THE PHYSICAL MODEL OF SCHRODINGER ELECTRON . SCHRODINGER CONVENIENT WAY FOR DESCRIPTION OF ITS QUANTUM BEHAVIOUR.

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Abstract

The physical model (PhsMdl) of a Schrodinger nonrelativistic quantized electron (SchEl) is built by means of a transition of the quadratic differential particle equation of Hamilton-Jacoby (QdrDfrPrtEqtHam/Jkb) into the quadratic differential wave equation of Schrodinger (QdrDfrWvEqtSch) in this work, which interprets the physical reason of its quantum (wave and stochastic) behaviour (QntWvBhv) by explanation of the physical reason which forces the classical Lorentz electron (LrEl) to participate in Furthian quantized stochastic oscillation motion (FrthStchOscMtn), which turn it into quantum SchEl. It is performed that this transition is realized by my consideration the Bohm's quantum potential as a kinetic energy of the forced FurthStchOscMtn of the SchEl's well spread (WllSpr) elementary electric charge (ElmElcChrg) close to a smooth thin trajectory of a classical LrEl. There exist as an essential analogy between the Furthian quantum stochastic trembling oscillation motion and the Brownian classical stochastic trembling motion so and between the description of their behaviours.

The object of this paper is to discuss and to bring a green light on the problems of the physical interpretation of the nonrelativistic quantized behaviour of the Schrodinger electron (SchEl), described by means of the nonrelativistic quantum mechanics (NrlQntMch) laws and its mathematical results. The purpose of the present work is to describe the felicitous physical model (PhsMdl) of the SchEl. An obvious physical model (PhsMdl) of the nonrelativistic quantized SchEl is built by means of some simple mathematical transformation of the known classical quadratic differential particle equation of Hamilton-Jacoby (ClsQdrDfrPrtEqtHam-Jkb) into the quantum quadratic differential wave equation of Schrodinger (QntQdrDfrWv EqtSch). After well physical substantiation it is performed that this transformation is realized by taking in a consideration the so called Bohm's quantum potential as a kinetic energy, what it is in reality, of the forced Furthian stochastic oscillation motion (FurthStchOscMtn) of the SchEl's well spread (WllSpr) elementary electric charge (ElmElcChrg) within the nearness to the trajectory of the classical Lorentz' electron (LrEl). In such a natural way this transition interprets the physical reason, exciting the quantum (stochastic corpuscular-wave) behaviour (QntWvBhv) by explanation of the physical reason which forces the classical LrEl to participate in a quantized FrthStchOscMtn, which turns it into the quantized SchEl. In this fashion the QntQdrDfrWvEqtSch is obtained through addition of the kinetic energy of the SchEl's FrthStchOscMtn, expressed the dispersion of its momentum or stochastic velocity u, to the ClsQdrDfrPrtEqtHam-Jkb. Therefore the nonrelativistic quantized Furthian random trembling circular oscillation motion with various radius values inside of different planes could be roughly determined with some mathematical calculation by means of the classical probabilities laws of both the classical stochastic theory (ClsStchThr) and Maxwell classical electrodynamics (ClsElcDnm).

Hence the resonant electric interaction (ElcIntAct) of the SchEl's WllSpr ElmElcChrg with the averaged electric intensity (ElcInt) of the StchVrtPhtns from the fluctuating vacuum (FlcVcm)(zero-point radiation field) determines the influence of its behavior because they creates their stochastically diverse harmonic circular oscillations with various radii within the neighborhood of the smooth narrow path of the LrEl, spreading and turning it into some wide rough cylindrically spread path and inducting the transition of the classical LrEl into the quantized SchEl. The ElcInt between the WllSpr ElmElcChrg of the SchEl and the ElcInt of the resultant quantized electromagnetic field (QntElcMgnFld) (zero-point field),determined by both the boundary conditions and the existent StchVrtPhtns, forced the WllSpr ElmElcChrg to participate within isotropic stochastically orientated in the three-dimensional space circular oscillation, the averaged kinetic energy, which every SchEl could obtained from the FlcVcm, may be obtained by the following formula, well known from NrlClsMch :

$$E_k \;=\; \frac{m\,\langle(\omega)^2\,(\delta r)^2\,\rangle}{2} \;=\; \frac{e^2}{\pi}\,\frac{m.C}{\hbar}\,\left\{\frac{\hbar}{m.C^2}\right\}^2 \int\limits_{\omega_{min}}^{\omega_{max}} \omega\, d\omega \tag{1}$$

