PHYSICAL MODEL OF SCHRODINGER ELECTRON. FAYNMAN CONVENIENT WAY IN MATHEMATICAL DESCRIPTION OF ITS QUANTUM BEHAVIOUR

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Abstract

The physical model (PhsMdl) of a nonrelativistic quantized Schrodinger's electron (SchrEl) is offered. The behaviour of the SchrEl's well spread (WllSpr) elementary electric charge (ElmElcChrg) had been understood by means of two independent and different in a frequency and size motions. The description of this resultant motion may be done by substitution of the classical Wiener continuous integral with the quantized Feynmam continuous integral. There are possibility to show by means of the existent not only formal but substantial analogy between the quadratic differential wave equation in partial derivatives of Schrodinger and quadratic differential particle equation in partial derivatives of Hamilton-Jacoby that the addition of a kinetic energy of the stochastic harmonic oscillation of some quantized micro particles to the kinetic energy of classical motion of the same micro particles formally determines their wave behaviour.It turns out the stochastic motion of the quantized micro particles powerfully to break up the smooth thin line of the classical motion of the same micro particle in many broad cylindrically spread path. The SchrEl participate in stochastically roughly determined circumferences within different flats and with different radii, with centres which are successively arranjed over short and very disorderly orientated lines. Therefore the quantized motion of some micro particle cannot be described by smooth thin well contured (focused) line, describing the classical motion of the macro particle.

1 Introduction

A physical model (PhsMdl) [1],[2] and [3] of the nonrelativistic quantized Schrodinger's electron (SchrEl) is offered in this work. In our obvious PhsMdl the SchrEl will be regarded as some well spread (WllSpr) elementary electric charge (ElmElcChrg), taking simultaneously part in two independent and different in size and frequency motions: A) Some classical motion of a Lorentz' electron (LrEl) along a smooth clear-cut thin classical trajectory realized in a consequence of some a known interaction (IntAct) of LrEl's over spread (OvrSpr) ElmElcChrg, magnetic dipole moment (MgnDplMmn) or bare mass with the intensity of some external classical fields (ClsFlds) as it is done within the Newton nonrelativistic classical mechanics (NrlClsMch) and Maxwell-Lorentz nonrelativistic classical electrodynamics (ClsElcDnm). B) The isotropic three-dimensional nonrelativistic quantized ((IstThrDmnNrlQnt) Furthian stochastic boson harmonic oscillations FrthStchBsnHrmOscs) of the SchrEl as a result of the permanent electric interaction (ElcIntAct) of its WllSpr ElmElcChrg with the electric intensity (ElcInt) of the resultant quantized electromagnetic field (QntElcMgnFld) of the stochastic virtual photons (StchVrtPhtns), stochasticly generated by dint of StchVrtPhtns ([31], [32] and [33]), exchanged

between the fluctuating vacuum (FlcVcm) and it. This Furthian quantized stochastic behaviour of the SchrEl is very similar to the Brownian classical stochastic behaviour of the ClsMicrPrt. But in a principle the exact description of the resultant behaviour of the SchrEl owing of its participation in both the mentioned motions could be done only by means of the nonrelativistic quantum mechanics' (NrlQntMch) and nonrelativistic classical electrodynamics' (ClsElcDnm) laws.

The description of some quantized micro particle QntMicrPrt behaviour, within the matrix presentation of the NrlQntMch, offered by Heisenberg, ([5]) have been accompanied with unfounded affirmation that its unknown motion cannot be described by dint of any its trajectory.Therefore for this purpose one must use the matrix elements of its operator, which in a reality presents a Fourie components of the same trajectory. Indeed, missunderstanding the cause for incommon dualistic behaviour of QntMcrPrts lets one erroneous applay an ansamble statistical commentary instead of a stochastic diffusible one of the probabibly interpretation of the modul square of its orbital wave function (OrbWvFnc) and gives some incorrect physical interpretation of the uncertainity relations of Heisenberg ([5]). Therefore the contiuity integral representation of the motion within the NrlQntMch have been missinterpretated as a natural generalization of the classical space-time trajectory. Some physical scientists have considered this representation as a giving some possibility for the construction of some trajectoty, which is compatible with the uncertainity relation of Heisenberg within Feynman's continuity integral representation of the NrlQntMch. But this is very incorrect allegation as because these continuity path integrals are calculated over all virtual possible trajectories in this area. In Feynman mathematical formalism ([22]) the transition of some QntMicrPrt from one space point into another one is characterized by no one trajectoty but by a greet number of possible trajectories, each of them insert by certain own contribution of the probability in its transition amplitude.

