Quasi-Elastic Barrier Distribution as a Tool for Investigating Unstable Nuclei

K. Hagino
Department of Physics, Tohoku University, Sendai 980-8578, Japan

and N. Rowley
Institut de Recherches Subatomiques, UMR7500, IN2P3-CNRS/Universtité Louis Pasteur, BP28, F-67037 Strasbourg Cedex 2, France

Received on 14 November, 2004

The method of fusion barrier distribution has been widely used to interpret the effect of nuclear structure on heavy-ion fusion reactions around the Coulomb barrier. We discuss a similar, but less well known, barrier distribution extracted from large-angle quasi-elastic scattering. We argue that this method has several advantages over the fusion barrier distribution, and offers an interesting tool for investigating unstable nuclei.

I. INTRODUCTION

It has been well recognized that heavy-ion collisions at energies around the Coulomb barrier are strongly affected by the internal structure of colliding nuclei [1, 2]. The couplings of the relative motion to the intrinsic degrees of freedom (such as collective inelastic excitations of the colliding nuclei and/or transfer processes) results in a single potential barrier being replaced by a number of distributed barriers. It is now well known that a barrier distribution can be extracted experimentally from the fusion excitation function $\sigma_{\rm fus}(E)$ by taking the second derivative of the product $E\sigma_{\rm fus}(E)$ with respect to the center-of-mass energy E, that is, $d^2(E\sigma_{\rm fus})/dE^2$ [3]. The extracted fusion barrier distributions have been found to be very sensitive to the structure of the colliding nuclei [1, 4], and thus the barrier distribution method has opened up the possibility of exploiting the heavy-ion fusion reaction as a "quantum tunneling microscope" in order to investigate both the static and dynamical properties of atomic nuclei.

The same barrier distribution interpretation can be applied to the scattering process as well. In particular, it was suggested in Ref. [5] that the same information as the fusion cross section may be obtained from the cross section for quasi-elastic scattering (a sum of elastic, inelastic, and transfer cross sections) at large angles. Timmers *et al.* proposed to use the first derivative of the ratio of the quasi-elastic cross section $\sigma_{\rm qel}$ to the Rutherford cross section σ_R with respect to energy, $-d(d\sigma_{\rm qel}/d\sigma_R)/dE$, as an alternative representation of the barrier distribution [6]. Their experimental data have revealed that the quasi-elastic barrier distribution is indeed similar to that for fusion, although the former may be somewhat smeared and thus less sensitive to nuclear structure effects (see also Refs.[7–9] for recent measurements). As an example, we show in Fig. 1 a comparison between the fusion and the quasi-elastic barrier distributions for the ^{16}O + ^{154}Sm system [10].

In this contribution, we undertake a detailed discussion of the properties of the quasi-elastic barrier distribution [10], which are less known than the fusion counterpart. We shall discuss possible advantagges for its exploitation, putting a particular emphasis on future experiments with radioactive beams.

II. QUASI-ELASTIC BARRIER DISTRIBUTIONS

Let us first discuss heavy-ion reactions between inert nuclei. The classical fusion cross section is given by,

$$\sigma^{cl}_{\rm fus}(E)=\pi R_b^2\left(1-\frac{B}{E}\right)\theta(E-B), \qquad (1)$$

where R_b and B are the barrier position and the barrier height, respectively. From this expression, it is clear that the first derivative of $E\sigma^{cl}_{\rm fus}$ is proportional to the classical penetrability for a 1-dimensional barrier of height B or eqivalently the s-wave penetrability,

$$\frac{d}{dE}[E\sigma^{cl}_{\rm fus}(E)]=\pi R_b^2\,\theta(E-B)=\pi R_b^2 P_{cl}(E), \qquad (2)$$

and the second derivative to a delta function,

$$\frac{d^2}{dE^2}[E\sigma^{cl}_{\rm fus}(E)]=\pi R_b^2\,\delta(E-B). \qquad (3)$$

In quantum mechanics, the tunneling effect smears the delta function in Eq. (3). If we define the fusion test function as

$$G_{\rm fus}(E)=\frac{1}{\pi R_b^2}\frac{d^2}{dE^2}[E\sigma_{\rm fus}(E)], \qquad (4)$$

this function has the following properties: i) it is symmetric around $E=B$, ii) it is centered on $E=B$, iii) its integral over E is unity, and iv) it has a relatively narrow width of around $\hbar\Omega\ln(3+\sqrt{8})/\pi\sim 0.56\hbar\Omega$, where $\hbar\Omega$ is the curvature of the Coulomb barrier.

