Antibunching effect of the radiation field in a microcavity with a mirror undergoing heavily damping oscillation

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The interaction between the radiation field in a microcavity with a mirror undergoing damping oscillation is investigated. Under the heavily damping cases, the mirror variables are adiabatically eliminated. The the stationary conditions of the system are discussed. The small fluctuation approximation around steady values is applied to analysis the antibunching effect of the cavity field. The antibunching condition is given under two limit cases.

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1. Introduction

There has been interest in the investigation of the non-classical behavior of the light field, for example, the squeezed state of the electromagnetic field [1–4]. A single mode or a multimode electromagnetic field is squeezed when the fluctuation of its one quadrature component is reduced to below the standard quantum limit. But the fluctuation of its conjugate partner will be enlarged in terms of the uncertainty relation. The squeezed electromagnetic field could be applied to the optical communication and ultrasensitive gravitational wave detection.

Recently, it has been shown that a cavity with free oscillating mirror might be employed as a model for squeezing. It was firstly proposed by Stenholm without taking into account the effect of the fluctuation on the oscillating mirror [5]. Since then, this model is generalized to the coupling of the system to the external world [6–8]. But up to now, another non-classical behavior of the radiation field in a microcavity with a movable mirror , such as antibunching effect, isn't still studied.

In this paper, we will discuss the antibunching effect of the radiation field in a microcavity with a mirror undergoing heavily damping oscillation. In section 2, we firstly give a quantizing Hamiltonian including external dissipative effect. The master equation of the reduced density matrix for the system is given. We convert the master equation into a c-number equation (the Fokker-Planck equation) by using the two-mode positive P representation. Then the stochastic equation corresponding to the Fokker-Planck equation is obtained. In section 3, we discuss the stability of the system under the good cavity limit and linearize the stochastic equation around stable values. In section 4, antibunching effect is investigated and antibunching condition of the system is given. Finally we give the conclusion of this paper.

2. Hamiltonian and Master equation

We firstly consider the interaction between a single mode cavity field and a movable mirror. Based on reference [6–8], we have a effective quantizing Hamiltonian

$$H_0 = \hbar\omega_c a^+ a + \hbar\omega_m b^+ b - \hbar g a^+ a(b^+ + b) + i\hbar(E(t)a^+ - E^*(t)a) \tag{1}$$

where $E(t)$ is proportional to the amplitude of the external driving field of the cavity mode. $E^*(t)$ is complex conjugate of $E(t)$. ω_c is the cavity frequency. $a(a^+)$ are annihilate (create) operators of the cavity field. ω_m is a frequency of the mirror. $b(b^+)$ are annihilate (create) operators of the mirror. $g = \frac{\omega_c}{L}(\frac{\hbar}{2m\omega_m})$. m is a mass of the mirror and L is the equilibrium cavity length.

If the cavity is bad, and the mirror is damped by the circumstance when it is moving, then the Hamiltonian (1) need correct to add the dassipative effect of the circumstance. So we have

$$H = H_0 + \hbar a^+\Gamma_1 + \hbar a\Gamma_1^+ + \hbar b^+\Gamma_2 + \hbar b\Gamma_2^+ \tag{2}$$

where $\Gamma_1(\Gamma_1^+)$ is the reservoir operators of the cavity. We also model the damping effect of the mirror as the result of the interaction between the mirror and the many harmonic oscillator. They satisfy the Markovian correlation function (where for simplicity, we only consider the zero temperature case):

$$\begin{aligned} <\Gamma_i(t)\Gamma_i^+(t')> &= \gamma_i\delta(t-t') \\ <\Gamma_i^+(t)\Gamma_i(t')> &= 0 \end{aligned} \tag{3}$$

with $i = 1, 2$. γ_1 is the decay rate of the cavity field. γ_2 is the damping coefficient of the motion mirror. The master equation of the reduced density matrix for the system is

$$\begin{aligned} \frac{\partial\rho}{\partial t} &= \frac{1}{i\hbar}[H_0,\rho] + \gamma_1(2a\rho a^+ - \rho a^+ a - a^+ a\rho) \\ &+ \gamma_2(2b\rho b^+ - \rho b^+ b - b^+ b\rho) \end{aligned} \tag{4}$$