2 Physical explanation of the essence of the electron physical model by analogy between the mathematical description of FrthQntMicrPrt behaviour and BrmClsMicrPrt behaviour.

In our obvious physical model (PhsMdl) [3] of the nonrelativistic quantized Schrodinger's electron (SchrEl) it will be regarded as some well spread (WllSpr) elementary electric charge (ElmElcChrg), taking simultaneously part in two different motions : A) The classical motion of a Lorentz' electron (LrEl) along an smooth well contoured thin classical trajectory realized in a consequence of a some known interaction (IntAct) of its over spread (OvrSpr) ElmElcChrg, magnetic dipole moment (MgnDplMmn) or bare mass with the intensity of some external classical fields (ClsFlds) as it is done in the Newton nonrelativistic classical mechanics (NrlClsMch) and Maxwell-Lorentz nonrelativistic classical electrodynamics (ClsElcDnm). B) The isotropic three-dimensional nonrelativistic quantized Furthian stochastic boson harmonic oscillations (IstThrDmnNrlQnt FrthStchBsnHrmOsc) of the SchrEl as a result of the permanent electric interaction (ElcIntAct) of its WllSpr ElmElcChrg with the electric strength of the resultant quantized electromagnetic field (QntElcMgnFld) of the stochastic virtual photons (StchVrtPhtns), generated y dint of StchVrtPhtns exchanged between the fluctuating vacuum (FlcVcm) and it. This Furthian quantized stochastic behaviour of the SchrEl is very similar to the Brownian classical stochastic behaviour of the ClsMacrPrt.

Indeed,it is well known that the classical motion of some Lorentz' electron (LrEl) as a classical macro particle (ClsMacrPrt) is well described by means of a clear-cut smooth narrow line, while the quantized motion of some Schrodinger's electron (SchrEl) as a quantum micro particle (QntMicrPrt) is well discribed is well described by sum of two line: the first one is a

distinct smooth thin classical line and the second one is many broad cylinricaly spread path. The SchrEl participates in stochastically roughly determined circumferences oscillations within different flats and with different radii, with centres which are successively arranjed over frequently broken line,short and amounted by random very disorderly orientated in space petty pieces lines. Therefore the quantized motion of some micro particle cannot be described by smooth thin well contured (focused) line. Therefore the quadratic differential wave equation of Schrodinger (QdrDfrWvEqtSchr) ([8]) may be obtained through an addition of the kinetic energy of Furth quantized stochastic harmonic oscillation motion ([21]), expressed by the dispersion of its imaginery momentum or stochastic osmotic velosity to the quadratic differential particle equation of Hamilton-Jacoby (QdrDfrPrtEqtHml/Jcb). Since then a transparent survey of the behaviour of a nonrelativistic quantized SchEl in our PhsMdls may be build by means of the substitution of classical Wiener's continuous integral ([17]) with quantized Feynman's continuous integral ([21] and [22]). Therefore it is necessary to take into consideration that Schrodinger in 1931 ([18]) and Furth in 1933 ([21]) had found some formal analogy between the quadratic differential diffusive equation of Focker-Plank (QdrDfrDfsEqtFcrPln) :

$$\frac{\partial W}{\partial t} = div(Wv) - D\,\Delta W \tag{1}$$

for the distribution function W of a probability density (DstFncPrbDns) of the free Brownian classical micro particle (BrnClsMicrPrt) in a motionless coordinate system in a respect to one and the quadratic differential wave equation of Schrodinger (QdrDfrWvEqtSch)

$$\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2\Delta}{2m}\,\Psi + V\,\Psi \tag{2}$$

for an orbital wave function (OrbWvFnc) Ψ of a free Furthian quantized micro particle (FrthQntMicrPrt) in a motionless coordinate system in respect to one.This similarity become particulary stricing at an absence of any external forces when U = 0 and v = 0.

