Causality and probabilistic interpretation of quantum mechanics*

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Abstract

It is shown that the probabilistic treatment of quantum mechanics can be coordinated with causality of all physical processes. The physical interpretation of quantum-mechanical phenomena such as process of measurement and collapse of quantum state is given.

The purpose of the present paper is the substantiation of a thesis that the probabilistic interpretation of quantum mechanics is quite compatible with the supposition about unique causality of all physical processes. The consideration is performed within the framework of the binary model (see papers [1]) of quantum mechanics in which it is supposed that there are separate material carriers of corpuscular and wave properties in quantum objects.

Following the basic idea of the papers [1] we shall consider that any quantum object has dynamical and phase degrees of freedom. Carriers of the dynamical degrees of freedom are local. Further they will refer to as nucleuses of the quantum object (to not confuse to atomic nucleuses). Carriers of the phase degrees of freedom are fields, which further will refer to as an information fields or a shell of a quantum object. These fields are spread in the space.

An elementary quantum object consists of one nucleus and a shell (information field) which are coherent each other. It is possible to assume, though it is not necessary, that nucleuses exist in pulsatory (in the time and the space) regime like some splashes of the information field. In this case in microscopic scale a nucleus will not have continuous trajectory in the space - time but in macroscopic scale the trajectory will exist.

Action onto a quantum object can be dynamical and phase. The dynamical action is accompanied by transmission of dynamical quantities. It acts onto the dynamical degrees of freedom, i.e. onto the nucleuses. Respectively, the nucleuses are responsible for corpuscular properties of the quantum object and they contain the information about observables quantities in the latent shape. The phase action is not accompanied by transmission of dynamical quantities (or by very small transmission) and it acts on the phase degrees of freedom. A classical measuring device reacts to the dynamical action of the quantum object. Therefore the device reacts to an elementary quantum object as onto one aggregate. But the concrete result of this reaction is defined by structure of the shell of the quantum object. This structure depends on state of the information field. Further we shall name it physical state of the quantum object and we shall designate by symbol φ .

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Now we try to formalize these physical ideas about the quantum object. In order to take into account the latency of the information about observables quantities we postulate, that the observables \hat{B} are Hermite elements of noncommutative involute algebra (*-algebras) \mathfrak{A} . Let's consider that to each physical state φ of a quantum object univalently there corresponds a functional on the algebra \mathfrak{A} . This functional we shall designate by the same symbol φ . The term "the physical state" will denote structure of the information field, and the functional, corresponding to this structure.

Thus, if $\hat{B} \in \mathfrak{A}$ and $\hat{B}^* = \hat{B}$ then $\varphi(\hat{B}) = B$ is a real number (value of the observable \hat{B}) which is obtained in the *concrete* measurement.

Let $\{\hat{Q}\}$ is some maximal set of mutually commuting Hermite elements of the algebra \mathfrak{A} , i.e. $\{\hat{Q}\}\subset\mathfrak{A}$ and

$$\begin{split} &\text{if } \hat{Q}_i, \ \hat{Q}_j \in \{\hat{Q}\} \text{ then } [\hat{Q}_i, \hat{Q}_j] = 0; \\ &\text{if } \hat{Q}_i \in \{\hat{Q}\}, \quad \hat{Q}_j \in \mathfrak{A} \text{ and } [\hat{Q}_i, \hat{Q}_j] = 0 \text{ then } \hat{Q}_j \in \{\hat{Q}\}; \\ &\text{if } \hat{Q}_i \in \{\hat{Q}\}, \quad \hat{Q}_j \notin \{\hat{Q}_j\} \text{ then } [\hat{Q}_i, \hat{Q}_j] \neq 0. \end{split}$$

The functional φ maps the set $\{\hat{Q}\}$ in a set of real numbers

$$\{\hat{Q}\} \stackrel{\varphi}{\longrightarrow} \{Q = \varphi(\hat{Q})\}.$$

For the different functionals φ_i , φ_j the sets $\{\varphi_i(\hat{Q})\}$, $\{\varphi_j(\hat{Q})\}$ can differ and can coincide. If for all $\hat{Q} \in \{\hat{Q}\}$ is valid $\varphi_i(\hat{Q}) = \varphi_j(\hat{Q}) = Q$ then we shall term the physical states φ_i and φ_j as $\{Q\}$ -equivalent. Let's denote by $\{\varphi\}_Q$ set of all $\{Q\}$ -equivalent physical states. Let's term the set $\{\varphi\}_Q$ as a quantum state and we shall designate Ψ_Q .

