

Probabilistic quantum cloning via Greenberger-Horne-Zeilinger states

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We propose a probabilistic quantum cloning scheme using Greenberger-Horne-Zeilinger states, Bell basis measurements, single-qubit unitary operations and generalized measurements, all of which are within the reach of current technology. Compared to another possible scheme via Tele-CNOT gate [D. Gottesman and I. L. Chuang, *Nature* **402**, 390 (1999)], the present scheme may be used in experiment to *clone the states of one particle to those of two different particles* with higher probability and less GHZ resources.

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I. INTRODUCTION

Quantum computers can solve problems that classical computers can never solve [1]. However, the practical implementation of such device need careful consideration of the minimum resource requirement and feasibility of quantum operation. The basic operation in quantum computer is unitary evolution, which can be performed using some single-qubit unitary operations and Controlled-NOT (C-NOT) gates [2]. While single-qubit unitary operation can be executed easily [3], the implementation of C-NOT operation between two particles (for example two photons) encounters great difficulty in experiment [4]. With linear optical devices (beam splitters, phase shifters, etc.), the C-NOT operations between the several quantum qubits (such as location and polarization) of a single photon is within the reach of current quantum optics technology [5], but nonlinear interactions are required for the construction of practical C-NOT gate of two particles [4]. Those nonlinear interactions are normally very weak, which forecloses the physical implementation of quantum logic gate.

To solve this problem, Gottesman and Chuang [6] suggested that a generalization of quantum teleportation [7]—using single-qubit operations [3], Bell-basis measurements [8] and certain entangled quantum states such as Greenberger-Horne-Zeilinger (GHZ) states [9]—is sufficient to construct a universal quantum computer and presented systematic constructions for an infinite class of reliable quantum gates (including Tele-C-NOT gate). Experimentally, quantum teleportation has been partially realized [10] and three-photon GHZ entanglement has

been observed [11]. Thus their construction of quantum gates offers possibilities for relaxing experimental constraints on realizing quantum computers.

Unfortunately, up to now there has been no way to experimentally distinguish all four of the Bell states, although some schemes do work for two of the four required cases—yielding at most a 50% absolute efficiency [8]. In Gottesman and Chuang's scheme, two GHZ states and three Bell-basis measurements are needed to perform a C-NOT operation, which yields 1/8 probability of success in experiment. To complete a unitary operator, many C-NOT gates may be needed, which makes the probability of success close to zero. Moreover, the creation efficiency of GHZ states is still not high in experiment now [11]. Therefore a practical experiment requires careful consideration of the minimum resource and the maximum probability of success.

In this paper we investigate the problem of probabilistic quantum cloning using GHZ states, Bell basis measurements, single-qubit unitary operations and generalized measurements. The single-qubit generalized measurement can be performed by the unitary transformation on the composite system of that qubit and the auxiliary probe with reduction measurement of the probe [12]. In an optical quantum circuit, the probe qubit can be represented as the location of a photon and such process can be implemented using only linear optical components (such as polarizing beam splitter and polarization rotation) [5]. We mention above that the construction of practical C-NOT between two particles is not within current experimental technology, but it does not prohibit

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the C-NOT operation between different degrees of freedom of one photon. This kind of C-NOT is allowed in linear optical circuit and is of different type from C-NOT between different particles [5]. So the single-qubit generalized measurement on the polarization can be performed with location as the probe.

Consider a sender Alice holds an one-qubit quantum state $|\phi\rangle$ and wishes to transmit identical copies to N associates (Bob, Claire, etc.). Quantum no-cloning theorem [13] implies that the copies cannot be perfect; but this result does not prohibit cloning strategies with a limited degree of success. Two most important cloning machines – universal [14–16] and state-dependent [17–19] – have been proposed by some authors. However, it is not available¹ for Alice to generate the copies locally using an appropriate quantum network [16,19,20] and then teleport each one to its recipient by means of teleportation due to the difficulty of executing C-NOT operation [4]. To avoid such difficulty, recently, Murao *et al.* [21] presented an optimal 1 to N universal quantum telecloning strategy via a $(2N)$ -particle entangled state. Such entanglement is difficult to prepare in experiment when N is large. Quantum probabilistic (state-dependent) cloning machine is designed to perfectly reproduce linear independent states secretly chosen from a finite set with non-zero probability [18–20]. The corresponding telecloning process can be executed via the Tele-C-NOT gates [6] according to the cloning strategies provided in [19,20]; but such procedure requires too many GHZ states and Bell basis measurements and can succeed with probability close to zero. The scheme we propose in this paper needs only $(N - 1)$ GHZ states and $(N - 1)$ Bell-basis measurements to implement $M \rightarrow N$ cloning. Although such process cannot reach the optimal probability as that in local situation, it may be used in current experiment to *cloning the states of one particle to those of two different particles* with higher probability and less GHZ resources.

The rest of the paper is organized as follows. In section II we discuss some strategies of probabilistic cloning and present the concept of *probability spectrum* to describe different strategies. Comparing two most important ones, we show that M entries $1 \rightarrow N$ cloning give more copies at the price of higher probability of failure than one $M \rightarrow N$ cloning. In Section III, we present the probabilistic telecloning process via three-particle entangled state and also show how to construct the entangled state from GHZ state by local operations. A summary is given in Section IV.

¹However, it is available to clone the states of one qubit of a single photon to two qubits of that photon using optical simulation [5].