$$\frac{\partial W}{\partial t} = -D\,\Delta W \tag{3}$$

and

$$i\,\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2\Delta}{2m}\,\Psi \tag{4}$$

The unimportant distinction between two equations consists in the existence of imaginary unit i in diffusivity factor of wave equation, i.e. the if diffusivity factor D of BrnClsMcrPrt has real value, the diffusivity factor D has imaginary value $\frac{i\hbar^2}{2m}$. They had found that there exists an essential coincidence between two presentations (3) and (4) if the coefficient of the diffusion D is equal of $\frac{i\hbar}{2m}$. Therefore Feynman has used for transition between two OrbWvFncs Ψ of some free FrthQntMcrPrt with different coordinates and times the following formula:

$$\Psi(x_1,t_1) = \int K(x_1,t_1|x_2,t_2)\,\Psi(x_2.t_2)\,dx_2 \tag{5}$$

in   analogous   of such the formula,which early had been used by Einstein ([15]),[20]),Smoluchovski ([16]) and Wiener ([17]) for the transition between two DstFncsPrbDns $W(\lambda,t)$ of a free BrnClsMicrPrt:

$$W(\lambda,t) = \int W(\lambda_o,t_o)\,P(\lambda_o,t_o|\lambda,t)\,d\lambda_o \tag{6}$$

The diffusivity D, which is very strongly dependent as on the viscosity and temperature of the solvent, so on the radius of the BrnClsPrt, can been determined by help of the DstFncsPrbDns $W(\lambda, t)$ by means means of the followig definition formula :

$$D(\lambda_o) = \left(\begin{array}{c} \text{Ł} i \, m \\ \Delta t \to 0 \end{array}\right) \cdots \int_a^b \frac{(\lambda - \lambda_o)^2}{2\Delta t} P(\lambda_o, t_o | \lambda, t) \, d\lambda \qquad (7)$$

It is neccessary to turn here our attention to satisfaing as from the functions of the hit probability $P(x_1, t_1 | x_3, t_3)$ and $P(x_3, t_3 | x_2, t_2)$, so from the functions $K(x_1, t_1 | x_3, t_3)$ and $K(x_3, t_3 | x_2, t_2)$ of the following M-change relations,which characterizes Markovian processes:

$$P(x_1, t_1 | x_2, t_2) = \int P(x_1, t_1 | x_3, t_3) \, P(x_3, t_3 | x_2, t_2) \, dx_3 \qquad (8)$$

and

$$K(x_1, t_1 | x_2, t_2) = \int K(x_1, t_1 | x_3, t_3) \, K(x_3, t_3 | x_2, t_2) \, dx_3 \qquad (9)$$

if the probability function K for the FrthQntMicrPrt within the NrlQntMch has the following well known form :

$$K(x_o, t_o | x, t) = \frac{\sqrt{m}}{\sqrt{2i\pi\hbar t}} \exp[-i \frac{mx^2}{2\hbar t}] \qquad (10)$$

which is analogous of the probability function P for the BrnClsMcrPrt within the StchClsMch having the following Gaussina exponential form :

$$P(x_o, t_o | x, t) = \frac{1}{\sqrt{4\pi\tau D}} \exp[-\frac{x^2}{4\pi D\tau}] \qquad (11)$$

But as we can see from (10) and (11) that one have no physical mean of some classical velocity, as if

$$\left(\begin{array}{c} \text{Ł} i \, m \\ t_n \to t_{n-1} \end{array}\right) \cdots \left(\frac{x_n - x_{n-1}}{t_n - t_{n-1}}\right) \neq \left(\begin{array}{c} \text{Ł} i \, m \\ t_{n+1} \to t_n \end{array}\right) \cdots \left(\frac{x_{n+1} - x_n}{t_{n+1} - t_n}\right) \qquad (12)$$

when the BrnClsMicrPrt participates within the BrnStchMtn. From here it follows that although Feynman speak very loudly about his using of the smallest action principle at description of the unknown uncommon behaviour of the QntMicrPrts, in a reality he go on very silently by dint of the matematical apparatus of the BrnStchMch, using the existent substantial analogy between the FrthStchMtn of the QntMicrPrt and well known BrnStchMtn of the BrnMicrPrt.