As usual we suppose that the upper limit ω_{max} is equal to double value of the energy at rest of the SchEl ($\omega_{max} \;=\; 2m.C^2$). As the contribution of the lower limit ω_{min} has negligible importance, we could suppose that $\omega_{min} \;=\; 0$. In this approximation we cam easily obtained from eq.(1) its following presentation :

$$E_k \;=\; \frac{2}{\pi}\cdot\frac{e^2}{C.\hbar}\,.\,m.C^2 \tag{2}$$

Hence the existence of the isotropic three-dimensional nonrelativistic Furthian QntStchBhv of the SchEl within the nonrelativistic quantum mechanics (NrlQntMchn) very strongly remind us about the classical StchBhv (ClsStchBhv) of some Brownian stochastic particle (BrnStchPrt). Thence the ElcIntAct of the SchEl's WllSpr ElmElcChrg (or a MgnIntAct of the neutron's MgnDplMm) with the resonantly averaged ElcInt (or MgnInt for neutral massive hadron) of the QntElcMgnFld of the existent StchVrtPhtns in the FlcVcm corresponds to the stochastic action of the fluctuating resultant force on account of many molecular impacts upon the BrnStchPrt at a time of its scattering. In our PhsMdl of the SchEl we explain its FrthQntStchBhv and one assist sorting the matter out the physical opinion of its parameter within the NrlQntMchn.

In above elaborate we have possibility to present the spatial distribution $\Upsilon\,(\varrho)$ of the ElcChrg of the WllSpr ElmElcChrg by dint of Kirchoff's presentation of $\delta(\varrho)$-function :

$$F\,(\varrho) \;=\; \left\{\frac{2}{3\,\pi}\right\}^{3/2}\,\left\{\frac{m.C}{\hbar}\right\}^3\,\exp\,\{-(\frac{\varrho}{\lambda_o})^2\} \tag{3}$$

Here we must point that the spatial distribution (3) of the SchEl's WllSpr ElmElcChrg is caused by the participation of the Dirac's electron's fine spread (FnSpr) ElmElcChrg in the isotropic three-dimensional relativistic quantized Schrodinger's self-consistent strong correlated fermion harmonic oscillation motion.The isotropic three-dimensional relativistic quantized Schrodinger's self-consistent strong correlated fermion harmonic oscillation motion of the FnSpr

ElmElcChrg of the DrEl may be correctly described by the three α (γ) matrixes of four order. but in approximation of change of the strongly correlated fermion harmonic oscillation by the incorrelated boson harmonic oscillation, we could used the well-known orbital wave function (OrbWvFnc) $\psi_o(\varrho)$ of the three-dimensional harmonic oscillator in its ground state, having the following analytical presentation :

$$\psi_o(\varrho) = \{\lambda_o\sqrt{\pi}\}^{-\frac{3}{2}} \exp\{-\frac{\varrho^2}{2\lambda_o^2}\} \tag{4}$$

where λ_o is the constant of the oscillation ($\lambda_o^2 = \frac{\hbar}{m\,\omega} = \frac{3}{2}.\{\frac{\hbar}{m.C}\}^2 = \frac{2}{3}.\langle\,\varrho^2\,\rangle$. After some cursory comparison it is easily to understand, that Kirchoff δ-function $F(\varrho)$ (3) is obtained from the OrbWvFnc $\psi_o(\varrho)$ (3) by means of the equation $F(\varrho) = |\,\psi_o(\varrho)\,|^2$, the well known from the NrlQntMch.

In order to obtain the averaged potential of the SchEl we must put into right side of Poison equation the spatial distribution of ElcChrg $\Upsilon(\varrho)$ of its WllSpr ElmElcChrg.In such a naturally way we have possibility to calculate roughly the averaged self-potential of the SchEl's WllSpr ElmElcChrg and to obtain its following excertional presentation :

$$V(\varrho) = \frac{-2.e}{\sqrt{\pi}.\varrho} \int_o^{(\frac{\varrho}{\lambda_o})} \exp(-x^2)\,dx \tag{5}$$

The potential energy of the electric self-action (ElcSlfAct) of the SchEl's WllSpr ElmElcChrg with the spatial distribution of its ElcChrg $\Upsilon(\rho)$ from (3) and its own potential $V(\rho)$ from (5) we are capable to determine by means of the following obvious presentation :

$$E_p = \frac{2e}{\sqrt{\pi}} \cdot \frac{4\,\pi\,e}{\pi\sqrt{\pi}\lambda_o} \int_o^\infty \exp(-u^2)\,\frac{u^2\,du}{u} \int_o^u \exp(-x^2)\,dx \tag{6}$$