We next ask ourselves the question of how best to define a similar test function for a scattering problem. In the pure classical approach, in the limit of a strong Coulomb field, the differential cross sections for elastic scattering at $\theta=\pi$ is given by,

$$\sigma^{cl}_{\rm el}(E,\pi)=\sigma_R(E,\pi)\,\theta(B-E), \qquad (5)$$

where $\sigma_R(E,\pi)$ is the Rutherford cross section. Thus, the ratio $\sigma^{cl}_{\rm el}(E,\pi)/\sigma_R(E,\pi)$ is the classical reflection probability $R(E)$ $(=1-P(E))$, and the appropriate test function for scattering is [6],

$$G_{\rm qel}(E)=-\frac{dR(E)}{dE}=-\frac{d}{dE}\left(\frac{\sigma_{\rm el}(E,\pi)}{\sigma_R(E,\pi)}\right). \qquad (6)$$

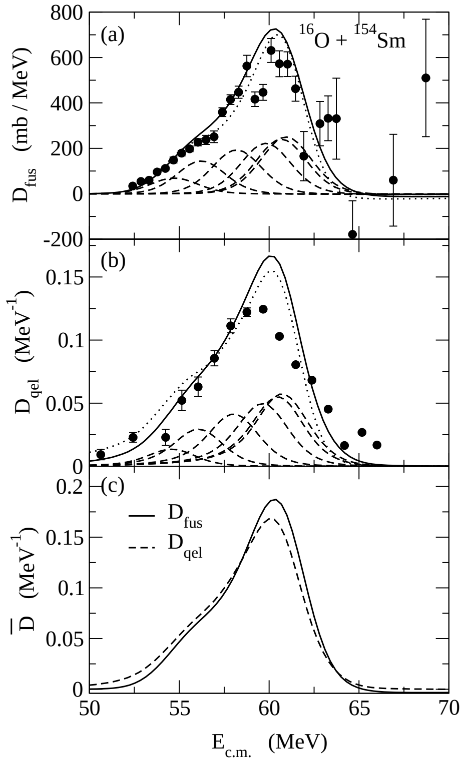

FIG. 1: (a) The fusion barrier distribution for the ^{16}O + ^{154}Sm reaction. The solid line is obtained with the orientation-integrated formula with $\beta_2 = 0.306$ and β_4= 0.05. The dashed lines indicate the contributions from the six individual eigenbarriers. These lines are obtained by using a Woods-Saxon potential with a surface diffuseness parameter a of 0.65 fm. The dotted line is the fusion barrier distribution calculated with a potential which has a = 1.05 fm. (b) Same as Fig. 1(a), but for the quasi-elastic barrier distribution. (c) Comparison between the barrier distribution for fusion (solid line) and that for quasi-elastic scattering (dashed line). These functions are both normalized to unit area in the energy interval between 50 and 70 MeV.

In realistic systems, due to the effect of nuclear distortion, the differential cross section deviates from the Rutherford cross section even at energies below the barrier. Using the semi-classical perturbation theory, we have derived a semi-classical formula for the backward scattering which takes into account the nuclear effect to the leading order [10]. The result for a scattering angle θ reads,

$$\frac{\sigma_{\rm el}(E,\theta)}{\sigma_R(E,\theta)} = \alpha(E,\lambda_c)\cdot|S(E,\lambda_c)|^2, \qquad (7)$$

where $S(E,\lambda_c)$ is the total (Coulomb + nuclear) S-matrix at energy E and angular momentum $\lambda_c = \eta\cot(\theta/2)$, with η being the usual Sommerfeld parameter. Note that $|S(E,\lambda_c)|^2$ is nothing but the reflection probability of the Coulomb barrier, $R(E)$. For $\theta = \pi$, λ_c is zero, and $|S(E,\lambda_c=0)|^2$ is given by

$$|S(E,\lambda_c=0)|^2 = R(E) = \frac{\exp\left[-\frac{2\pi}{\hbar\Omega}(E-B)\right]}{1+\exp\left[-\frac{2\pi}{\hbar\Omega}(E-B)\right]} \qquad (8)$$