We shall take one fundamental supposition touching the physical state: each physical state is unique, i.e. there are no two identical states in the world and the physical states never repeat. It is possible to consider the physical state is determined by all previous history of the concrete physical object and this history for each object is individual. In particular, in two different experiments we necessarily deal with two different physical states. Different physical states φ_i , φ_j correspond to different functionals $\varphi_i()$, $\varphi_j()$, i.e. always there will be such observable \hat{B} , that $\varphi_i(\hat{B}) \neq \varphi_j(\hat{B})$. From this supposition follows, that each physical state φ_i can be exhibited in form of the functional $\varphi_i()$ no more, than in one experiment.

For each observable \hat{A} we shall introduce concept "an actual state". It is a physical state, in which the observable \hat{A} was measured or will be measured. A set of such states we shall denote by $[\varphi]^{\hat{A}}$. A set $\{Q\}$ of equivalent states, actual for an observable \hat{A} , we shall designate $\{\varphi\}_Q^{\hat{A}}$. Following the standard quantum mechanics, we shall consider that only mutually commuting observables can be measured in one experiment.

The functional φ is not linear. However we shall require, that on Hermite elements of algebra $\mathfrak A$ a functional $\varphi(\)$, corresponding to actual states, would satisfy to the following postulates:

(1) 1)
$$\varphi(\lambda \hat{I}) = \lambda$$
, \hat{I} is unity of algebra \mathfrak{A} , λ is real number;

$$2) \quad \varphi(\hat{A}+\hat{B})=\varphi(\hat{A})+\varphi(\hat{B}), \ \varphi(\hat{A}\hat{B})=\varphi(\hat{A})\varphi(\hat{B}), \ \text{if} \ [\hat{A},\hat{B}]=0;$$

3)
$$\sup_{\alpha} \varphi(\hat{A}^*\hat{A}) > 0$$
, if $\hat{A} \neq 0$; $\varphi(\hat{A}^*\hat{A}) = 0$, if $\hat{A} = 0$;

4) for each set
$$\{Q\}$$
 and each $\hat{A} \in \mathfrak{A}$
there is $\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \varphi_i(\hat{A}) \equiv \Psi_Q(\hat{A}),$

where $\{\varphi_1, \ldots, \varphi_n\}$ is a random sample of the set $\{\varphi\}_Q^{\hat{A}}$;

5) for every
$$\hat{A}$$
, $\hat{B} \in \mathfrak{A}$ $\Psi_Q(\hat{A} + \hat{B}) = \Psi(\hat{A}) + \Psi(\hat{B})$.

We shall extend the functionals $\varphi()$ onto anti-Hermitian elements of the algebra \mathfrak{A} with the help of the equality $\varphi(i\hat{A})=i\varphi(\hat{A})$. The functional $\Psi_Q()$, appearing in the fourth postulate, has a meaning of a functional $\varphi()$ which is averaged over all $\{Q\}$ -equivalent actual states. Symbolically it can be represented in the form of Monte-Carlo integral over the actual states:

$$\Psi_Q(\hat{A}) = \int_{\varphi \in \{\varphi\}_Q^{\hat{A}}} d\mu(\varphi) \, \varphi(\hat{A}).$$

We shall note that

(2)
$$\int_{\varphi \in \{\varphi\}_Q^{\hat{A}}} d\mu(\varphi) = 1.$$

Let us connect the functional $\Psi_Q(\)$ with each quantum state Ψ_Q . The fourth postulate assumes that this functional does not depend on a concrete random sample. Further the term "a quantum state Ψ_Q " we shall use both for the set $\{\varphi\}_Q$ of physical states and for the corresponding functional $\Psi_Q(\)$ (the quantum average).