II. STRATEGIES OF PROBABILISTIC CLONING

Generally, the most useful states are $|\phi_{\pm}(\theta)\rangle = \cos\theta|1\rangle \pm \sin\theta|0\rangle$ in quantum information theory. Given M initial copies, Alice need not to always execute the cloning operation by taking these copies as a whole. Suppose Alice divides the M copies into m different kinds of shares, each of which includes ϑ_i entries $k_i \rightarrow N_i$ cloning processes. For different kinds of shares, one of the two parameters k_i and N_i should be different. These parameters should satisfy

$$\sum_{i=1}^m k_i \vartheta_i = M. \quad (2.1)$$

The probability of obtaining x copies for Alice can be represented as

$$P(x) = \sum_{i=1}^m \sum_{g_i N_i = x} C_{\vartheta_i}^{g_i} \gamma_{k_i N_i}^{g_i} (1 - \gamma_{k_i N_i})^{\vartheta_i - g_i}, \quad (2.2)$$

where $C_{\vartheta_i}^{g_i} = \vartheta_i! / g_i! (\vartheta_i - g_i)!$, g_i denotes successful cloning attempts in ϑ_i same processes, $\gamma_{k_i N_i}$ is the success probability of $k_i \rightarrow N_i$ cloning, which is

$$\gamma_{k_i N_i} = \frac{1 - \cos^{k_i} 2\theta}{1 - \cos^{N_i} 2\theta}. \quad (2.3)$$

$P(x)$ is the discrete function of x and can be represented as a series of discrete lines in the $P(x) - x$ plane, which we called as *Probability Spectrum*. Different probabilistic cloning strategies are corresponding to different *Probability Spectrums*.

Two important parameters can be obtained from *Probability Spectrum*, that is, the expected value of the output copies number E and the probability of failure F , which are defined as

$$E\{k_i, N_i, \vartheta_i\} = \sum_{i=1}^m \sum_{x=0}^{\vartheta_i N_i} x P(x), \quad (2.4)$$

$$F\{k_i, N_i, \vartheta_i, K\} = \sum_{x=0}^{K-1} P(x). \quad (2.5)$$

It is regarded as failure if the copies number Alice attains less than the cloning goal K . When M is large, above two

parameters can well describe different cloning strategies. In the following, we discuss two most important cloning strategies (the cloning goal $K = N$):

- (1) cloning the M copies as a whole ($M \rightarrow N$).
- (2) cloning each copy respectively ($M \times (1 \rightarrow N)$).

The second is included for it is the strategy we choose in probabilistic telecloning process. Comparing above two strategies with the two parameters E and F , we find the second give more copies at the price of higher probability of failure. In fact, if Alice choose the second strategy, the cloning attempts may succeed for two or more initial copies, thus Alice may have chance to get more than N copies. The expected values for the two different strategies can be represented as

$$E_1 = N\gamma_{MN}, \quad (2.6)$$

$$\begin{aligned} E_2 &= \sum_{k=0}^M kNC_M^k \gamma_{1N}^k (1 - \gamma_{1N})^{M-k} \\ &= MN\gamma_{1N} \sum_{k=1}^M C_{M-1}^{k-1} \gamma_{1N}^{k-1} (1 - \gamma_{1N})^{(M-1)-(k-1)} \\ &= MN\gamma_{1N}, \end{aligned} \quad (2.7)$$

where $2 \leq M \leq N$. Denote $t = \cos 2\theta$, we get $\Delta E = E_2 - E_1 = N\widetilde{\Delta E} / (1 - t^N)$, where $\widetilde{\Delta E} = M - Mt - 1 + t^M$. Obviously $0 \leq t \leq 1$. When $t = 0$, $\widetilde{\Delta E} = M - 1 > 0$. If $t = 1$, $\gamma_{MN} = \frac{M}{N}$ and $\Delta E = \widetilde{\Delta E} = 0$. When $t \neq 1$, $\frac{d\widetilde{\Delta E}}{dt} = -M + Mt^{M-1} < 0$, thus $\widetilde{\Delta E}$ is monotonously decreasing and always greater than or equal to zero, that is

$$E_1 \leq E_2, \quad (2.8)$$

with equality only for $|\varphi_+(\theta)\rangle = |\varphi_-(\theta)\rangle$ ($t = 1$). ΔE is very large when M is large. The expected values for different M, N are plotted in Fig. 1.

Fig. 1

The failure probabilities of above two strategies are

$$F_1 = 1 - \gamma_{MN} = t^M (1 - t^{N-M}) / (1 - t^N), \quad (2.9)$$

$$F_2 = (1 - \gamma_{1N})^M = ((t - t^N) / (1 - t^N))^M, \quad (2.10)$$

respectively. Note the fact that for any $a_i \geq 0$, $(\prod_{i=1}^n a_i)^{\frac{1}{n}} \leq \frac{1}{n} (\sum_{i=1}^n a_i)$ with equality only for $a_1 = a_2 = \dots = a_n$, we derive

$$\begin{aligned} F_1 &= \frac{t^M}{(1 - t^N)^M} (1 - t^N)^{M-1} (1 - t^{N-M}) \\ &\leq \frac{t^M}{(1 - t^N)^M} \left(1 - \frac{t^{N-M} + (M-1)t^N}{M}\right)^M \\ &\leq \frac{t^M}{(1 - t^N)^M} (1 - t^{N-1})^M = F_2 \end{aligned} \quad (2.11)$$

with equality only for $t = 0$ or 1 ($\theta = \pi/4$ or 0). The failure probabilities for different M, N are illustrated in Fig. 2.

Fig. 2

Now that the two different strategies have both advantage and shortage, Alice should choose one according to her need. If she need more copies, she can adopt $1 \rightarrow N$ strategy. If she wishes to obtain the copies with greater success probability, she should choose $M \rightarrow N$ cloning process.