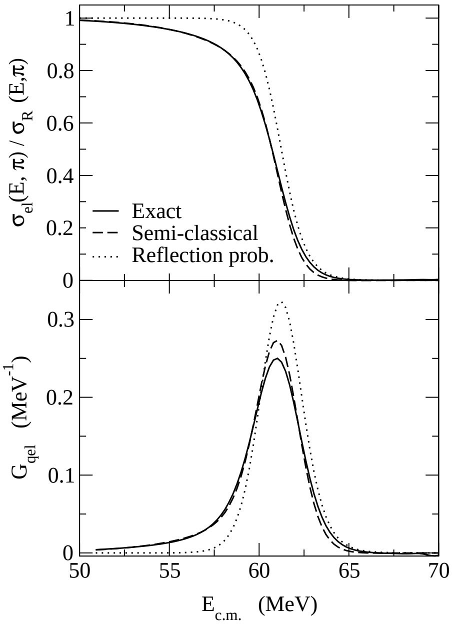

FIG. 2: The ratio of elastic scattering to the Rutherford cross section at $\theta = \pi$ (upper panel) and the quasi-elastic test function $G_{\rm qel}(E) = -d/dE(\sigma_{\rm el}/\sigma_R)$ (lower panel) for the ^{16}O + ^{144}Sm reaction.

in the parabolic approximation. $\alpha(E,\lambda_c)$ in Eq. (7) is given by

$$\alpha(E,\lambda_c) = 1+\frac{V_N(r_c)}{ka}\frac{\sqrt{2a\pi k\eta}}{E} \qquad (9)$$

$$\times\left[1-\frac{r_c}{Z_PZ_Te^2}\cdot 2V_N(r_c)\left(\frac{r_c}{a}-1\right)\right], \qquad (10)$$

where $k=\sqrt{2\mu E/\hbar^2}$, with μ being the reduced mass for the colliding system. The nuclear potential $V_N(r_c)$ is evaluated at the Coulomb turning point $r_c = (\eta+\sqrt{\eta^2+\lambda_c^2})/k$, and a is the diffuseness parameter in the nuclear potential.

Figure 2 shows an example of the excitation function of the cross sections and the corresponding quasi-elastic test function, $G_{\rm qel}$ at $\theta=\pi$ for the ^{16}O + ^{144}Sm reaction. Because of the nuclear distortion factor $\alpha(E,\lambda_c)$, the quasi-elastic test function behaves a little less simply than that for fusion. Nevertheless, the quasi-elastic test function $G_{\rm qel}(E)$ behaves rather similarly to the fusion test function $G_{\rm fus}(E)$. In particular, both functions have a similar, relatively narrow, width, and their integral over E is unity. We may thus consider that the quasi-elastic test function is an excellent analogue of the one for fusion, and we exploit this fact in studying barrier structures in heavy-ion scattering.

In the presence of the channel couplings, the fusion and the quasi-elastic cross sections may be given as a weighted sum of the cross sections for uncoupled eigenchannels,

$$\sigma_{\rm fus}(E) = \sum_\alpha w_\alpha\sigma_{\rm fus}^{(\alpha)}(E), \qquad (11)$$

$$\sigma_{\rm qel}(E,\theta) = \sum_\alpha w_\alpha\sigma_{\rm el}^{(\alpha)}(E,\theta), \qquad (12)$$

where $\sigma_{\rm fus}^{(\alpha)}(E)$ and $\sigma_{\rm el}^{(\alpha)}(E,\theta)$ are the fusion and the elastic cross sections for a potential in the eigenchannel α. These equations immediately lead to the expressions for the barrier distribution in terms of the test functions,

$$D_{\rm fus}(E) = \frac{d^2}{dE^2}[E\sigma_{\rm fus}(E)] = \sum_\alpha w_\alpha \pi R_{b,\alpha}^2 G_{\rm fus}^{(\alpha)}(E), \quad (13)$$

$$D_{\rm qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\rm qel}(E,\pi)}{\sigma_R(E,\pi)}\right) = \sum_\alpha w_\alpha G_{\rm qel}^{(\alpha)}(E). \quad (14)$$

III. ADVANTAGES OVER FUSION BARRIER DISTRIBUTIONS

There are certain attractive experimental advantages to measuring the quasi-elastic cross section $\sigma_{\rm qel}$ rather than the fusion cross sections $\sigma_{\rm fus}$ to extract a representation of the barrier distribution. These are: i) less accuracy is required in the data for taking the first derivative rather than the second derivative, ii) whereas measuring the fusion cross section requires specialized recoil separators (electrostatic deflector/velocity filter) usually of low acceptance and efficiency, the measurement of $\sigma_{\rm qel}$ needs only very simple charged-particle detectors, not necessarily possessing good resolution either in energy or in charge, and iii) several effective energies can be measured at a single-beam energy, since, in the semi-classical approximation, each scattering angle corresponds to scattering at a certain angular momentum, and the cross section can be scaled in energy by taking into account the centrifugal correction. Estimating the centrifugal potential at the Coulomb turning point r_c, the effective energy may be expressed as [6]

$$E_{\rm eff} \sim E - \frac{\lambda_c^2\hbar^2}{2\mu r_c^2} = 2E\frac{\sin(\theta/2)}{1+\sin(\theta/2)}. \quad (15)$$