As a generalization of the equalityes (8) and (9) they have shown that the probability function describes the probabiility of some free QntMacrPrt (BrnClsPrt) to move from the point x_o in the time moment t_o to the point x in the time t, passing through the interval of some virtual trajectory between the points a and b ,is clearly defined as a product from the probaibility of same free BrnClsPrt to move from the point x_o in the time moment t_o to the point x_1 in the time moment t_1, passing through the interval of some virtual trajectory between the points a_1 and b_1, times the probability of same free BrnClsPrt to move from the point x_1 in the time moment t_1 to the point x_2 in the time moment t_2, passing through the interval of some virtual trajectory between the points a_1 and b_1, times the probability of same free BrnClsPrt to move from the point x_2 in the time moment t_2,passing through the interval of some virtual trajectory between the points a_2 and b_2 and so on,times the probability of same free BrnClsPrt

to move from the point x_n in the time moment t_n, passing through the interval of some virtual trajectory between the points a_n and b_n, after their integration in respect of all the intermediate variables over their intervals :

$$P(x_o, t_o | x, t) = \int \ldots \int P(x_o, t_o | x_1, t_1) \, P(x_1, t_1 | x_2, t_2)$$
$$P(x_2, t_2 | x_3, t_3) \ldots P(x_n, t_n | x, t) \, dx_1, dx_2, dx_3, \ldots dx_n \tag{13}$$

and analogous

$$K(x_o, t_o | x, t) = \int \ldots \int K(x_o, t_o | x_1, x_1) \, K(x_1, t_1 | x_2, t_2)$$
$$K(x_2, t_2 | x_3, t_3) \ldots K(x_n, t_n | x, t) \, dx_1, dx_2, dx_3, \ldots dx_n \tag{14}$$

However,it is very important to understand why the form of probability function $K_{i,j}$ of two independent events,having property of a product of their own probability function K_i and K_j, has exponential connection with the action function $S_{i,j}(r,t)$ of a free QntMicrPrt, having property of a sum of two independent events. Therefore the form (10), written by Feynman, coinsidences with the gaussian exponent (11), very early writing down for description of the probability $P(r_o, t_o | r, t)$ to find some BrnClsPrt after a time interval $\tau = t - t_o$ of a distance $x = r - r_o$. Hence if the DstFncPrbDns W may has positive real value only, the OrbWvFnc Ψ may has a complex value. This means that for some part of the OrbWvFnc Ψ may exist a total analogy between both the QdrPrtEqn and their solutions. Indeed, if the exponent and normalization factor of the DstFncPrbDns W have real value only, the exponent and normalization factor of the OrbWvFnc Ψ may have complex value.

Indeed,if we suppose that

$$E = \frac{\{\bar{p}\}^2}{2m} + \frac{\{\delta p\}^2}{2m} + U(r) \tag{15}$$

then within a quasiclassical approximatin many physicists presume that the SchrEl's OrbWvFnc Ψ may been written in the following two forms : a) within the classical accessible area :

$$\Psi(r,t) = \left\{ C_1 \, \exp\{\frac{i}{\hbar} \int p \, dx\} \, \exp\{-\frac{1}{2} \ln p\} + C_2, \exp\{\frac{-i}{\hbar} \int p \, dx\} \, \exp\{-\frac{1}{2} \ln p\} \right\} \tag{16}$$

and b) within the classical accessible area :

$$\Psi(r,t) = C_1 \, \exp\{-\frac{1}{\hbar} \int p \, dx\} \, \exp\{-\frac{1}{2} \ln p\} \tag{17}$$

where

$$p = \sqrt{2m\{E - U\}} = \sqrt{\{\bar{p}\}^2 + \{\delta p\}^2} \tag{18}$$

if $\bar{p} = \langle |p| \rangle$ and $\{\delta p\}^2 = \{p - \bar{p}\}^2$. When $U \geq E$ then