The twofold integration can be easily execute by integration by parts. In such a way after elementary calculation we can obtain the following obvious result :

$$E_p = \frac{4\,e^2}{\lambda_o\pi} \int_o^\infty \exp(-x^2)\,dx = \sqrt{\frac{2}{\pi}}\,\frac{e^2}{\lambda_o} = \frac{2}{\sqrt{3\pi}} \cdot \frac{e^2}{C.\hbar}\,.\,m.C^2 \tag{7}$$

In further after some cursory comparison of the eqs.(2) and (7) we could understand that the value of the averaged kinetic energy, which the SchEl obtain from the FlcVcm at its stochastic circular oscillations, is equal of the value of the potential energy of its ElcSlfAct between spatial density of its WllSpr ElmElcChrg and its own averaged potential. This equality is no accidental nature and for certain have important significant. After this comparison we can understand why the potential energy of the own averaged electric potential has no contribution into the rest energy of the DfEl. It turns out that every electron obtains the potential energy of the ElcSlfAct of its WllSpr ElmElcChrg by its own averaged potential in form of the kinetic energy on account of its participation in the isotropic three-dimensional nonrelativistic quantized stochastic boson harmonic oscillations from the FlcVcm at the interaction of its WllSpr ElmElcChrg with the ElcInt of the StchVrtPhtns. Moreover, we can easily understand that the participation of the WllSpr ElmElcChrg of the SchEl in the isotropic three-dimensional nonrelativistic quantized stochastic boson harmonic oscillations not only takes its illocalizing energy from the FlcVcm, ensuring with this the stability of its ground state in H-atom, but at this as well as all this oscillation create its additional MchMmn and MgnDplMmn, and this ElcIntAct its tunnelling through some potential barriers and causes the shift of its energy levels in atoms.Therefore all this experimental observed phenomena in the long run demonstrate the real participate of the SchEl in the isotropic three-dimensional nonrelativistic quantized stochastic boson harmonic oscillations as a result of the ElcIntAct of its WllSpr ElmElcChrg with the EctInt of the resultant QntElcMgnFld of the existent StchVrtPhtns.

Although till now nobody know what the McrPrt means, all the same there exists a possibility for a consideration of an unusual behaviour of a QntMcrPrt by means of a transparent surveyed PhsMdl of the SchEl. In our PhsMdl the SchEl will be treated as a well spread (WllSpr) ElmElcChrg, taking simultaneously part in two different motions: A/The classical motion of the LrEl along an well contoured smooth and thin trajectory realized in a consequence of some classical interaction (ClsIntAct) of its over spread (OvrSpr) ElmElcChrg, bare mass or magnetic dipole moment (MgnDplMm) with some external classical fields (ClsFlds), described by well known laws of the Newton nonrelativistic classical mechanics (NrlClsMch). This motion may be finically described by virtue of the laws of both the NrlClsMch and the classical electrodynamics (ClsElcDnm); B/The isotropic three-dimensional nonrelativistic quantized (IstThrDmnNrlQnt) Furthian stochastic boson harmonic oscillation motion (FrthStchBznHrmOscMtn) of the SchEl as a result of the permanent ElcIntAct of the electric intensity (ElcInt) of the self-consistent resultant QntElcMgnFld of all the StchVrtPhtns, existing within the FlcVcm and generated by dint of the VrtPhtn's stochastic exchange between them. The SchEl's motion and its unusual quantized behaviour, described in the NrlQntMch may be easily understood by assuming it as a forced random trembling oscillation motion (RndTrmMtn) upon a stochastic joggle influence of the StchVrtPhtns scattering from some BrnClsPrt. Therefore the RndTrmMtn can be approximately described through some determining calculations by means of both the laws of the Maxwell ClsElcDnm and the probable laws of the classical stochastic theory (ClsStchThr). But in a principle the exact description of the SchEl's uncommon behaviour can be carry into a practice by means only of the NrlQntMch's laws and ClsElcDnm s ones.