III. PROBABILISTIC TELECLONING PROCESS

Suppose Alice holds M copies of one-qubit quantum state $|\phi\rangle_X$ that is secretly chosen from the set $\{|\phi_{\pm}(\theta)\rangle = \cos\theta|1\rangle \pm \sin\theta|0\rangle\}$ and wishes to clone it to N associates (Bob, Claire, etc.). In local situation, she can do so using the unitary-reduction operation – a combination of unitary evolution together with measurements – on the $N + 1$ qubit (N qubit of the cloning system and a probe to determine whether the cloning is successful) with maximum success probability [19] $\gamma_{MN} = (1 - \cos^M 2\theta) / (1 - \cos^N 2\theta)$. This unitary-reduction operator can be decomposed into the interaction between two particles using a special unitary gate [19]:

$$D(\theta_1, \theta_2) |\phi_{\pm}(\theta_3)\rangle |1\rangle = |\phi_{\pm}(\theta_1)\rangle |\phi_{\pm}(\theta_2)\rangle. \quad (3.1)$$

with $\cos 2\theta_3 = \cos 2\theta_1 \cos 2\theta_2$ and $0 \leq \theta_j \leq \pi/4$, which suffice to determine θ_3 uniquely. This operation $D^\dagger(\theta_1, \theta_2)$ transforms the information describing the initial states $|\phi_{\pm}(\theta_1)\rangle |\phi_{\pm}(\theta_2)\rangle$ into one qubit $|\phi_{\pm}(\theta_3)\rangle$. With such pairwise interaction, the initial states $|\phi_{\pm}(\theta)\rangle^{\otimes M}$ can be transferred into states $|\phi_{\pm}(\theta_M)\rangle |0\rangle^{\otimes (M-1)}$ using corresponding operator $D_M = D_1(\theta_{M-1}, \theta_1) D_2(\theta_{M-2}, \theta_1), \dots, D_{M-1}(\theta_1, \theta_1)$, where $D_j(\theta_{M-j}, \theta_1)$ is denoted as the operator $D(\theta_{M-j}, \theta_1)$ acts on particles $(1, j+1)$ and θ_j is determined by $\cos 2\theta_j = \cos^j 2\theta$. This operator is unitary and D_M^\dagger can perform the reverse transformation. Thus we only need to transfer the states $|\phi_{\pm}(\theta_M)\rangle$ to the appropriate form $|\phi_{\pm}(\theta_N)\rangle$ to obtain $|\phi_{\pm}(\theta)\rangle^{\otimes N}$ using operation D_N^\dagger (with similar definition as D_M^\dagger). This process can be accomplished by a unitary-reduction operation

$$U |\phi_{\pm}(\theta_M)\rangle_1 |P_0\rangle = \sqrt{\gamma} |\phi_{\pm}(\theta_N)\rangle_1 |P_0\rangle + \sqrt{1 - \gamma} |1\rangle_1 |P_1\rangle, \quad (3.2)$$

where $|P_0\rangle$ and $|P_1\rangle$ are the orthogonal bases of the probe system. If a postselective measurement of probe P results in $|P_0\rangle$, the transformation is successful, otherwise the cloning attempt has failed and the result is discarded. U is unitary and the transformation probability $\gamma = \gamma_{MN}$. U is a qubit 1 controlling probe P rotation $R_y(2\omega) = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix}$ with $\omega = \arccos \sqrt{\frac{(1-\cos^M 2\theta)(1+\cos^N 2\theta)}{(1+\cos^M 2\theta)(1-\cos^N 2\theta)}}$.

Operations D_M and D_N^\dagger involve the interactions of two particles which is difficult to implement in current experiment. In this paper, we adopt $M \times (1 \rightarrow N)$ strategy to substitute D_M and transfer M copies of the states $|\phi_\pm(\theta)\rangle$ to $|\phi_\pm(\theta_N)\rangle$ respectively using similar unitary-reduction operation as that in Eq. (3.2). To substitute the operation D_N^\dagger , we use three-particle entanglement to implement the operator $D_j(\theta_{N-j}, \theta_1)$, which acts as

$$D_j(\theta_{N-j}, \theta_1) |\phi_\pm(\theta_{N-j+1})\rangle |1\rangle = |\phi_\pm(\theta_{N-j})\rangle |\phi_\pm(\theta_1)\rangle. \quad (3.4)$$

Assume Alice and the j -th associate C_j share a three-particle entangled state $|\psi^j\rangle_{SAC_j}$ as a starting resource. This state must be chosen so that, after Alice performs local Bell measurements and informs C_j of the results, she and C_j can obtain the state $|\phi_\pm(\theta_{N-j})\rangle_A |\phi_\pm(\theta_1)\rangle_{C_j}$ by using only local operation. Denote $|\varphi_i^j\rangle = D_j(\theta_{N-j}, \theta_1) |i\rangle |1\rangle$, $i \in \{0, 1\}$, a choice of $|\psi^j\rangle_{SAC_j}$ with these properties may be the three-particle state

$$|\psi^j\rangle_{SAC_j} = \frac{1}{\sqrt{2}} \left(|0\rangle_S |\varphi_1^j\rangle_{AC_j} - |1\rangle_S |\varphi_0^j\rangle_{AC_j} \right), \quad (3.5)$$

where S represents a single qubit held by Alice, which we should refer to as the ‘‘port’’ qubit. The tensor product of $|\psi^j\rangle_{SAC_j}$ with the state $|\phi_\pm(\theta_{N-j+1})\rangle_X = h_j |1\rangle \pm t_j |0\rangle$ ($h_j = \cos\theta_{N-j+1}$, $t_j = \sin\theta_{N-j+1}$) held by Alice is four-qubit state. Rewriting it in a form that singles out the Bell basis of qubit X and S , we get