Therefore, one expects that the function $-d/dE(\sigma_{\rm el}/\sigma_R)$ evaluated at an angle θ will correspond to the quasi-elastic test function (6) at the effective energy given by eq. (15).

This last point not only improves the efficiency of the experiment, but also allows the use of a cyclotron accelerator where the relatively small energy steps required for barrier distribution experiments cannot be obtained from the machine itself [7]. Moreover, these advantages all point to greater ease of measurement with low-intensity exotic beams, which will be discussed in the next section.

In order to check the scaling property of the quasi-elastic test function with respect to the angular momentum, Fig. 3 compares the functions $\sigma_{\rm el}/\sigma_R$ (upper panel) and $-d/dE(\sigma_{\rm el}/\sigma_R)$ (lower panel) obtained at two different scattering angles. The solid line is evaluated at $\theta = \pi$, while the dotted line at $\theta = 160^o$. The dashed line is the same as the dotted line, but shifted in energy by $E_{\rm eff} - E$. Evidently, the scaling does work well, both at energies below and above the Coulomb barrier, although it becomes less good as the scattering angle decreases [10].

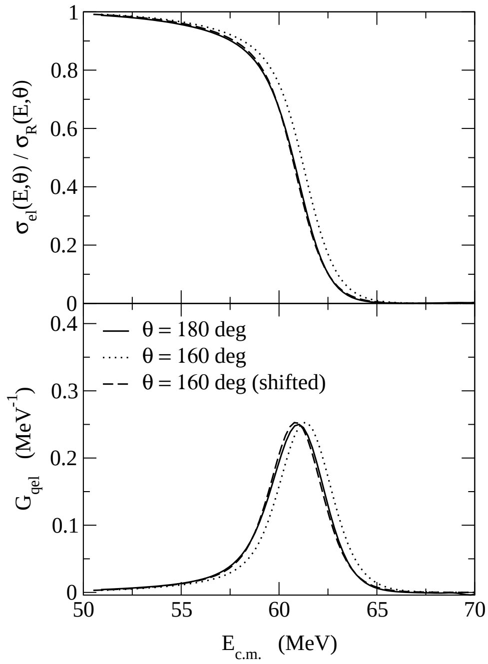

FIG. 3: Comparison of the ratio $\sigma_{\rm el}/\sigma_R$ (upper panel) and its energy derivative $-d/dE(\sigma_{\rm el}/\sigma_R)$ (lower panel) evaluated at two different scattering angles.

IV. QUASI-ELASTIC SCATTERING WITH RADIOACTIVE BEAMS

Low-energy radioactive beams have become increasingly available in recent years, and heavy-ion fusion reactions involving neutron-rich nuclei have been performed for a few systems [11–13]. New generation facilities have been under construction at several laboratories, and many more reaction measurements with exotic beams at low energies will be performed in the near future. Although it would still be difficult to perform high-precision measurements of fusion cross sections with radioactive beams, the measurement of the quasi-elastic barrier distribution, which can be obtained much more easily than the fusion counterpart as we discussed in the previous section, may be feasible. Since the quasi-elastic barrier distribution contains similar information as the fusion barrier distribution, the quasi-elastic measurements at backward angles may open up a novel way to probe the structure of exotic neutron-rich nuclei.

In order to demonstrate the usefulness of the study of the quasi-elastic barrier distribution with radioactive beams, we take as an example the reaction ^{32}Mg and ^{208}Pb, where the quadrupole collectivity of the neutron-rich ^{32}Mg remains to be clarified experimentally. Fig. 4 shows the excitation function of the quasi-elastic scattering (upper panel) and the quasi-elastic barrier distribution (lower panel) for this system. The solid and dashed lines are results of coupled-channels calculations where ^{32}Mg is assumed to be a rotational or a vibrational nucleus, respectively. We include the quadrupole excitations in ^{32}Mg up to the second member (that is, the first 4^+ state in the rotational band for the rotational coupling, or the double