$$|p| = i\sqrt{2m\{U - E\}} = i\sqrt{\{\bar{p}\}^2 + \{\delta p\}^2} \tag{19}$$

Then the imaginary part of the exponent (the real part S_1 of the action function S),

$$\{\frac{i}{\hbar}\} \int p \, dx\} \quad and \quad \{\frac{-i}{\hbar}\} \int p \, dx\} \tag{20}$$

will describe the classical motion along distinct smooth narrow line and the real part of the exponent (the imaginary part S_2 of the action function S)

$$\frac{1}{2}\ln(p) \tag{21}$$

will describe the Furthian stochastic motion (FrthStchMtn) along a frequently broken of petty strongly disorientated small pieces closely to the smooth and distinc thin line of the classical motion. Therefore the FrthStchMtn will erode the clear-cut smooth thin line and the total motion of the QntMcrPrt will be spread in a wide path. As the energy E, the averaged momentum $\bar{p}$ and the dispersion $\sqrt{(\delta p)^2}$ are determined by the OrbWvFnc Ψ of the QntMicrPrt then the possibility decrease of its discovery within the appointed area will be also determined. When the potential U is bigger then the total energy E of some QntMicrPrt, then the momentum p must be substituted by the $i|p|$. As a result of that we can suppose that the unusual dualistic behaviour of the QntMicrPrt within the NrlQntMch can be described by dint of the following mutual conjugated physical quantities :

$$r_j = \bar{r}_j + \delta r_j \quad and \quad p_j = \bar{p}_j + \delta p_j \tag{22}$$

The upper supposition shows us way for some part of the QntMicrPrt's OrbWvFnc may exist a total analogy between the presentations of both the QdrPrtDfrEdts and their solutions.In this way we understand why the behaviour of the QntMicrPrt must be described by a OrbWvFnc Ψ , although the behaviour of the ClsMacrPrt may be described only by a clear-cut smooth thin line.

Indeed, the first, it is known from quantum electrodynamics (QntElcDnm), that when the energy of some QntMicrPrt has a complex value, then its real part describes the real energy of the particle, while its imaginary part describes its disappearance in the time, i.e.the time of its decay into another QntMicrPrts; in the second, it is known from quantum mechanics theory (QntMchThr) of the Solid State, that the real part of the momentum of the QntMicrPrt describes its averaged current part, while the imaginary part of the momentum of the QntMcrPrt describes its disappearance in the space, i.e. the decrement of the probability the QntMcrPrt to come in inside of the potential barier U.

Hence, although that Feynman speak loudly about the principle of the smallest action function, but he uses always the mathematical apparatus of the Brownian stochastic motion (BrnStchMtn), as the imaginary part of the action S of the FrthQntMcrPrt takes the form of the real part of the exponent of the DstFncPrbDns W of BrnClsMicrPrt, which describes its BrnStchMtn.We can impressively see this discrepancy between interpretation and using the mathematical apparatus of the ClsStchMch, particulary at the derivation of the Schrodinger wave equation by using of some formulas from the BrnStchMtn theory with the consideration the potential role.

$$\Psi(x, t + \epsilon) = \frac{1}{A}\int_V \exp[\frac{im\eta^2}{2\hbar\epsilon}]\,\Psi(x + \eta, t)\,d\eta \tag{23}$$

Indeed, if Feynman has used in a reality the principle of the smallest action, he would not have expand in a power only the potential exponent and keeping of this part of the action, which describes the kinetic energy of the SchEl's FrthStchMtn. But Feynman whould expand the Lagrangian exponent, as there is distinction between the kinetic and potential energies, which in according with the smallest action principle must compensate each other. The expansion of the potential exponent only :

$$\exp[\frac{i\epsilon}{\hbar}U(x + \frac{\eta}{2}, t)] \cong [1 - \frac{i\epsilon}{\hbar}U(x + \frac{\eta}{2}, t)] \tag{24}$$

means that Feynmam keeps the kinetic energy exponent $exp[\frac{imn^2}{2\hbar\epsilon}]$ for averaging the interaction of the FrthQntMcrPrt by means of the DstFncPrbDns W, assuming that the QntMicrPrt perform the FrthQntStchMtn. I think that Feynman has expanded the potential exponent because it compensates the kinetic energy of the NtnClsMtn, which participates in a QvdDfrClsEqtHml-Jcb and as last it would break semi-group properties of the Gausian exponential distribution.