$$\begin{aligned} & |\Omega^{\pm j}\rangle_{XSAC_j} \quad (3.6) \\ &= -\frac{1}{2} |\Psi^-\rangle_{XS} \left(h_j |\varphi_1^j\rangle_{AC_j} \pm t_j |\varphi_0^j\rangle_{AC_j} \right) \\ &+ \frac{1}{2} |\Psi^+\rangle_{XS} \left(h_j |\varphi_1^j\rangle_{AC_j} \mp t_j |\varphi_0^j\rangle_{AC_j} \right) \\ &\pm \frac{1}{2} |\Phi^-\rangle_{XS} \left(t_j |\varphi_1^j\rangle_{AC_j} \pm h_j |\varphi_0^j\rangle_{AC_j} \right) \\ &\pm \frac{1}{2} |\Phi^+\rangle_{XS} \left(t_j |\varphi_1^j\rangle_{AC_j} \mp h_j |\varphi_0^j\rangle_{AC_j} \right), \end{aligned}$$

where $|\Psi^\pm\rangle_{XS} = \frac{1}{\sqrt{2}} (|01\rangle_{XS} \pm |10\rangle_{XS})$, $|\Phi^\pm\rangle_{XS} = \frac{1}{\sqrt{2}} (|00\rangle_{XS} \pm |11\rangle_{XS})$ are the Bell basis of the two-qubit

system $X \otimes S$. The telecloning process can now be accomplished by the following procedure.

(i) Alice performs a Bell-basis measurement of qubits X and S , obtaining one of the four results $|\Psi^\pm\rangle_{XS}$, $|\Phi^\pm\rangle_{XS}$.

(ii) Alice use different strategies according to different measurement results. If the result is $|\Psi^-\rangle_{XS}$, the subsystem AC_j is projected precisely into the state $h_j |\varphi_1^j\rangle_{AC_j} \pm t_j |\varphi_0^j\rangle_{AC_j} = |\phi_\pm(\theta_{N-j})\rangle_A |\phi_\pm(\theta_1)\rangle_{C_j}$. If $|\Psi^+\rangle_{XS}$ is obtained, $\sigma_z \otimes \sigma_z$ must be performed on system AC_j since $|\varphi_0^j\rangle_{AC_j}$ and $|\varphi_1^j\rangle_{AC_j}$ obey the following simple symmetry:

$$\sigma_z \otimes \sigma_z |\varphi_i^j\rangle_{AC_j} = (-1)^{i+1} |\varphi_i^j\rangle_{AC_j}. \quad (3.7)$$

With above operations, the states of system AC_j are transferred to $|\phi_\pm(\theta_{N-j})\rangle_A |\phi_\pm(\theta_1)\rangle_{C_j}$, just as operation $D_j(\theta_{N-j}, \theta_1)$ functions.

(iii) In the case one of the other two Bell states $|\Phi^\pm\rangle_{XP}$ is obtained, the corresponding states are entangled states. For example, if measurement result is $|\Phi^-\rangle_{XP}$, the remained states can be written as $|\alpha_\pm\rangle = \frac{\pm 1}{\sin 2\theta_{N-j+1}} (|\phi_\pm(\theta_{N-j})\rangle |\phi_\pm(\theta_1)\rangle - \cos 2\theta_{N-j+1} |\phi_\mp(\theta_{N-j})\rangle |\phi_\mp(\theta_1)\rangle)$, which lie in the subspace spanned by states $\{|\phi_+(\theta_{N-j})\rangle |\phi_+(\theta_1)\rangle, |\phi_-(\theta_{N-j})\rangle |\phi_-(\theta_1)\rangle\}$. The inner-products show that $|\alpha_\pm\rangle$ are orthogonal to $|\phi_\mp(\theta_{N-j})\rangle |\phi_\mp(\theta_1)\rangle$. So they are entangled states unless $|\phi_+(\theta_{N-j})\rangle |\phi_+(\theta_1)\rangle$ are orthogonal to $|\phi_-(\theta_{N-j})\rangle |\phi_-(\theta_1)\rangle$, which means $|\phi_\pm(\theta)\rangle$ are orthogonal. When $|\phi_\pm(\theta_1)\rangle$ are not orthogonal, Alice and C_j must disentangle the states to the needed states $|\phi_\pm(\theta_{N-j})\rangle |\phi_\pm(\theta_1)\rangle$ simultaneously using only local operations and classical communication (LQCC). Unfortunately, this process cannot be deterministic although both transformation $|\alpha_+\rangle \rightarrow |\phi_+(\theta_{N-j})\rangle |\phi_+(\theta_1)\rangle$ and $|\alpha_-\rangle \rightarrow |\phi_-(\theta_{N-j})\rangle |\phi_-(\theta_1)\rangle$ can be deterministically executed according to Nielsen theorem [22]. In fact, suppose there exists a process H to accomplish so using only LQCC, the evolution equation of the composite system of particles A, C_j and the local auxiliary particles G^A, G^{C_j} can be expressed as

$$\begin{aligned} & H |\alpha_\pm\rangle |G_0^A\rangle |G_0^{C_j}\rangle \quad (3.8) \\ &= \sum_{i=1}^h \sum_{k=1}^l \sqrt{\eta_{ik}} |\phi_\pm(\theta_{N-j})\rangle |\phi_\pm(\theta_1)\rangle |G_i^A\rangle |G_k^{C_j}\rangle. \end{aligned}$$

H is a linear operation, thus we get

$$\begin{aligned} & H |\phi_\pm(\theta_{N-j})\rangle |\phi_\pm(\theta_1)\rangle |G_0^A\rangle |G_0^{C_j}\rangle \quad (3.9) \\ &= |\alpha_\pm\rangle \sum_{i=1}^h \sum_{k=1}^l \sqrt{\eta_{ik}} |G_i^A\rangle |G_k^{C_j}\rangle. \end{aligned}$$

Operation H use only local operations and classical communications which cannot enhance the entanglement. Obviously no entanglement exists in the left side of Eq. (3.9), but the right side is an entangled state between particle A , C_j . Thus such process H does not exist. However, consider current experiment technology, only two Bell basis $|\Psi^\pm\rangle$ of the four can be identified by interferometric schemes, with the others $|\Phi^\pm\rangle$ giving the same detection signal [8], so we only need to consider $|\Psi^\pm\rangle$ in our protocol.