In an accordance of the analogy between the Furthian quantum stochastic trembling oscillation motion and the Brownian classical stochastic trembling motion and the description of their behaviours (of the BrnClsPrts and of the FrthQntPrts) with a deep physical understanding of the Furthian random trembling oscillation motion (FrthRndTrmOscMtn), we must determine both as the value V_j^- of the BrnClsMcrPrt's (FrthQntMcrPrt) velocity before the moment t of the scattering time of some molecule (LwEnr-StchVrtPhtn) from one (its OvrSpr ElmElcChrg), so the value V_j^+ of its velocity after the same moment t of the scattering time by means of the following definitions:

$$V_j^-(r,t) \,=\, Lim_{Dt\to o}\left(\frac{r_j(t)-r_j(t-Dt)}{Dt}\right)\,;\; V_j^+(r,t) \,=\, Lim_{Dt\to o}\left(\frac{r_j(t.+Dt)-r_j(t)}{Dt}\right)\,; \quad (8)$$

In addition we may determine two new velocities v_j and u_j by dint of the following simple equations :

$$2\,V_j \,=\, \left[V_j^+ \,+\, V_j^-\right]\,; \quad , \quad 2\,i\,U_j \,=\, \left[V_j^+ \,-\, V_j^-\right]\,; \qquad (9)$$

In conformity with the eqs.(9) it is obviously followed that the current velocity, having a real value V, in reality describes the regular drift of the BrnClsMcrPrt (FrthQntMcrPrt) and the osmotic velocity, having a imagine value iU, in reality describes nonrelativistic Brawnian classical (Furthian quantized) stochastic trembling harmonic oscillations. Afterwards by virtue of the well-known definition equations:

$$2m\,V_j \,=\, m\,\left[V_j^+ \,+\, V_j^-\right] \,=\, 2\,\nabla_j\,S_1\,; \quad and \quad 2i\,m\,U_j \,=\, m\,\left[V_j^+ \,-\, V_j^-\right] \,=\, 2\,i\,\nabla_j\,S_2,; \quad (10)$$

one can obtain following presentation of the SchEl's OrbWvFnc $\Psi(r,t)$:

$$\Psi(r,t) \,=\, \exp\left(\frac{iS_1}{\hbar} \,-\, \frac{S_2}{\hbar}\right) \,=\, B\,\exp\left(\frac{iS_1}{\hbar}\right) \tag{11}$$

It is easily to verify the results (10) and (11). In effect one be obtained by means of the following natural equations:

$$m\,V_j^+\,\Psi \,=\, -i\,\hbar\nabla_j\,\exp\left(\frac{-iS_1}{\hbar} \,-\, \frac{S_2}{\hbar}\right) \,=\, (\,\nabla_j\,S_1 \,+\, i\,\nabla_j\,S_2\,)\;\Psi \quad ; \tag{12}$$

$$m\,V_j^-\,\Psi^+ \,=\, +i\,\hbar\nabla_j\exp\left(\frac{iS_1}{\hbar} \,-\, \frac{S_2}{\hbar}\right) \,=\, (\,\nabla_j\,S_1 \,-\, i\,\nabla_j\,S_2\,)\;\Psi^+ \quad ; \tag{13}$$