3   Calculation of the minimal dispersions of some dynamical variables of QntMicrPrts as a result of their participation in the FrthQntStchMtn

We attempt in what follows to show that the smalles values of some dynamical variable dispersions may been determined as a result of their participation in the FrthQnttchMtn,using their definition by Feynman.Hence when Feynman has discussed about the time dependence of the velocity of some QntMicrPrt

$$v_n^+ = \left(\begin{array}{c}\text{Ł}im\\ \epsilon \to 0\end{array}\right)\frac{(x_{n+1} - x_n)}{(t_{n+1} - t_n)} = (v + iu), \tag{25}$$

and

$$v_n^- = \left(\begin{array}{c}\text{Ł}im\\ \epsilon \to 0\end{array}\right)\frac{(x_n - x_{n-1})}{(t_n - t_{n-1})} = (v - iu), \tag{26}$$

where

$$t_{n+1} = t_n + \epsilon \quad and \quad t_{n-1} = t_n - \epsilon, \tag{27}$$

, he has assumed that it is

$$u^2 \approx \left(\frac{2D}{\epsilon}\right) \tag{28}$$

which may be only if

$$\mid x_{n+1} - x_n \mid\cong\mid x_n - x_{n-1} \mid\approx \sqrt{(2D\epsilon)} = \sqrt{\left(\frac{\hbar\epsilon}{m}\right)}, \tag{29}$$

, as it must be at FrthStchMtn. Indeed, if

$$\langle(\Delta p)^2\rangle) = \frac{1}{2}(\Delta p)^2 = \frac{(m\Delta x)^2}{2(\epsilon)^2} = \frac{m\hbar}{2\epsilon}, \tag{30}$$

, and if

$$\langle(\Delta x)^2\rangle = \frac{1}{2}(\Delta x)^2, \tag{31}$$

, then

$$\langle(\Delta p)^2\rangle \times \langle(\Delta x)^2\rangle = \left(\frac{\hbar}{2}\right)^2, \tag{32}$$

Further when Feynman has discussed about the kinetic energy of the QntMcrPrt, he has asserted that instead the known expression:

$$\frac{m(x_{n+1} - x_n)^2}{(2\epsilon)^2} + \frac{m(x_n - x_{n-1})^2}{(2\epsilon)^2} \tag{33}$$

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we must use the following expression:

$$2\frac{m}{2} \times \frac{(x_{n+1} - x_n)}{\epsilon} \times \frac{(x_n - x_{n-1})}{\epsilon}, \tag{34}$$

Indeed, if from eqns.(25) and (26) we have : $v^+ = (v + iu)$ and $v^- = (v - iu)$ then

$$\left(\frac{m(x_{n+1} - x_n)^2}{(2\epsilon)^2}\right) + \left(\frac{m(x_n - x_{n-1})^2}{(2\epsilon)^2}\right) = \frac{mv^2}{2} - \frac{mu^2}{2}, \tag{35}$$

which is wrong, but

$$\frac{m}{2} \times \frac{(x_{n+1} - x_n)}{\epsilon} \times \frac{(x_n - x_{n-1})}{\epsilon} = \frac{mv^2}{2} + \frac{mu^2}{2}, \tag{36}$$

, which is correct. Moreover, if both

$$(\Delta E) = \frac{mu^2}{2} \tag{37}$$

, where

$$u^2 = \frac{\langle (\Delta x)^2 \rangle)}{(\epsilon)^2} = \frac{(\Delta x)^2}{2(\epsilon)^2} = \frac{\hbar}{2m\epsilon}, \tag{38}$$

and from (27) $\quad \Delta t = \epsilon$ then we have immediately :

$$\langle (\Delta E)^2 \rangle \times \langle (\Delta t)^2 \rangle = \left(\frac{\hbar}{2}\right)^2. \tag{39}$$