After Alice obtain the state $|\phi_\pm(\theta_{N-j})\rangle_A$, she take it as the input states $|\phi_\pm(\theta_{N-j})\rangle_X$ and use another three-particle entangled state $|\psi^{j+1}\rangle$ to obtain the states $|\phi_\pm(\theta_{N-(j+1)})\rangle_A |\phi_\pm(\theta_1)\rangle_{C_{j+1}}$ between Alice and C_{j+1} , etc.,. In the last process, if Alice wishes to transmit the copies to the associates C_{N-1} and C_N , the system A should be on the side C_N . With the series transformations, the associates C_1, C_2, \dots, C_N obtain the states $|\phi_\pm(\theta_1)\rangle_{C_j}$ respectively and they finish the telecloning process.

In the following, we show how to prepare the three-particle entangled state $|\psi^j\rangle$ represented in Eq. (3.5) by LQCC using GHZ state as resource. Consider Alice and C_j initially share a GHZ state $|\xi\rangle_{SAC_j} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$, to implement the telecloning process, they must transfer it to the suitable state using only LQCC. First a local unitary operation $R_y^S(\pi/2) \otimes R_y^A(-\pi/2) \otimes R_y^{C_j}(-\pi/2)$ is performed to transfer $|\xi\rangle_{SAC_j}$ to $|\xi'\rangle_{SAC_j} = \frac{1}{4} \left((|0\rangle - |1\rangle)_S (|1\rangle + |0\rangle)_{AC_j}^{\otimes 2} + (|0\rangle + |1\rangle)_S (|1\rangle - |0\rangle)_{AC_j}^{\otimes 2} \right)$. To obtain required states, local generalized measurement (POVM) is needed, which is described by operators M_m on corresponding system, satisfying the completeness relation $\sum_m M_m^\dagger M_m = I$. After the measurement, the results (classical communication) are sent to other system, who performs a local quantum operation ε_m on its system according to the requirement of the transforma-

tion task. The operation ε_m is conditional on the result m and may be non-unitary.

However, it is difficult to perform the operation ε_m according to classical communication in experiment. In the following, we introduce a method to prepare the initial state by systems S , A and C_j performing local operations respectively without classical communication. In our protocol, there are two possible final states and both of them can be used for telecloning with same Bell states $|\Psi^\pm\rangle$ measured. Define operations M_{jim} ($i = 1, 2, 3, m = 0, 1$) on S , A , and C_j system with matrix representations $M_{j10} = \begin{pmatrix} \sin\theta_{N-j+1} & 0 \\ 0 & \cos\theta_{N-j+1} \end{pmatrix}$, $M_{j11} = \begin{pmatrix} \cos\theta_{N-j+1} & 0 \\ 0 & \sin\theta_{N-j+1} \end{pmatrix}$, $M_{j20} = \begin{pmatrix} \sin\theta_{N-j} & 0 \\ 0 & \cos\theta_{N-j} \end{pmatrix}$, $M_{j21} = \begin{pmatrix} \cos\theta_{N-j} & 0 \\ 0 & \sin\theta_{N-j} \end{pmatrix}$, $M_{j30} = \begin{pmatrix} \sin\theta_1 & 0 \\ 0 & \cos\theta_1 \end{pmatrix}$, $M_{j31} = \begin{pmatrix} \cos\theta_1 & 0 \\ 0 & \sin\theta_1 \end{pmatrix}$ on the basis $|0\rangle, |1\rangle$ respectively. Note that $M_{j10}^\dagger M_{j10} + M_{j11}^\dagger M_{j11} = I$, therefore those define a generalized measurement on each system, which may be implemented using standard techniques involving only projective measurements and unitary transforms [12]. If we consider a probe P to assist the generalized measurement $M_0 = \begin{pmatrix} \sin\theta & 0 \\ 0 & \cos\theta \end{pmatrix}$, $M_1 = \begin{pmatrix} \cos\theta & 0 \\ 0 & \sin\theta \end{pmatrix}$, the unitary operator acting on the particle and the probe can be represented as $\begin{pmatrix} R_y(-\pi+2\theta) & 0 \\ 0 & R_y(-2\theta) \end{pmatrix}$ on the basis $\{|0P_0\rangle, |0P_1\rangle, |1P_0\rangle, |1P_1\rangle\}$, where $R_y(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$ is a rotation by θ around \hat{y} . If the measurement result gives $m = 1$ for a system, then a rotation σ_x is performed on this system. Let $|\xi_{(-1)^{k+p+t}}\rangle_{SAC_j}$ denote the state after the measurement and local σ_x , given that outcome k, p, t occurred for A, C_j, S system respectively, then

$$|\xi_{(-1)^{k+p+t}}\rangle_{SAC_j} = \begin{cases} \frac{1}{\sqrt{2}} \left(|0\rangle_S |\varphi_1^j\rangle_{AC_j} - |1\rangle_S |\varphi_0^j\rangle_{AC_j} \right), & \text{when } (-1)^{k+p+t} = 1 \\ \kappa \left(\frac{t_j}{h_j} |0\rangle_S |\varphi_0^j\rangle_{AC_j} - \frac{h_j}{t_j} |1\rangle_S |\varphi_1^j\rangle_{AC_j} \right), & \text{when } (-1)^{k+p+t} = -1 \end{cases} \quad (3.10)$$