In this fashion the QntQdrDfrEqtSch is obtained through addition of the kinetic energy of the SchEl's FrthRndTrmMtn, expressed with the dispersion of its momentum or stochastic velocity, to the ClsQdrDfrEqtHam-Jkb. Hence the classical motion of the LrEl is described by a smooth narrow path, which is determined from its classical real part S_1 of the complex action $S[r,t]$ and its derivatives, but the Furthian quantized stochastic motion of the SchEl is described by a rough cylindrically spread broad path, which is determined correctly from its imaginary part S_2 represented by the module of its orbital wave function (OrbWvFnc) $\Psi(r,t)$ and operators. Consequently, the quantum motion is described by rough broad path, which is determined from the quantum action $S[r,t]$ and its derivatives by the orbital wave function (OrbWvFnc) $\Psi(r,t)$ and operators. It turns out, that if the action function $S(r,t)$ has only a real value S_1, then the micro particle (McrPrt) moves along a classical well contoured smooth and narrow path ; but when if the action function $S(r,t)$ has only imaginary value S_2, then the McrPrt moves on a its trajectory, cylindrically spread and turned into wide path of the cylindrical form with differ radii and centers, being on small pieces from stochastically broken line ; when the action function has a complex value $S(r,t)$, then the McrPrt moves in the quantized dual form : indeed as the real part S_1 of the action function and its derivative determine the classical motion and its current velocity v and the imaginary part S_2 of the complex action function $S(r,t)$ and its derivative determine the stochastic motion and its osmotic velocity u. This spread of the smooth thin curve through its cylindrically spread and turned into wide path of the cylindrical form with differ radii and centers, being on petty breaking of small pieces makes the trajectory in rough and road path, which forces us to put the OrbWvFnc $\Psi(r,t)$ description of the SchEl's behaviour.Hence the classical motion of the LrEl is described by a smooth narrow path, which is determined from its classical real part S_1 of the complex action $S(r,t)$ and its derivatives, but the Furthian stochastic quantum oscillation motion of the SchEl is described by a wide rough path, which is mathematical correctly determined from its imaginary part S_2 represented by the module of its orbital wave function (OrbWvFnc) $\Psi(r,t)$ and operators. It turns out, when the action function $S(r,t)$ has only a real value S_1, then the NtnMcrPrt moves along its classical well contoured smooth and narrow path; when the action function $S(r,t)$ has only an imaginary value S_2, then the BrnMcrPrt moves stochastically on a frequently broken and very scattered orientated line of small pieces; when the action function S has a complex value, then the QntMcrPrt moves in a quantized dual form: as the real part S_1 of the action function S and its derivatives determine the classical motion and its current velocity v and the imaginary part S_2 of the action function S and its derivatives determine the forced stochastic motion and its spreading (osmotic) velocity u. This spreading of the thin and smooth classical trajectory through its wide path of the cylindrical form with differ radii and centers, being on often breaking of small pieces forces us to put the OrbWvFnc $\Psi(r,t)$ for description of the SchEl's behaviour.

Indeed, it is well known that the imaginary part of the energy of the McrPrt describes its decay in the time and the imaginary part of the velocity of the McrPrt describes its going out from the classical trajectory in the space, which is forbidden for the free motion of the ClsMcrPrt. Therefore the module quadrate of the SchEl's OrbWvFnc $\Psi(r,t)\mid\Psi(r,t)\mid^2$, where hasn't any imaginary part (i.s.has no real part S_1 of its action function S), describes only its probability for its discovering (location) in a very small area of the space,close by the space point having coordinates r,in the moment t of the time.The fluctuating alternation of the imaginary parts of the SchEl's energy and momentum (quantities of motion) may be considered as a result of continuous exchange of some parts of its energy and momentum at the uninterrupted alternative absorption and emission of the stochastic virtual photons (StchVrtPhtns) within the fluctuating vacuum (FlcVcm).

In a consequence of what was asserted above in order to obtain the QntQdrDfr WvEqn of Sch we must add to the kinetic energy $\frac{(\nabla_l S_1)^2}{2m}$ of the NtnClsPrt in the following ClsQdrDifPrtEqt of Hml-Jcb

$$-\frac{\partial S_1}{\partial t}=\frac{(\nabla_j S_1)^2}{2m}+U\,; \tag{14}$$

the kinetic energy $\frac{(\nabla_l S_2)^2}{2m}$ of the BrnClsPrt. In such the natural way we obtain the following analytic presentation of the QntQdrDfrWvEqt of Sch :

$$-\frac{\partial S_1}{\partial t}=\frac{(\nabla_j S_1)^2}{2m}+\frac{(\nabla_j S_2)^2}{2m}+U\,; \tag{15}$$

It is obviously to understand that the first term $\frac{(\nabla_l S_1)^2}{2m}$ in the eq.(15) describes the kinetic energy of the regular translation motion of the NtnClsPrt with its current velocity $v_l=\frac{\nabla_l S_1}{m}$ and the second term $\frac{(\nabla_l S_2)^2}{2m}$ describes the kinetic energy of the random trembling oscillation motion (RndTrmOscMtn) of the BrnClsPrt with its osmotic velocity $u_l=\frac{\nabla_l S_2}{m}$. Therefore we can rewrite the expression (15) in the following form:

$$E_t=\frac{m\,v^2}{2}+\frac{m\,u^2}{2}+U=\frac{\langle\,P\,\rangle^2}{2\,m}+\frac{\langle(\Delta\,P)^2\rangle}{2\,m}+U\,; \tag{16}$$