Further the values of the dispersion $\langle (\Delta P_r)^2 \rangle$ and $\langle (\Delta L_j)^2 \rangle$ can been determined by virtue of the uncertainity relations of Heisenberg :

$$\langle (\Delta P_r)^2 \rangle \times \langle (\Delta r)^2 \rangle \geq \frac{\hbar^2}{4} \tag{40}$$

$$\langle (\Delta L_x)^2 \rangle \times \langle (\Delta L_y)^2 \rangle \geq \frac{\hbar^2}{4} \times \langle (L_z)^2 \rangle \tag{41}$$

$$\langle (\Delta L_y)^2 \rangle \times \langle (\Delta L_z)^2 \rangle \geq \frac{\hbar^2}{4} \times \langle (\Delta L_x)^2 \rangle \tag{42}$$

and

$$\langle (\Delta L_z)^2 \rangle \times \langle (\Delta L_x)^2 \rangle \geq \frac{\hbar^2}{4} \times \langle (\Delta L_y)^2 \rangle \tag{43}$$

Thence the distersion $\langle (\Delta P_r)^2 \rangle$ will really have its minimal value at the maximal value of the $\langle (\Delta r)^2 \rangle$, which is $\approx \langle r^2 \rangle$. In such a way we obtained that the minimal value of the dispersion $\langle (\Delta P_r)^2 \rangle$ can been determined by the following well known inequality :

$$\langle (\Delta P_r)^2 \rangle \geq \frac{\hbar^2}{4} \times \langle (\Delta r)^2 \rangle \tag{44}$$

When the SchEl is placed within an external potential with the cylyndrical symmetry then the direction of the axis z of the our coordinate system coincideswith the direction of the angular MchMmn,then $\langle (L_z) \rangle$ therefore by means of (41) we can obtain that :

$$\langle (\Delta L_x)^2 \rangle = \langle (\Delta L_y)^2 \rangle = \frac{l\hbar^2}{2} \tag{45}$$

As by means of the inequalities (42) and (45) we can obtain that $\langle (\Delta L_z)^2 \rangle \approx \frac{\hbar^2}{4}$, then we can obtain that: at $\langle (L_z) \rangle \neq 0$,i.e. at an existence of the cylindrical symmetry $(\langle (\Delta L_x)^2 \rangle = \langle (\Delta L_y)^2 \rangle = \frac{l\hbar^2}{2}$

$$\langle (L)^2 \rangle = (\langle L_z \rangle)^2 + \langle (\Delta L_x)^2 \rangle +, \langle (\Delta L_y)^2 \rangle + \langle (\Delta L_z)^2 \rangle = (l\hbar)^2 + l\hbar^2 + \frac{\hbar^2}{4} = (l\hbar + \frac{\hbar}{2})^2 \tag{46}$$

and at $\langle L_z \rangle = 0$,i.e. at an existence of the spherical symmetry $(\langle (\Delta L_x)^2 \rangle = \langle (\Delta L_y)^2 \rangle = \langle (\Delta L_z)^2 \rangle = \frac{\hbar^2}{4})$

$$\langle (\Delta L_x)^2 \rangle +, \langle (\Delta L_y)^2 \rangle + \langle (\Delta L_z)^2 \rangle = \frac{3\hbar^2}{4} \tag{47}$$

Realy, the obtained upperesults may been obtained by means of the formal transfer from the three-dimensional QdrPrtDfrWvEqtSchr for the spherical part $R(r)$ of the SchrEl's OrbWvFnc Ψ,depend only from r, written in a spherical coordinate system:

$$\frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + \left[\frac{2m}{\hbar^2} \{ E - U(r) \} - \frac{l(l+1)}{r^2} \right] R(r) = 0 \tag{48}$$

to two dimensional QdrPrtDfrWvEqtSchr for the cylindrical part $\Phi(\rho)$ of the SchrEl's OrbWvFnc Ψ, depend only from ρ, written in a cylindrical coordinate system :

$$\frac{d^2 \Phi}{d(\rho)^2} + \frac{1}{\rho} \frac{d\Phi}{d\rho} + \left[\frac{2m}{\hbar^2} \{ E - U(\rho) \} - \frac{(l+1/2)^2}{(\rho)^2} \right] \Phi(\rho) = 0 \tag{49}$$