We could convert the operator eq.(4) into a c-number equation (the Fokker-Planck Equation) by using the two-mode positive P representation [9] . This representation ensures that the P function exists as a well-behaved distribution which is singular when the usual Glauber-Sudarshan P representation was applied. That is:

$$\begin{aligned} \frac{\partial P}{\partial t} &= [\frac{\partial}{\partial\alpha_1}(i\omega_c\alpha_1 - ig\alpha_1\alpha_2^+ - ig\alpha_1\alpha_2 + \gamma_1\alpha_1 - E(t)) \\ &+ \frac{\partial}{\partial\alpha_1^+}(-i\omega_c\alpha_1^+ + ig\alpha_1^+\alpha_2^+ + ig\alpha_1^+\alpha_2 + \gamma_1\alpha_1^+ - E^*(t)) \\ &+ \frac{\partial}{\partial\alpha_2}(i\omega_m\alpha_2 - ig\alpha_1^+\alpha_1 + \gamma_2\alpha_2) \\ &+ \frac{\partial}{\partial\alpha_2^+}(-i\omega_m\alpha_2^+ + ig\alpha_1^+\alpha_1 + \gamma_2\alpha_2^+) \\ &+ \frac{\partial}{\partial\alpha_1}\frac{\partial}{\partial\alpha_2}ig\alpha_1 - \frac{\partial}{\partial\alpha_1^+}\frac{\partial}{\partial\alpha_2^+}ig\alpha_1^+]P \end{aligned} \tag{5}$$

where α_1 and α_1^+, and α_2 and α_2^+ are no longer complex conjugate of each other, instead they are independent complex variables. In terms of the Ito ruler, the Fokker-Planck eq.(5) is equivalent to the following set of the stochastic equations.

$$\frac{\partial\alpha_1}{\partial t} = -i\omega_c\alpha_1 + ig\alpha_1\alpha_2^+ + ig\alpha_1\alpha_2 - \gamma_1\alpha_1 + E(t) + \Gamma_{\alpha_1} \tag{6a}$$

$$\frac{\partial\alpha_1^+}{\partial t} = i\omega_c\alpha_1^+ - ig\alpha_1^+\alpha_2^+ - ig\alpha_1^+\alpha_2 - \gamma_1\alpha_1^+ + E^*(t) + \Gamma_{\alpha_1^+} \tag{6b}$$

$$\frac{\partial\alpha_2}{\partial t} = -i\omega_m\alpha_2 + ig\alpha_1^+\alpha_1 - \gamma_2\alpha_2 + \Gamma_{\alpha_2} \tag{6c}$$

$$\frac{\partial\alpha_2^+}{\partial t} = i\omega_m\alpha_2^+ - ig\alpha_1^+\alpha_1 - \gamma_2\alpha_2^+ + \Gamma_{\alpha_2^+} \tag{6d}$$

where Γ_{α_i} and $\Gamma_{\alpha_i^+}$ are Gaussian random variables with zero mean. Their correlation functions satisfy:

$$<\Gamma_{\alpha_1}(t)\Gamma_{\alpha_2}(t')> \;= <\Gamma_{\alpha_2}(t)\Gamma_{\alpha_1}(t')>= ig\alpha_1\delta(t-t') \tag{7a}$$

$$<\Gamma_{\alpha_1^+}(t)\Gamma_{\alpha_2^+}(t')> \;= <\Gamma_{\alpha_2^+}(t)\Gamma_{\alpha_1^+}(t')>= -ig\alpha_1^+\delta(t-t') \tag{7b}$$

$$<\Gamma_{\alpha_l}(t)\Gamma_{\alpha_m^+}(t')> \;= <\Gamma_{\alpha_m^+}(t)\Gamma_{\alpha_l}(t')>= 0 \tag{7c}$$

with $l = 1$ or 2 and $m = 1$ or 2. The above correlation functions could be obtained from the matrix elements of the diffusion matrix of eq.(5) by using Ito ruler.

3. Adiabatic elimination of the mirror variable and stability analysis

In the good-cavity limit, that is, the damping coefficient of the mirror is much larger than the decay rate of the cavity field ($\gamma_2 \gg \gamma_1$). ˚This means that the decay time of the cavity field is much longer than the decay time of the mirror. So the mirror variables could be adiabatically eliminated from eqs.(6a-6b) . Firstly we find a steady-state of the mirror variables . In the case of the steady state, $\dot{\alpha}_2 = 0$ and $\dot{\alpha}_2^+ = 0$. So we have from eqs.(6c-6d)

$$-i\omega_m\alpha_2 + ig\alpha_1^+\alpha_1 - \gamma_2\alpha_2 + \Gamma_{\alpha_2} = 0 \tag{8a}$$
$$i\omega_m\alpha_2^+ - ig\alpha_1^+\alpha_1 - \gamma_2\alpha_2^+ + \Gamma_{\alpha_2^+} = 0 \tag{8b}$$

From eqs.(8a-8b), we obtain:

$$\alpha_2 = \frac{\Gamma_{\alpha_2} + ig\alpha_1\alpha_1^+}{\gamma_2 + i\omega_m} \tag{9a}$$
$$\alpha_2^+ = \frac{\Gamma_{\alpha_2^+} - ig\alpha_1\alpha_1^+}{\gamma_2 - i\omega_m} \tag{9b}$$