Let $\hat{A}^*\hat{A} = \hat{B} \in \{\hat{Q}\}$. If $\varphi \in \{\varphi\}_Q$, then $\varphi(\hat{A}^*\hat{A}) = B$, where $B \in \{Q\}$. Therefore

$$\Psi_Q(\hat{A}^*\hat{A}) = B = \varphi(\hat{A}^*\hat{A})\Big|_{\varphi \in \{\varphi\}_Q}.$$

From here

$$\|\hat{A}\|^2 \equiv \sup_{Q} \Psi_Q(\hat{A}^* \hat{A}) = \sup_{\varphi \in [\varphi]^{\hat{A}}} \varphi(\hat{A}^* \hat{A}) > 0, \text{ if } \hat{A} \neq 0.$$

As $\Psi_Q(\hat{A}^*\hat{B})$ is a linear (in $\hat{A}^*\hat{B}$) positive semidefinite functional then the Cauchy-Bunkyakovsky-Schwarz inequality is valid (see for example [2])

$$\Psi_Q(\hat{A}^*\hat{B})\Psi_Q(\hat{B}^*\hat{A}) \le \Psi_Q(\hat{A}^*\hat{A})\Psi_Q(\hat{B}^*\hat{B}).$$

Therefore for $\Psi_Q(\hat{A}^*\hat{A})$ the postulates for square of seminorm of the element \hat{A} are fulfilled. Respectively it is possible to accept $\|\hat{A}\|$ for the norm of \hat{A} . The algebra \mathfrak{A} becomes C^* -algebra at such definition of the norm. According to the Gelfand-Naumark theorem (see [2]) every C^* -algebra can be realized as algebra of linear operators in some Hilbert space. Thus, the proposed here construction of quantum mechanics permit the standard realization.

As against usual scheme of quantum mechanics in the proposed construction one additional element is present. It is the physical state φ and the corresponding nonlinear functional $\varphi()$. The functional $\varphi()$ describes results of individual measurement in a concrete experiment, and the functional $\Psi_Q()$ describes average value of the observable in a series of experiments, performed in identical conditions from the point of view of the observer, i.e. at the same quantum state. The fact of existence of the functionals $\varphi()$ and $\Psi_Q()$, satisfying to the enumerated postulates, is proved by all complex of quantum experiments. The standard quantum mechanics is busy in problems describing by the functionals $\Psi_Q()$. However now single quantum phenomena take a great meaning. For example, such phenomena underlie an operation of quantum computer. Therefore it is desirable to supplement the formalism of the standard quantum mechanics by positions, which would allow considering the single quantum phenomena. On the other hand, it is extremely desirable, that such expanded

formalism did not give rise to deductions, which would not agree with deductions of the standard quantum mechanics.

Namely for sufficing this condition the postulate of uniqueness of each physical state is accepted. It follows this postulate that the physical state can not be univalently fixed. Really, to fix the functional $\varphi()$, we should know its value $\varphi(\hat{B})$ for all independent observables \hat{B} . Physically it is not realizable. In one experiment we can find $\varphi(\hat{B}_i)$ only for mutually commuting observables \hat{B}_i , and in different experiments we necessarily deal with different functionals $\varphi()$.

The most hard fixing of the functional $\varphi()$, which can physically be realized, consists in reference it to some set $\{\varphi\}_Q$, i.e. to a certain quantum state. For this purpose it is enough to perform measurements of mutually commuting observables. It is possible to be restricted only by independent measurements. In principle it can be done in one experiment. Thus we can not have the complete information about a physical state of a concrete physical object in essence. The maximal observable and controllable information on the physical object is concentrated in the quantum state.