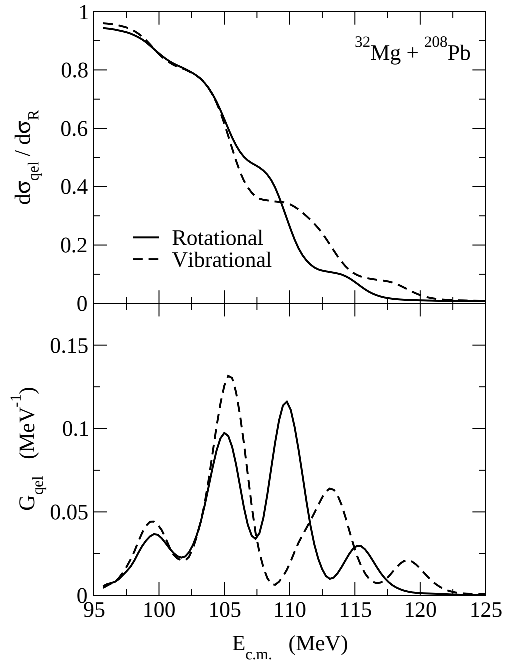

FIG. 4: The excitation function for quasi-elastic scattering (upper panel) and the quasi-elastic barrier distribution (lower panel) for the ^{32}Mg + ^{208}Pb reaction around the Coulomb barrier. The solid and the dashed lines are the results of coupled-channels calculations which assume that ^{32}Mg is a rotational and a vibrational nucleus, respectively. The single octupole-phonon excitation in ^{208}Pb is also included in the calculations.

phonon state for the vibrational coupling). In addition, we include the single octupole phonon excitation at 2.615 MeV in ^{208}Pb. We use a version of the computer code CCFULL [14] in order to integrate the coupled-channels equations. One clearly sees well separated peaks in the quasi-elastic barrier distribution both for the rotational and for the vibrational couplings. Moreover, the two lines are considerably different at energies around and above the Coulomb barrier, although the two results are rather similar below the barrier. We can thus expect that the quasi-elastic barrier distribution can indeed be utilized to discriminate between the rotational and the vibrational nature of the quadrupole collectivity in ^{32}Mg, although these results might be somewhat perturbed by other effects which are not considered in the present calculations, such as double octupole-phonon excitations in the target, transfer processes or hexadecapole deformations.

We mention that the distorted-wave Born approximation (DWBA) yields identical results for both rotational and vibrational couplings (to first order). In order to discriminate whether the transitions are vibration-like or rotation-like, at least second-step processes (reorientation and/or couplings to higher members) are necessary. The coupling effect plays a more important role in low-energy reactions than at high and intermediate energies. Therefore, we expect that quasi-elastic scattering around the Coulomb barrier will provide a useful means to allow the detailed study of the structure of neutron-rich nuclei in the near future.

Acknowledgments

This work was supported by the Grant-in-Aid for Scientific Research, Contract No. 16740139, from the Japan Society for the Promotions of Science.

---

[1] M. Dasgupta, D.J. Hinde, N. Rowley, and A.M. Stefanini, Annu. Rev. Nucl. Part. Sci. **48**, 401 (1998).
[2] A.B. Balantekin and N. Takigawa, Rev. Mod. Phys. **70**, 77 (1998).
[3] N. Rowley, G.R. Satchler, and P.H. Stelson, Phys. Lett. B **254**, 25 (1991).
[4] J.R. Leigh *et al.*, Phys. Rev. C**52**, 3151 (1995).
[5] M.V. Andres, N. Rowley, and M.A. Nagarajan, Phys. Lett. B **202**, 292 (1988).
[6] H. Timmers *et al.*, Nucl. Phys. **A584**, 190 (1995).
[7] E. Piasecki *et al.*, Phys. Rev. C**65**, 054611 (2002).
[8] D.S. Monteiro *et al.*, Nucl. Phys. **A725**, 60 (2003).
[9] R.F. Simoes *et al.*, Phys. Lett. **B527**, 187 (2002).
[10] K. Hagino and N. Rowley, Phys. Rev. C**69**, 054610 (2004).
[11] C. Signorini *et al.*, Nucl. Phys. **A735**, 329 (2004).
[12] J.F. Liang *et al.*, Phys. Rev. Lett. **91**, 152701 (2003).
[13] R. Raabe *et al.*, Nature **431**, 823 (2004).
[14] K. Hagino, N. Rowley, and A.T. Kruppa, Comp. Phys. Comm. **123**, 143 (1999).