There is necessity to point here, that the formal transfer from the equation (48) to the equation (49) can been realized by virtue of the exchange of r and $R(r)$ with ρ and $\frac{\Phi(\rho)}{\sqrt{\rho}}$ in the corresponding way. Further it is well known that the presentation of the QdrPrtDfrWvEqtSchr for the SchEl's total OrbWvFnc $\Psi(\rho, \varphi, z)$ has the following well known form :

$$\frac{\partial^2 \Psi}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \Psi}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \Psi}{\partial \varphi^2} + + \frac{\partial^2 \Psi}{\partial z^2} + \frac{2m}{\hbar^2} \{ E - U(\rho, z) \} \Psi = 0 \tag{50}$$

Then in a result of the comparison of the eq.(49) with the eq.(50) we can obtain the averaged value of the total OrbMchMnt's square $< L^2 >$ in the NrlQntMch must coincide with its value $\hbar(l + 1/2)$ determined by the eq. (46).In such a way we obtain the average value of the total orbital mechanical moment (OrbMchMmn) in a square $\langle L^2 \rangle$ of a SchEl in the cylindrical coordinate.From above it is followed that the value $\langle L^2 \rangle = \hbar^2 l(l+1)$, which can be obtained in the spherical coordinate, taking no into account the part $\langle (\Delta P_r)^2 \rangle = \frac{\hbar^2}{4r^2}$ .Indeed,after all the part of the product $\langle (\Delta P_r^2) \rangle \times \langle r^2 \rangle$ may be considered as the dispersion $\langle (\Delta L_r)^2 \rangle$ of $\langle (L_r)^2 \rangle$ along axis r, when the axis z coincides with the radius-vector r. In such a way we can write the SchEl's OrbWvFnc as a result of an upper discussion in the following presentations :

$$\Psi(\rho, \varphi, z) = \Psi_l(\rho, \varphi, z) \exp(i\frac{\varphi}{2}) = \Psi_l(\rho, z) \exp(i\frac{(2l+1)\varphi}{2}) \tag{51}$$

4   Conclution

The realized above investigation shows that when the SchEl is moving in Coulomb potentiale of the NclElcChrg of some H-like atom, then the stability of its ground state is ensured by the existence of the SchEl's kinetic energy of its FrthStchMtn, generated as a result of the continuous ElcIntAct of its BlrElmElcChrg with QntElcMgnFld of stoch astic created virtual photons (StchVrtPhtns) within the fluctuating vacuum (FlcVcm). Besides that it could be easily shown that not only the SchEl's localized energy, ensuring a stability of its ground state within H-atoms, but as well as all those are following: the existence of its additional MchMm and MgnDplMm,the SchEl's tunnelling through the potential barrier and the shifts of its energy level in an atoms - are natural and incontestable manifestations of its effective participation in the FrthStchMtn too. However there exist an essential difference between Brownian classical stochastic motion (BrnClsStchMtn) of some BrnClsPrt within NrlClsMch and Furth's quantum stochastic motion (FrthQntStchMtn) of some QntMicrPrt within NrlQntMch as the result of the existent difference between both moving cause: stochastic scattering of some atoms or molecules from another BrnClsPrt and the ElcIntAct of the SchEl's BlrElmElcChrg with the ElcInt of LwEn-StchVrtPhtns, generated stochasticaly in the FlcVcm through continuous exchanges of the LwEn-StchVrtPhtns between the SchEl's WllSpr ElmElcChrg and the FlcVcm .

In such a natural way we had ability to obtain the minimal value of the dispersion product, determined by the Heisenberg uncertainty relation. Hence we can come to a conclusion that the dispersions of the dynamical parameters of the quantized micro particles are natural result of their forced motions owing to ElcMgnIntAct of its WllSpr ElmElcChrg or MgnDplMm with the resultant strengths of the ElcFld or MgnFld of QntElcMgnFlds of StchVrtPhtns at its FrthQntStchMtn through the FlcVcm.

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