At the same time, the quantum states have some subjective element. From the standard quantum mechanics it is well known that any quantum state can be represented in form of superposition of quantum states which are fixed by one maximal set $\{\hat{Q}\}$ of mutually commuting observables. In the proposed here construction it corresponds to the fact that one physical state can belong to the different quantum states $\{\varphi\}_Q$ and $\{\varphi\}_R$. I.e. $\varphi \in \{\varphi\}_Q \cap \{\varphi\}_Q$, where the states $\{\varphi\}_Q$ are classified by values of the set $\{\hat{Q}\}$ of mutually commuting observables \hat{Q} , and $\{\varphi\}_R$ are classified by values of observables $\hat{R} \in \{\hat{R}\}$. The observables \hat{Q} and \hat{R} do not commute among themselves. Then depending on what set $\{\hat{Q}\}$ or $\{\hat{R}\}$) we shall choose for classification the physical state φ will be referred either to the quantum state $\{\varphi\}_Q$ or to the quantum state $\{\varphi\}_R$.

For example, let a spin-free particle decays into two particles A and B with spins 1/2 which scatter at large distance. Let's measure a projection of spin onto the axis z for the particle A. Let result will be $S_z(A)$. Then using the conservation law, we can state that for the particle B with absolute probability the projection of spin onto the axis z is equal $S_z(B) = -S_z(A)$. It denotes that the quantum state of the particle B corresponds to such value of the projection of spin onto axis z. However for the particle A we could measure the projection of spin onto axis x. Let result would be $S_x(A)$. Then we could state that the particle B is in the quantum state, which corresponds to the value $S_x(B) = -S_x(A)$ of the observable $\hat{S}_x(B)$.

As any physical operations with the particle B are not fulfilled, the physical state in both cases will be same $\varphi \in \{\varphi\}_{-S_z(A)} \cap \{\varphi\}_{-S_x(A)}$. The different quantum states of the particle B are related only to our subjective choice of the device for measurement of the physical state of the particle A.

This example is the description of the experiment proposed by Bohm [3] for demonstrating of the Einstein-Podolsky-Rosen paradox [4]. In proposed here treatment any paradox is absent. The physical state φ particles B, which is an objective reality, does not depend on our manipulations with the remote particle A. Any transmission of an action on distance is absent. The described experiment is an example of indirect measurement at which the information about state of the quantum object is obtained without physical action onto it. Usually interpretation of the indirect experiment arouses the greatest difficulties, first of all, bound with concept of collapse of quantum state (reduction of wave function). In adduced example by our wish we "channelize" the particle B into the quantum state $\{\varphi\}_{-S_z(A)}$, or

into the quantum state $\{\varphi\}_{-S_x(A)}$, physically not acting onto the particle B. Such collapse can be named subjective (or passive) in contrast to objective (active) collapse, which is related to the actual physical action onto the quantum object. About the objective collapse we shall talk later. Here we shall note that the subjective collapse is related to physical impossibility for us to receive the complete objective information about the quantum object (the complete information about the physical state). We should be content by the partial information (quantum state). It depends on our desire what concrete part we prefer to be satisfied by.

It is possible also to use this experiment for demonstrating that, strictly speaking, it would be possible to receive larger the information about state of quantum object, than ascertaining of its membership to this or that quantum state. In the experiment we can measure the projection of spin onto the axis z for the particle A and the projection onto the axis x for the particle B. In this case we can establish that the physical state φ of the particles B belongs to intersection of the corresponding quantum states

(3)
$$\varphi \in \{\varphi\}_{-S_z(A)} \cap \{\varphi\}_{S_x(B)}.$$

But this information has specific character. It refer to the past, more precisely, to the restricted interval in the past from the moment of decay of the spin-free particle to the moment of measurement of the projection of spin of the particle B. In this moment the information described by the equation (3) will be garbled and will be useless for the further monitoring (or control) of the particle B.