where $\kappa = \sqrt{\frac{1 - \cos^2 2\theta_{N-j+1}}{2(1 + \cos^2 2\theta_{N-j+1})}}$. The probability to obtain the first state $|\xi_1\rangle_{SAC_j}$ is $p_1 = \frac{\sin^2 2\theta_{N-j+1}}{2}$ and the second $|\xi_{-1}\rangle_{SAC_j}$ is $p_{-1} = \frac{1 + \cos^2 2\theta_{N-j+1}}{2}$. The first state in Eq. (3.10) is exactly the state in Eq. (3.5) and the second state can also be used for telecloning. In fact, the combined states of systems $XSAC_j$ can be rewritten in a form that singles out the Bell basis of qubit X and S as

$$|\psi^{\pm j}\rangle'_{XSAC_j} = \mp \frac{\kappa}{\sqrt{2}} |\Psi^-\rangle_{XS} \left(h_j |\varphi_1^j\rangle_{AC_j} \pm t_j |\varphi_0^j\rangle_{AC_j} \right) \pm \frac{\kappa}{\sqrt{2}} |\Psi^+\rangle_{XS} \left(h_j |\varphi_1^j\rangle_{AC_j} \mp t_j |\varphi_0^j\rangle_{AC_j} \right) + \frac{\eta}{\sqrt{2}} |\Phi^-\rangle_{XS} \left(h_j^3 |\varphi_1^j\rangle_{AC_j} \pm t_j^3 |\varphi_0^j\rangle_{AC_j} \right) \quad (3.11)$$

$$-\frac{\eta}{\sqrt{2}} |\Phi^+\rangle_{XS} \left(h_j^3 |\varphi_1^j\rangle_{AC_j} \mp t_j^3 |\varphi_0^j\rangle_{AC_j} \right)$$

where $\eta = \frac{2\kappa}{\sin 2\theta_{N-j+1}}$. Obviously the first two terms can be transferred to the target states using same unitary operations as those in Eq. (3.6) and states $h_j^3 |\varphi_0^j\rangle_{AC_j} \pm t_j^3 |\varphi_1^j\rangle_{AC_j} = |\phi_{\pm}(\theta_{N-j})\rangle |\phi_{\pm}(\theta_1)\rangle + \cos 2\theta_{N-j+1} |\phi_{\mp}(\theta_{N-j})\rangle |\phi_{\mp}(\theta_1)\rangle$ need not be considered.

The probabilistic quantum cloning process via GHZ states is illustrated in Fig. 3(A) and Fig. 3(B) for the case $M = 1, N = 2$.

Fig. 3(A) and Fig. 3(B)

The unitary-reduction operation U in Eq. (3.2) and the generalized measurements M_{jim} can be implemented using linear optical components, i.e., polarizing beam splitter (PBS) and polarization rotation (PR). In Ref. [5], Cerf *et al.* constructed the location controlling polarization (LCP) NOT gate using a PR. A general LCP unitary rotation can also be executed similarly. The polarization controlling location (PCL) NOT gate is performed by the use of a PBS. However, a PCL unitary rotation need two PBS and some PR since direct rotation of location qubit is impossible. Generally, a PCL unitary rotation can be represented as $V = \begin{pmatrix} R_y(\xi) & 0 \\ 0 & R_y(\chi) \end{pmatrix}$ on the orthogonal basis $\{|0\rangle|P_0\rangle, |0\rangle|P_1\rangle, |1\rangle|P_0\rangle, |1\rangle|P_1\rangle\}$, with $|0\rangle, |1\rangle$ denoted as the polarization qubit and $|P_0\rangle, |P_1\rangle$ as the location qubit. V can be decomposed into $V = V_1 V_2 V_3 V_2 V_1$, where V_1 is a LCP-NOT gate, V_2 is a PCL-NOT gate and V_3 represents a LCP unitary operation that performs $R_y(\xi)$ on the polarization qubit if the location qubit is on $|P_0\rangle$, and $R_y(-\chi)$ if the location qubit on $|P_1\rangle$. So operation V can be implemented using linear optical components as that in Fig. 4.

Fig. 4

Each generalized measurement M gives two output paths 0 and 1 and eight possible results may be output for the three photons while they only represent two possible final states $|\xi_1\rangle_{SAC_j}$ and $|\xi_{-1}\rangle_{SAC_j}$. By the use of fiber the two paths for each M can be convert into one, which means tracing out over the location qubit, and the final state of the three photons turns into the mixed state $\rho_{SAC_j} = p_1 |\xi_1\rangle \langle \xi_1| + p_{-1} |\xi_{-1}\rangle \langle \xi_{-1}|$. However, after Bell basis measurement of the tensor product state $|\phi_{\pm}(\theta_{N-j+1})\rangle_X \langle \phi_{\pm}(\theta_{N-j+1})| \otimes \rho_{SAC_j}$, the final states are still $h_j |\varphi_1^j\rangle_{AC_j} \pm t_j |\varphi_0^j\rangle_{AC_j}$ and $h_j |\varphi_1^j\rangle_{AC_j} \mp t_j |\varphi_0^j\rangle_{AC_j}$ corresponding to $|\Psi^-\rangle_{XS}$ and $|\Psi^+\rangle_{XS}$ because of Eq. (3.6) and (3.11).