We substitute eqs.(9a-9b) into eqs.(6a-6b) and eliminate the mirror variables. Then we have:

$$\frac{\partial\alpha_1}{\partial t} = -i\omega_c\alpha_1 + iG(\omega_m)\alpha_1^+\alpha_1^2 - \gamma_1\alpha_1 + \Gamma \tag{10a}$$
$$\frac{\partial\alpha_1^+}{\partial t} = i\omega_c\alpha_1^+ - iG(\omega_m)\alpha_1^{+2}\alpha_1 - \gamma_1\alpha_1^+ + \Gamma^+ \tag{10b}$$

with $G(\omega_m) = \frac{2g^2\omega_m}{\gamma_2+\omega_m^2}$ and

$$\Gamma = \frac{ig\alpha_1}{\gamma_2 + i\omega_m}\Gamma_{\alpha_2} + \frac{ig\alpha_1}{\gamma_2 - i\omega_m}\Gamma_{\alpha_2^+} + \Gamma_{\alpha_1} \tag{11a}$$
$$\Gamma^+ = -\frac{ig\alpha_1^+}{\gamma_2 + i\omega_m}\Gamma_{\alpha_2} - \frac{ig\alpha_1^+}{\gamma_2 - i\omega_m}\Gamma_{\alpha_2^+} + \Gamma_{\alpha_1^+} \tag{11b}$$

Using eqs.(7a-7c) and eqs.(11a-11b), we calculate the correlation function

$$< \Gamma(t)\Gamma^+(t') > \; = < \Gamma^+(t)\Gamma^+(t) >= \frac{2\gamma_2 g^2 n}{\gamma_2^2 + \omega_m^2}\delta(t-t') \tag{12a}$$
$$< \Gamma^+(t)\Gamma^+(t') > \; = -\frac{2g^2\alpha_1^{+2}}{\gamma_2 - i\omega_m}\delta(t-t') \tag{12b}$$
$$< \Gamma(t)\Gamma(t') > \; = -\frac{2g^2\alpha_1^2}{\gamma_2 + i\omega_m}\delta(t-t') \tag{12c}$$

The eqs.(10a-10b) are difficult to be solved. In general, we are interested in the properties of the steady state. So we denote the steady values of α_1 and α_1^+ by α_0 and α_0^+ respectively. We assume that the system has a small approximation around the steady values, namely

$$\begin{cases} \alpha_1(t) = \alpha_0 + \delta\alpha_1(t) \\ \alpha_1^+(t) = \alpha_0^+ + \delta\alpha_1^+(t) \end{cases} \tag{13}$$

So non-linear eqs.(10a-10b) are simplified into the following linear equations around the steady values.

$$\frac{\partial}{\partial t}\begin{pmatrix} \delta\alpha_1 \\ \delta\alpha_1^+ \end{pmatrix} = \begin{pmatrix} -i\omega_c - \gamma_1 + i2G(\omega_m)\alpha_0^+\alpha_0 & iG(\omega_m)\alpha_0^2 \\ -iG(\omega_m)\alpha_0^{+2} & i\omega_c - \gamma_1 + i2G(\omega_m)\alpha_0^+\alpha_0 \end{pmatrix}\begin{pmatrix} \delta\alpha_1 \\ \delta\alpha_1^+ \end{pmatrix} + \begin{pmatrix} -\frac{2g^2\alpha_0^2}{\gamma_2+i\omega_m} & \frac{2\gamma_2 g^2 n}{\gamma_2^2+\omega_m^2} \\ \frac{2\gamma_2 g^2 n}{\gamma_2^2+\omega_m^2} & -\frac{2g^2\alpha_0^{+2}}{\gamma_2-i\omega_m} \end{pmatrix}^{\frac{1}{2}}\begin{pmatrix} \eta_1(t) \\ \eta_2(t) \end{pmatrix} \tag{14}$$

where $n = \alpha_0^+\alpha_0$ and $\eta_i(t)$ statisfy delta correlation function:

$$< \eta_i(t) > \; = 0 \tag{15a}$$
$$< \eta_i(t)\eta_j(t') > \; = \delta_{ij}\delta(t-t') \tag{15b}$$

We abbreviate eq.(8) as following:

$$\frac{\partial}{\partial t}\delta\vec{\alpha}(t) = -A\vec{\alpha}(t) + D^{\frac{1}{2}}\vec{\eta}(t) \tag{16}$$

But one of the important feature of eq.(14) is whether the steady solutions are stable, that is, when α_1 and α_1^+ somewhat deviate from α_0 and α_0^+, whether they will still return to steady values. So we neglect the fluctuation forces of eq.(16) and have:

$$\frac{\partial}{\partial t}\delta\vec{\alpha}(t) = -A\vec{\alpha}(t) \tag{17}$$