Now we shall discuss dynamics and temporal evolution of quantum object. As was already spoken, the action onto quantum object can be dynamical and phase. By hypothesis elementary quantum object is nonlocal due to the shell. The different parts of the quantum object can undergo different exterior action and lose mutual coherence. In turn, it should result in disintegration of the quantum object, since all constituent parts of the elementary quantum object must be coherent by hypothesis. As the elementary quantum objects are rather stable structures, there should be a cause, which hinders loss of the coherence. Let's assume that such cause is a strong phase interaction within the elementary quantum object. It recovers coherence. Let's assume also that at the microscopic level the direct phase action onto quantum object is much feebler than dynamical action.

At the direct phase action the exterior objects act onto phase degrees of freedom of quantum object directly. However indirect phase action is possible. It is carried out as follows. The exterior objects dynamically act onto dynamical degrees of freedom (onto nucleus) of the quantum object. Further this action is transmitted to phase degrees of freedom through strong interior phase interaction. It brings to reorganization of the shell of the quantum object, i.e. to modification of its physical state. Such phase action is not weak as against dynamical.

Neglecting the direct phase action, we shall consider that the evolution of a physical state of quantum object is determined by dynamical action, and it is controlled by a dynamical equation of motion, which is coordinated with usual quantum-mechanical equation of motion. Namely we shall assume that the physical state φ evolves in the time so, that the functional, corresponding to this state, $\varphi(\hat{A})$ $(\hat{A} \in \mathfrak{A})$ varies as follows:

(4)
$$\varphi_0(\hat{A}) \to \varphi_t(\hat{A}) \equiv \varphi_0(\hat{A}(t)),$$

where

(5)
$$\frac{d\,\hat{A}(t)}{d\,t} = \frac{i}{\hbar} \left[\hat{H}, \hat{A}(t) \right], \qquad \hat{A}(0) = \hat{A}.$$

Here \hat{H} is a usual Hamiltonian (considered as an element of the algebra \mathfrak{A}) of the quantum object.

The equations (4) and (5) quite unequivocally describe temporal evolution of the physical state. Therefore when only dynamical action is accounted (it corresponds to that that von Neumann [5] names as action of the second type) physical processes are strictly determinable. Other matter, that with the help of observations we can determine the initial value $\varphi_0(\hat{A})$ of the functional (physical state) only to within its membership to some quantum state $\{\varphi\}_Q$. Therefore majority of our predictions about the further observed dates for the considered quantum object can be only probabilistic.

Now we will turn to consideration of direct phase action. In the previous reasoning we considered that they can be neglected. A situation however is possible when it cannot be done. This situation is realized when the very large number of exterior objects act the quantum object. As the nucleus is local, it feels action of small number of the exterior objects if the long-range action is absent. As opposed to this, the shell having nonlocal structure feels action of the large number of the exterior objects. The mass character of the action can cancel weakness of the separate action. It happens only in that case when the separate weak actions do not cancel each other. It is possible to assume that exactly such situation is realized at action of a classical measuring device onto a quantum object. I.e. distinctive feature of a measuring device is that its separate microscopic elements exerts synchronous direct phase action onto the quantum object.

The typical classical measuring device comprises analyzer and detector. The analyzer is a classical device with one inlet and several exits. In the analyzer due to of direct phase action the united shell of quantum object decomposes onto several coherent constituents. Symbolically we shall figure it so:

$$\varphi \to \varphi_1 \oplus \varphi_2 \oplus \ldots \oplus \varphi_i \oplus \ldots$$

I.e. the united structure (the physical state φ) decomposes onto the direct (coherent) sum of constituents φ_i . Inside the analyzer everyone evolves somehow, but all constituents preserve a mutual coherence. Therefore, if in the further the constituents will incorporate they will be able to interfere among themselves.

The constituent φ_i quits the analyzer through *i*-th exit. Everyone (*i*-th) exit corresponds to a particular value A_i of certain observable \hat{A} (or of several mutually commuting observables). Thus, the analyzer is a point of branching of the initial physical state φ . The part of the shell, falling into the *i*-th branch, belongs to a set $[\varphi]^{A_i}$, which contains all information fields φ' such, that the corresponding functionals satisfy to equality $\varphi'(\hat{A}) = A_i$.

