z_{pp'}(t),$$

where (compare to eq.(2a)):

$$z_{pp'}(t) = \sum_j |\beta_j|^2 \exp(-\imath t(\lambda_{p0j} - \lambda_{p'0j})/\hbar)$$

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