Let us compare the efficiency of above telecloning process and that using Tele-C-NOT gates [6]. To complete a Tele-C-NOT operation, two GHZ states and three Bell

basis measurement are need, which yields 1/8 probability. Performing a $D_j(\theta_{N-j}, \theta_1)$ operation needs three C-NOT gates [19,20], that is, Alice only has probability of $\frac{1}{512}$ to succeed. While our protocol use one GHZ states and yields the probability

$$p = p_1 \times \frac{1}{2} + p_{-1} \times \kappa^2 \quad (3.12)$$

$$= \frac{\sin^2 2\theta_{N-j+1}}{2} = \frac{1 - \cos^2(N-j+1) 2\theta}{2}.$$

When θ is not too small, the success probability is not too low. If we do not consider the preparation of three-particle entanglement states, the efficiency of Tele- $D_j(\theta_{N-j}, \theta_1)$ is 50%, which is exactly the efficiency of Bell measurement. If we have enough GHZ states, we can prepare enough required three-particle entangled states. In the initial information compress process, we adopt the $M \times (1 \rightarrow N)$ cloning strategy. Using this strategy, more than one $|\phi_{\pm}(\theta_N)\rangle$ can be obtained. So if the Tele- $D_j(\theta_{N-j}, \theta_1)$ operation fails to one $|\phi_{\pm}(\theta_N)\rangle$, we have chance to use another and that increases the success probability. The overall cloning probability of our protocol (not include that in states preparation) can be represented as

$$P = \sum_{k=1}^M C_M^k \gamma_{1N}^k (1 - \gamma_{1N})^{M-k} \left(1 - \left(1 - \left(\frac{1}{2} \right)^{N-1} \right)^k \right). \quad (3.13)$$

P decreases with the increase of N , therefore we often adopt $1 \rightarrow 2$ cloning strategy in practice.

Up to this point, our discussion has assumed that the initially shared three-partite entangled states are pure GHZ states. Suppose, however, that $|\xi\rangle_{SAC_j}$ is corrupted a little by decoherence before it is made available to the systems S, A and C_j , so they receive a density matrix σ instead. What can we say about the final states and the probabilities of success? We argue that the final states and the probabilities do not change too much if the windages of initial states are not too large.

We discuss this problem using the *trace distance*, a metric on Hermitian operators defined by $T(A, B) \equiv \text{Tr}(|A - B|)$, where $|X|$ denotes the positive square root of the Hermitian matrix X^2 . The trace distance is a quantity with a well-defined *operational meaning* as the probability of making an error distinguishing two states [24]. In this sense it may reflect the possible physical approximation between the states: the value of the trace distance smaller, the two states more similar. A direct example is that for pure states ψ and ϕ the trace distance and the fidelity are related by a simple formula,

$$T(\psi, \phi) = 2\sqrt{1 - F(\psi, \phi)}. \quad (3.14)$$

Ruskai [23] has shown that the trace distance contracts under physical processes. More precisely, if ϖ and σ

are any two density operators, and if $\varpi' \equiv \mathcal{E}(\varpi)$ and $\sigma' \equiv \mathcal{E}(\sigma)$ denote states after some physical process represented by the (trace-preserving) quantum operation \mathcal{E} occurs, then

$$T(\varpi', \sigma') \leq T(\varpi, \sigma). \quad (3.15)$$

So, after the telecloning process, the change of the final states is limited by the trace distance between initial states $|\xi\rangle_{SAC_j}$, $\langle\xi|$ and σ , and the continuity of probability also promises the less alteration of the successful probabilities represented by Eq. (3.12) and Eq. (3.13). Of course, the final states may not be the pure cloning states we required at this situation. It may be a mixed states resembling the cloning states with the accuracy dependent on the windage of the initial states.

Such telecloning process can also be accomplished using a multiparticle entangled state, similar as that has been shown in [21]. The quality of our method is that only three-particle entanglement is used. In this scheme, we use local generalized measurements and Bell basis measurement to avoid the interactions between particles, so it may be feasible in current experiment condition.

IV. SUMMARY

In summary, we have presented a probabilistic quantum cloning scheme using GHZ states, Bell basis measurements, single-qubit unitary operations and generalized measurements, all of which are within the reach of current technology. We considered different strategies and propose the concept of *Probability Spectrum* to describe them. For two most important, we show that M entries $1 \rightarrow N$ cloning process give more copies than one $M \rightarrow N$ process at the price of higher probability of failure. Compared to another possible scheme via Tele-CNOT [6] gate, our scheme may be feasible in experiment to *clone the states of one particle to those of two different particles* with higher probability and less GHZ resource.

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Figure Captions:

Fig. 1: The expected values of copy number for the two different strategies. Angle θ is corresponding to initial states set $\{\cos\theta|1\rangle \pm \sin\theta|0\rangle\}$. Here Solid line, Dashed line, Dotted line and Dash-Dotted line denote $10 \times (1 \rightarrow 20)$, $1 \times (10 \rightarrow 20)$, $2 \times (1 \rightarrow 3)$ and $1 \times (2 \rightarrow 3)$ cloning strategies respectively.

Fig. 2: The failure probabilities for the two different strategies. The four kinds of lines represent the same strategies as those in Fig. 1.

Fig. 3(A): The logic network of $1 \rightarrow 2$ probabilistic cloning via GHZ state. Alice and her associate C_1 , C_2 initially share a GHZ state consisting of the qubit S (the port), C_1 and C_2 (outputs, or ‘copy qubits’). Alice successfully transforms the initial states $\cos \theta |1\rangle_X \pm \sin \theta |0\rangle_X$ to $\cos \theta_2 |1\rangle_X \pm \sin \theta_2 |0\rangle_X$ if the probe (the location qubit of the photon X) results in $|P_0\rangle$, where the parameters $\cos^2 2\theta_2 = \cos 2\theta$, $\omega = \arccos \sqrt{\frac{(1+\cos^2 2\theta)}{(1+\cos 2\theta)^2}}$. Using the unitary rotation $R_y(\varsigma)$ and generalized measurement $M(\theta)$, Alice and C_1 , C_2 transform GHZ state to the required three-particle entangled state in the form

Eq. (3.10). Then Alice performs a Bell measurement of the port S along with ‘input’ qubit X and has 25% probability to obtain $|\Psi^-\rangle$ or $|\Psi^+\rangle$ respectively; subsequently, the receivers C_1 and C_2 do no operation or σ_x rotations on the output qubits, obtaining two perfect quantum clones. The implementation of generalized measurement $M(\theta)$ is illustrated in Fig. 3(B).