If only the dynamical action onto the quantum object is taken into account point of branching of the shell is a point of bifurcation for motion of the nucleus. In this point the interaction of the nucleus with the information field plays a role of "random" force, which guides the nucleus along one of the branch. Let's consider that the nucleus is retracted into the branch, for which

(6)
$$\varphi \in [\varphi]^{A_i},$$

where φ is information field of the quantum object before points of branching.

Such motion through the analyzer is admissible for the shell not changing structure. Let's consider that the nucleus should be in a resonance with neighbouring part of the shell. Then such motion is admissible for nucleus for which the resonant condition does not vary. Actually the structure of the shell varies at the analyzer. Therefore equation (6) is necessary

to consider, as the requirement of an invariance of the resonant condition for the nucleus at the point of bifurcation. The equation (6) is certain condition of continuity for the motion of the nucleus. This equation guarantees that at the point of bifurcation the evolution of the quantum object is uniquely determinated by its physical state φ .

The observable evolution has probabilistic character. Firstly, it is not controlled by a dynamical equation of motion (the bifurcation point). Secondly, the physical state φ before the bifurcation point can be fixed only to within membership φ to certain quantum state $\{\varphi\}_Q$. Due to the equation (6) the probability W_i of falling into the *i*-th branch for the nucleus is determined by the equality

(7)
$$W_i = \int_{\varphi \in \{\varphi\}_Q \cap [\varphi]^{A_i}} d\mu(\varphi).$$

Now we shall discuss the detector. It is a classical object, which has strong dynamical and phase interaction with quantum object. The detector is in a macroscopically unstable state. As a result of dynamical action of nucleus of the quantum object it goes out equilibrium. A catastrophic process, which makes macroscopically observable result, develops in it.

The phase action of quantum object onto the detector is proportioned onto large number of microscopical constituents of the detector and does not give macroscopically observable effect. Thus, the detector macroscopically reacts only to nucleus of the quantum object, i.e. it reacts to the quantum object as onto one aggregate.

If the detector is located at the *i*-th branch then it works with probability W_i (formula (7)). The nucleus of quantum object falls into the *i*-th branch with such probability. If the detector has worked, it denotes $\varphi \in [\varphi]^{A_i}$, i.e. $\varphi(\hat{A}) = A_i$. Thus, on the one hand, value of the functional $\varphi()$ really characterizes result of individual measurement. On the other hand, using formula (7) for average value < A > of the observable \hat{A} we can receive

$$< A> = \sum_i W_i A_i = \sum_i \int_{\varphi \in \{\varphi\}_Q \cap [\varphi]^{A_i}} d\mu(\varphi) \, \varphi(\hat{A}) = \int_{\varphi \in \{\varphi\}_Q} d\mu(\varphi) \, \varphi(\hat{A}) = \Psi_Q(\hat{A}).$$

It agrees with deductions of the standard quantum mechanics and with property (2) of the functional $\Psi_Q($).

The inverse action of the detector onto the quantum object can go along two scenarios. The first scenario is realized when the nucleus is at the branch where there is the detector. The detector dynamically acts onto the nucleus and strongly (directly and indirectly) onto that part of the shell of the quantum object, which has fallen into the *i*-th branch. Due to this action these parts of the quantum object lose coherence with those parts of the shell, which have fallen into other branches. As a result, firstly, they lose possibility to interfere with the part of the shell, which has passed through the *i*-th branch. Secondly, only this part of the shell remains in the structure of the quantum object, as only it does not lose coherence with the nucleus due to strong interior phase interaction.

Thus, there is sharp reorganization of the shell of the quantum object (of its physical state). In standard quantum mechanics this phenomenon is treated as a collapse of the quantum state. Here this phenomenon can be named objective or active collapse. By the exterior displays the objective collapse is quite similar to the earlier described passive collapse, but the physical essences of these phenomena are completely different.