After elementary physical obviously suppositions some new facts have been brought to light. Therefore the upper investigation entitles us to make the explicit assertion that the most important difference between the QntQdrDfr WvEqt of Sch and the ClsQdrDfrPrtEqt of Hml-Jcb is exhibited by the existence of the kinetic energy of the FrthRndTrmOscMtn in the first one. Therefore when the SchEl is appointed in the Coulomb's potential of the atomic nucleus fine spread (FnSpr) electric charge (ElcChrg) Ze its total energy may be written in the following form :

$$\langle\,E_t\,\rangle=\frac{1}{2\,m}\left[\langle P_r\rangle^2+\frac{\langle L\rangle^2}{\langle r\rangle^2}\right]+\frac{1}{2\,m}\left[\langle(\Delta P_r)^2\rangle+\frac{\langle(\Delta L)^2\rangle}{\langle r\rangle^2}\right]-\frac{Ze^2}{\langle r\rangle} \tag{17}$$

As any SchEl has eigenvalues $n_r=0$ and $l=0$ in a case of its ground state, so it follows that $\langle P_r\rangle=0$ and $\langle L\rangle=0$. As a consistency with the eq.(19) the eigenvalue of the SchEl's total energy E_t^o in its ground state in some H-like atom is contained only by two parts :

$$\langle\,E_t^o\,\rangle=\frac{1}{2\,m}\left[\langle(\Delta P_r)^2\rangle+\frac{\langle(\Delta L)^2\rangle}{(\langle r\rangle)^2}\right]-\frac{Ze^2}{\langle r\rangle} \tag{18}$$

Further the values of the dispersions $\langle(\Delta P_r)^2\rangle$ and $\langle(\Delta L)^2\rangle$ can be determined by virtue of the Heisenberg Uncertainty Relations (HsnUncRlt):

$$\langle(\Delta P_r)^2\rangle \times \langle(\Delta r)^2\rangle \geq \frac{\hbar^2}{4} \tag{19}$$

$$\langle(\Delta L_x)^2\rangle \times \langle(\Delta L_y)^2\rangle \geq \frac{\hbar^2}{4} \langle(\Delta L_z)^2\rangle \tag{20}$$

Thence the dispersion $\langle(\Delta P_r)^2\rangle$ will really have its minimal value at the maximal value of the $\langle(\Delta r)^2\rangle == \langle r\rangle^2$.In this way the minimal dispersion value of the $\langle(\Delta P_r)^2\rangle$ can be determined by the following equation :

$$\langle(\Delta P_r)^2\rangle = \frac{\hbar^2}{4\langle r^2\rangle} \tag{21}$$

As the SchEl's ground state has a spherical symmetry at $l = 0$, then the following equalities take place :

$$\langle(\Delta L_x)^2\rangle = \langle(\Delta L_y)^2\rangle = \langle(\Delta L_z)^2\rangle ; \tag{22}$$

Hence we can obtain minimal values of the dispersions (22) through division of the eq.(19) with the corresponding equation from the eq. (22). In that a way we obtain the following result :

$$\langle(\Delta L_x)^2\rangle + \langle(\Delta L_y)^2\rangle + \langle(\Delta L_z)^2\rangle = \frac{3\hbar^2}{4} \tag{23}$$

Just now we are in a position to rewrite the expression (19) in the handy form as it is well-known :

$$E_t^o = \frac{1}{2\,m}\left[\frac{\hbar^2}{4r^2} + \frac{3\hbar^2}{4r^2}\right] - \frac{Z\,e^2}{r} = \frac{1}{2}\,\frac{\hbar^2}{m\,r^2} - \frac{Z\,e^2}{r} ; \tag{24}$$

Subsequently the minimal value of the E_t^o may be determined by minimization of the expression (23) in respect of the radius r. In such a way we could obtain the minimizing equality :

$$\frac{\partial E_t^o}{\partial r}\Big|_{r=r_o} = \frac{-\hbar^2}{m\,r^3} + \frac{Z\,e^2}{r^2} = 0 ; \tag{25}$$