Fig. 3(B): The implementation of generalized measurement $M(\theta)$ in Fig. 3(A). The location qubit of the photon is adopted as the probe P .

Fig. 4: Optical simulation of PCL unitary rotation by the use of two polarizing beam splitters and some polarizing rotators, where PR1 performs operation $R_y(\xi)$ and PR2 executes operation $R_y(-\chi)$.

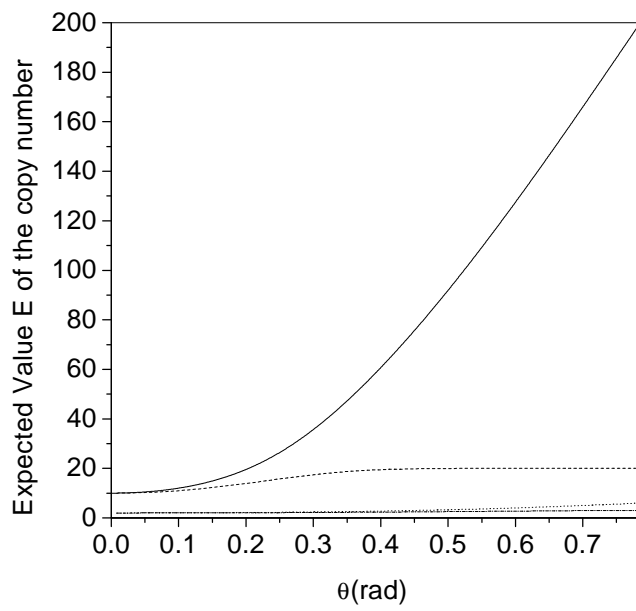


Fig. 1

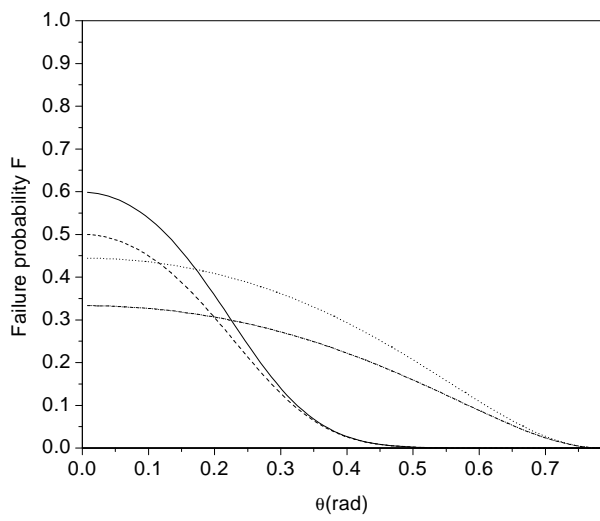


Fig.2

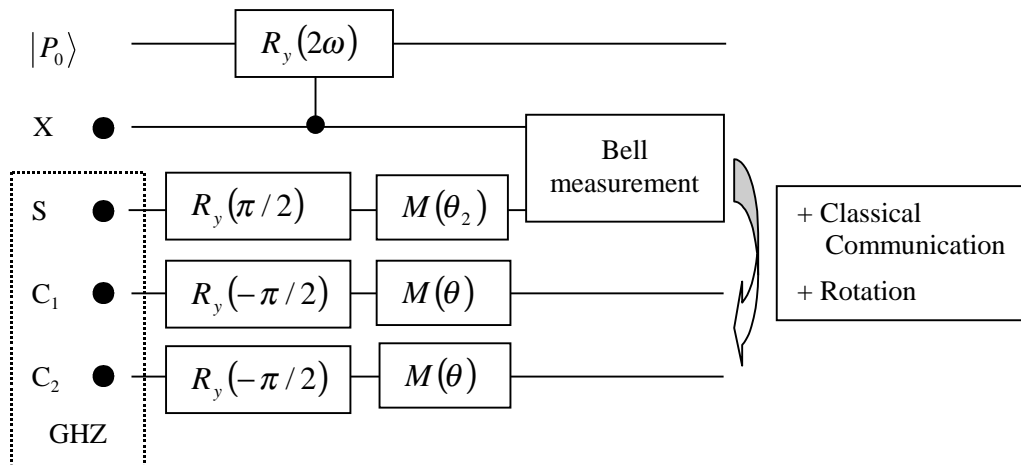


Fig. 3 (A)

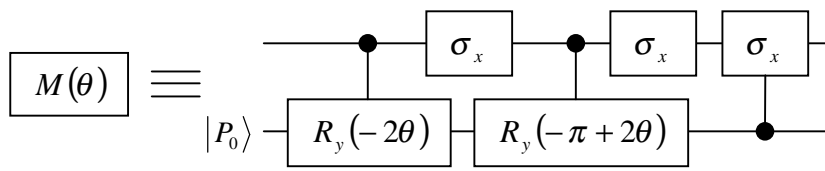


Fig. 3(B)

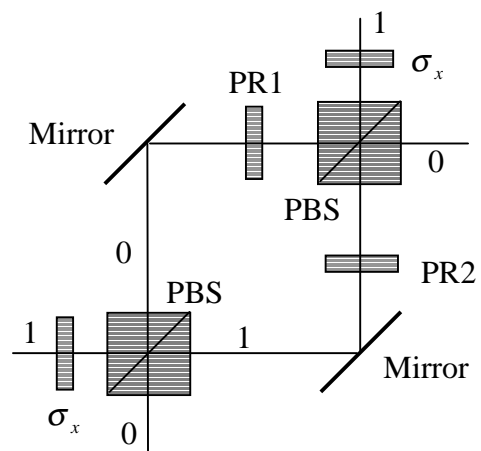


Fig. 4