It is necessary to note, that the modification of the physical state and, as the consequence, its quantum state happens due to actual modification of the part of the quantum object, which is at the detector. But not as a result of vanishing (of reduction) of those parts,

which are not at the detector. Nothing happens with them. Nevertheless, they cease to be constituents of the quantum object. In this case the action of the detector onto the quantum object is dynamical and phase. Either first type, or the second type of interaction can predominate. If the overwhelming contribution gives the dynamical action then this contribution can be described by the dynamical equations (4), (5).

If overwhelming or the essential contribution gives direct phase action then this contribution can not be described by the dynamical equations. By terminology of von Neumann it is interaction of the first type. However in this case (as opposed to the von Neumann's opinion) the physical evolution of the quantum object is uniquely determined by structure of that part of the shell, which has hit the detector. Other matter, that we have only the information, which there is in the equation $\varphi_i \in \{\varphi\}_Q \cap \{\varphi\}^{A_i}$ for this part φ_i of the shell. Therefore we can do only probabilistic predictions. Most probably, if the direct phase action plays main role then the modified part φ_i' of the shell will belong to the set $\{\varphi\}_{Q}^{A_i}$, but it will cease to belong to the set $\{\varphi\}_Q$. In favour of such supposition speaks experiment, as a particular quantum state is practically prepared usually so.

Let's consider now second scenario, when the nucleus falls into that branch, in which the detector is absent. For simplicity of reasoning we shall consider that the analyzer has only two branches. The detector is located in the second branch, and the nucleus falls into the first branch. In this case the detector does not work (negative experiment). However the standard quantum mechanics states that there is a collapse of a quantum state also. Let's look, how it can be justified within the framework of considered here model.

In this case at the analyzer the field φ decomposes onto coherent constituents φ_1 and φ_2 : $\varphi \to \varphi_1 \oplus \varphi_2$. The field φ_2 falls into operative zone of the detector. There this part of the shell undergoes strong direct action of the detector. As a result of this action φ_2 loses a coherence with the nucleus and φ_1 . Therefore φ_2 ceases to be a part of the shell of the quantum object. Now the quantum object will have physical state φ_1 . There is a modification of the physical state of the quantum object. This modification is not controlled by the dynamical equations (objective collapse of a quantum state). In this case we quite definitely can state, that $\varphi_1 \in [\varphi]^{A_1}$.

Let's note that both at the first and second scenario, on the one hand, as a result of action of the detector there is a (objective) collapse of the quantum state of the quantum object, on the other hand, a long-range action of the detector is absent.

The proposed scheme of quantum mechanics gives obvious and almost classical explanation of the most fundamental quantum phenomena. The scheme is free from any paradoxes. However there is a problem, maybe this scheme one of variants of scheme with hidden parameters. In a certain measure it is so, but the reasonings, over which schemes with the hidden parameters are rejected, is not correct in this case. The famous proof of von Neumann [5] about impossibility of the hidden parameters in quantum mechanics essentially founds on linearity of quantum mechanics. One of main elements, functional $\varphi(\)$, is not linear in the scheme, proposed here. Therefore proof of von Neumann does not concern the present case.

Other, not less famous, argument against schemes with the hidden parameters is Bell inequality [6]. We shall reproduce a typical deduction of this inequality. Let a quantum object Q (particle with spin 0 in the elementary variant of experiment) decays into two objects A and B (particles with spins 1/2). The objects A and B scatter on large distance and hit detectors D(A) and D(B), respectively, in which the measurements are independent. The object A has a set of observables \hat{A}_a (double projection of spin onto the direction a). The observables corresponding to different values of an index a, are not simultaneously

measurable. Each of observables can take two values ± 1 . In a concrete experiment the device D(A) measures an observable \hat{A}_a with a particular index a. For the object B everything is similar.

Let's assume that a quantum object Q has a hidden parameter λ . In each individual event the parameter λ has a particular value. The distribution of events according to the parameter λ is characterized by a measure $\mu(\lambda)$ with usual properties $\mu(\lambda) \geq 0$, $\int d\mu(\lambda) = 1$.