Thence we can obtain the value of the SchEl's orbital radius r in its ground state of an H-like atom as a result of the minimizing eq.(23)

$$r_o = \frac{\hbar^2}{2me^2} = \frac{a_o}{Z} ; \tag{26}$$

Here a_o is the Bohr's radius of the SchEl's ground state in the H-like atom.Further we can obtain the averaged value of the SchEl's total energy E_t^o when it occupies its ground state by the substitution of the following equilibrium value of the orbital radius r from the eq.(22):

$$\langle\, E_t^o \,\rangle = -\frac{mZ^2e^4}{2\hbar^2} ; \tag{27}$$

Since then it is easily to understand by means of upper account that if he ClsMcrPrt s motion is going along the clear definitized smooth thin trajectory in accordance with the NrlClsMch,then the QntMcrPrt's motion is perform in the form of the RndTrbOscMtn rough broad roadway near classical one of any NtnClsPrt within NrlClsMch. As a result of that we can suppose that the unusual dualistic behaviour of QntMcrPrt can be described by dint of the following physical quantities within NrlQntMch :

$$r_j = \bar{r}_j + \delta r_j \quad ; \quad p_j = \bar{p}_j + \delta p_j \quad ; \tag{28}$$

Indeed, because of existence of $\delta r_j \neq 0$ and $\delta p_j \neq 0$ within the NrlQntMch the value of the MchMm's square $\langle L^2 \rangle$ of the SchEl is different from the value of averaged MchMmn's square $\langle L \rangle^2$ of the LrEl in the NrlClsMch. Really,by dint of the Heisenberg Commutation Relations (HsnCmtRlt) :

$$L_x L_y - L_y L_x = i \hbar L_z ; \; L_y L_z - L_z L_y = i \hbar L_x ; \; L_z L_x - L_x L_z = i \hbar L_y ; \tag{29}$$

we can write two analogous inequalities : the inequality (20) and the following corresponding inequality :

$$\langle (\Delta L_y)^2 \rangle \times \langle (\Delta L_z)^2 \rangle \geq \langle (\Delta L_x)^2 \rangle \quad ; \quad \langle (\Delta L_z)^2 \rangle \times \langle (\Delta L_x)^2 \rangle \geq \langle (\Delta L_y)^2 \rangle \tag{30}$$

We can suppose in following that in a case when the SchEl is placed in the external potential of cylindrical symmetry its MchMn's component along the axis Z has averaged value $< L_z >$ $= l\hbar$ In a spite of that the averaged value of the MchMn's square must be determined by the following equality :

$$\langle L^2 \rangle = (\langle L_z \rangle)^2 + \langle (\Delta L_x)^2 \rangle + \langle (\Delta L_y)^2 \rangle + \langle (\Delta L_z)^2 \rangle ; \tag{31}$$

Further the values of the quantities $\langle (\Delta L_x)^2 \rangle$ $\langle (\Delta L_y)^2 \rangle$ and $\langle (\Delta L_z)^2 \rangle$ can be determined by virtue of the inequalities (29) and (30) in the following form :

$$\langle (\Delta L_x)^2 \rangle = \langle (\Delta L_y)^2 \rangle = \frac{l\hbar^2}{2} \quad and \quad \langle (\Delta L_z)^2 \rangle = \frac{\hbar^2}{4} \tag{32}$$

Then it is quite naturally that we must obtain the averaged value of the MchMn's square at experiment,which is well-founded by my physical point of view:

$$\langle L^2 \rangle = l^2 \hbar^2 + \frac{l\hbar^2}{2} + \frac{l\hbar^2}{2} + \frac{\hbar^2}{4} = \hbar^2 \, (l + \frac{1}{2})^2 \tag{33}$$

I think my successful picturesque example illustrates very exactly the extraordinary situation of the QntMcrPrt within the NrlQntMch. Hence the difference between the NtnClsBhv of the NtnClsMcrPrt, described by the laws of the NtnClsMch, the BrnStchBch of the BrnClsMcrPrt, described by the laws of the ClsStchMch, and the FrthStchBhv of the FrthQntMcrPrt, described by the laws of the NrlQntMch may be roughly understand by means of three different values of the action function S. It turns out, when the action function $S(r,t)$ has only a real value S_1, then the NtnMcrPrt moves along its classical well contured smooth and narrow path ; when the action function $S(r,t)$ has only an imaginary value S_2, then the BrnMcrPrt moves stochastically on a frequently broken and very scattered orientated line of small pieces ; when the action function S has a complex value, then the QntMcrPrt moves in the quantized dual form : as the real part S_1 of the action function S and its derivatives determine the classical motion and its current velocity v and the imaginary part S_2 of the action function S and its derivatives determine the forced stochastic motion and its spreading (osmotic) velocity u. This spreading of the thin and smooth classical trajectory through wide path of the cylindrical form with differ radii and centers, being on often breaking of small pieces forces us to put the OrbWvFnc $\Psi(r,t)$ for description of the SchEl's behaviour.

R E F E R E N C E S

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