In order to investigate the stationary of the system, we seek the solutions of the form $e^{-\lambda t}$ of eq.(17). The eigenvalues λ are determined by the equation

$$|A - \lambda I| = 0 \tag{18}$$

where I is a identity matrix. The solutions of the eq.(18) are

$$\lambda = \gamma_1 \pm \sqrt{(\omega_c - 3G(\omega_m)n)(\omega_c - G(\omega_m)n)} \tag{19}$$

This equation indicates if $G(\omega_m)n \leq \omega_c \leq 3G(\omega_m)n$, the real part of the eigen-solution of the eq.(18) is positive. The system is stable. When $\omega_c \leq nG(\omega_m)$ or $\omega_c \geq 3nG(\omega_m)$, only if $\gamma_1 \geq \sqrt{(\omega_c - 3G(\omega_m)n)(\omega_c - G(\omega_m)n)}$ then the system is also stable. So only we chose the proper parameter, the system always may reach to stability. The small fluctuation approximation is appreciate.

4. Second-order correlation function

The intensity correlation is another quantity of the experimental interest besides the first order optical coherence. It's truly photon correlation measurement. Theoretically, people have defined a second-order correlation function to investigate the joint photocount probability of detecting the arrival of a photon at one time and other ptoton at another time. For zero time delay, The second order correlation function is [11]

$$g^{(2)}(0) = \frac{< a^+a^+aa >}{< a^+a >^2} \tag{20}$$

Under the positive P representation we keep the second order terms of $\frac{\delta\alpha}{\alpha}$, then eq(20) becomes into:

$$g^{(2)}(0) == \frac{< \alpha^{+2}\alpha^2 >}{< \alpha^+\alpha >^2} = 1 + \frac{< \delta\alpha^2 >}{\alpha^2} + \frac{< \delta\alpha^{+2} >}{\alpha^{+2}} + 4\frac{< \delta\alpha^+\delta\alpha >}{\alpha^+\alpha} \tag{21}$$

In order to calculate the correlation functions $< (\delta\alpha)^2 >$ et.al, we need calculate matrix:

$$C = \begin{pmatrix} < \delta\alpha\delta\alpha > & < \delta\alpha\delta\alpha^+ > \\ < \delta\alpha^+\delta\alpha > & < \delta\alpha^+\delta\alpha^+ > \end{pmatrix} \tag{22}$$

The matrix C could be obtained by calculating of $\delta\alpha_1$ and $\delta\alpha_1^+$ from the linearized eq.(14). It is also could be given by [10]:

$$C = \frac{D.Det(A) + (A - ITr(A))D(A - ITr(A))^T}{2Tr(A)Det(A)}. \tag{23}$$

after tedious calculation, we have:

$$C_{11} = C_{22}^* = \frac{\alpha_1^2[2M\gamma_1^2 + 2NG(B + i\gamma_1) - i2\gamma_1 BM - G^2n^2(M + M^*)]}{4\gamma_1(\gamma_1^2 + B^2 - G^2n^2)} \tag{24a}$$

$$C_{12} = C_{21} = \frac{2N(\gamma_1^2 + B^2) + i\gamma_1 Gn^2(M^* - M) - GBn^2(M + M^*)}{4\gamma_1(\gamma_1^2 + B^2 - G^2n^2)} \tag{24b}$$

with

$$M = -\frac{2g^2}{\gamma_2 + i\omega_m} \quad N = \frac{2\gamma_2 g^2 n}{\gamma_2^2 + \omega_m^2} \quad B = \omega_c - 2Gn \tag{25}$$

So the second order correlation function is:

$$g(0) = 1 + \frac{G[(\omega_c - 2nG)(2n\gamma_2\omega_c - n^2\gamma_2 G - \gamma_1\omega_m) + n(\gamma_1^2\gamma_2 + n^2G^2\gamma_2 - 2\gamma_1\omega_m Gn)]}{n\gamma_1\omega_m[\gamma_1^2 - (\omega_c + 5nG)(\omega_c - nG)]} \tag{26}$$

From this equation, we see that the antibunching of the cavity field appear when the second term of the eq.(26) is a negative number. This condition could be satisfied after we chose some proper parameters. Now we consider two limit cases. In the case of the strong cavity field, we only keep the terms including n^3. The second term of eq.(26) is positive, no antibunching appears. But under case of the weak cavity field, namely when $n \to 0$, the cavity field presents antibunching behavior.

5. Conclusion

The interaction between the radiation field in a microcavity with a movable mirror undergoing heavily damping oscillation is investigated. Under the heavily damping cases, the mirror variables are adiabatically eliminated. The stationary condition of the system is given. The small fluctuation approximation around steady values is applied to analysis the antibunching behavior of the cavity field. In the case of the strong cavity field, no antibunching appears. But under the case of the weak cavity field, cavity field presents the antibunching effect.

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