All magnitudes, connected with individual event, depend on the parameter λ . In particular, the values of observables \hat{A}_a and \hat{B}_b , obtained in a concrete experiment, are functions A_a , B_b of the parameter λ . For individual event the correlation of observables \hat{A}_a and \hat{B}_b is characterized by the magnitude $A_a(\lambda)B_b(\lambda)$. The average value of the magnitude is referred to as correlation function E(a,b):

$$E(a,b) = \int d\mu(\lambda) A_a(\lambda) B_b(\lambda).$$

Giving various values to the indexes a and b and taking into account that

(8)
$$A_a(\lambda) = \pm 1, \quad B_b(\lambda) = \pm 1,$$

we shall obtain the following inequality

(9)
$$|E(a,b) - E(a,b')| + |E(a',b) + E(a',b')| \le$$

$$\le \int d\mu(\lambda) \left[|A_a(\lambda)| |B_b(\lambda) - B_{b'}(\lambda)| + |A_{a'}(\lambda)| |B_b(\lambda) + B_{b'}(\lambda)| \right] =$$

$$= \int d\mu(\lambda) \left[|B_b(\lambda) - B_{b'}(\lambda)| + |B_b(\lambda) + B_{b'}(\lambda)| \right].$$

In the right-hand side of the formula (9), due to equalities (8), one of the expressions

(10)
$$|B_b(\lambda) - B_{b'}(\lambda)|, \qquad |B_b(\lambda) + B_{b'}(\lambda)|$$

is equal to zero, and another is equal to two for each value of λ . From here we obtain the Bell inequality

$$|E(a,b) - E(a,b')| + |E(a',b) + E(a',b')| \le 2.$$

The correlation function E(a, b) is easily calculated within the framework of the standard quantum mechanics. In particular, when A and B are particles with spin 1/2

(12)
$$E(a,b) = -\cos\theta_{ab}, \quad \theta_{ab} \text{ is an angle between } a \text{ and } b.$$

It is easy to verify that there are directions a, b, a', b', for which formulas (11) and (12) contradict each other.

It would seem it is possible to repeat this derivation in the proposed here model, having made replacements of type $A(\lambda) \to \varphi(\hat{A})$, $B(\lambda) \to \varphi(\hat{B})$, $\int d\mu(\lambda) \dots \to \int_{\varphi \in \{\varphi\}_Q^{\hat{A}\hat{B}}} d\mu(\varphi) \varphi(\dots)$. However this opinion is erroneous. For derivation of the Bell inequality it is essential, that in the left-hand side of the inequality (9) it is possible to represent all terms in the form of united integral over one parameter λ . It is not valid for the quantum average substituting this integral, as it is necessary to integrate over actual states in it. The elements, appearing in different correlation functions, $\hat{A}_a\hat{B}_b$, $\hat{A}_{a'}\hat{B}_b$, $\hat{A}_a\hat{B}_{b'}$, $\hat{A}_a\hat{B}_{b'}$, $\hat{A}_{a'}\hat{B}_{b'}$, do not commute among themselves. Therefore sets of actual states, corresponding to these operators, do not intersect. In

derivation of the inequality (11) we tacitly supposed that expression (10) exist for each λ . However there is no physical state φ , which would be by actual state both for the observable \hat{B}_b and for the observable $\hat{B}_{b'}$.

In summary it is necessary especially to note, that the present paper is not at all attempt to formulate a new rival theory to quantum mechanics. The proposed scheme does not contradict any statement of the standard quantum mechanics. Maybe some theses gain slightly other physical interpretation. All deductions of the standard quantum mechanics are valid in the described scheme. At the same time there are additional elements (for example, functional $\varphi(\)$, equality (1)) in this scheme. They allow to include individual events in the domain of its application.

Strictly speaking, the formalism of the standard quantum mechanics assumes that the classical relations are reproduced only for average values of quantum observables. Meantime the practice shows that such relations are reproduced in each individual experiment. Of course, it concerns only those observables, what can be measured in one experiment. In the proposed approach this fact is consequence of the equalities (1).

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