Bloch oscillations of optical NOON states

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We show that when photons in NOON states undergo Bloch oscillations, they exhibit a periodic transition between spatially bunched and antibunched states. The period of the bunching/antibunching oscillation is N times faster than the period of the oscillation of the photon density, manifesting the unique coherence properties of NOON states. The transition occurs even when the photons are well separated in space.

When electrons in crystalline potentials are subjected to uniform external fields, classical mechanics predicts that they will exhibit Ohmic transport. Remarkably, in 1928 F. Bloch predicted that the quantum coherence properties of the electrons prevent their transport [1, 2]. He showed that the electrons dynamically localize and undergo periodic oscillations in space. Bloch oscillations (BOs) manifest the wave properties of the electrons, and therefore appear in other systems of waves in tilted periodic potentials. BOs were observed for electronic wavepackets in semiconductor supperlattices [3], matter waves in optical lattices [4] and light waves in tilted waveguide lattices [5–7].

In optics, BOs manifest the classical wave properties of light, and not its quantum (particle) nature. Recently, quantum properties of light propagating in waveguide lattices have been studied, predicting the emergence of non trivial photon correlations [8, 9]. Such non-classical correlations between photon pairs were experimentally observed in homogeneous waveguide lattices [10]. BOs of a single photon in tilted lattices were shown to follow the dynamics of coherent states [11]. Non-classical features of BOs of light in a two-band model were studied by Longhi, who showed that the probability to detect photon pairs in different bands oscillates nonclassically [12].

In this paper we study BOs for spatially entangled states of light propagating in waveguide lattices. We consider light fields initiated in a superposition of N photons in site μ' or in site ν', $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|N\rangle_{\mu'}|0\rangle_{\nu'} + e^{i\varphi}|0\rangle_{\mu'}|N\rangle_{\nu'}\right)$. Such superpositions, coined NOON states, exhibit fascinating quantum interference properties. NOON states are considered the optimal quantum states of light for quantum meteorology applications such as quantum lithography and quantum imaging [13]. Here we show that when NOON states undergo BOs, the nature of the correlations between the photons oscillate between spatially bunched and antibunched states. We find that the period of the oscillations is inversely proportional to the photon number N, resembling the λ/N oscillations of NOON states in Mach-Zehnder interferometers. Interestingly, the oscillation period is also inversely proportional to the initial separation of the two input sites $\mu'-\nu'$. A unique feature of the NOON state BOs is that the transition between the bunched and antibunched states can happen even when the photons are separated by many lattice sites.

We consider the simplest waveguide structure which exhibits BOs, a one dimensional lattice of single mode waveguides which are evanescently coupled. The light propagation in the lattice is determined by two parameters: the rate of the phase accumulation in the waveguides (the propagation constant) and the tunneling rate between neighboring sites (the coupling constant) [14]. The propagation of the fields in waveguide lattices is described by the tight-binding model, and was used to demonstrate many optical analogues of solid-state phenomena [15]. BO are observed when the coupling constants of all the waveguides are identical and the propagation constants depend linearly on the waveguide position [5, 6]. To study the propagation of non-classical light in such a structures we quantize the fields in the lattice. Since each of the waveguides supports a single mode, the field in waveguide μ is represented by the bosonic creation and annihilation operators $a_\mu^\dagger$ and a_μ, which satisfy the commutation relations $[a_\mu, a_\nu^\dagger] = \delta_{\mu,\nu}$. The operators evolve according to the Heisenberg equations [9]:

$$i\frac{\partial a_\mu^\dagger}{\partial z} = \mu B a_\mu^\dagger + C\left(a_{\mu+1}^\dagger + a_{\mu-1}^\dagger\right). \tag{1}$$

Here z is the spatial coordinate along the propagation axis, C is the coupling constant and B is the difference in the propagation constants of neighboring sites. The evolution of the creation/annihilation operators is calculated using the Green function $U_{\mu,\mu'}(z)$ of Eq. (1), $a_\mu^\dagger(z) = \sum_{\mu'} U_{\mu,\mu'}(z) a_{\mu'}^\dagger(z=0)$ [9]. The unitary transformation $U_{\mu,\mu'}(z)$ describes the amplitude for the transition of a single photon from waveguide μ' to waveguide μ. The Green function of Eq. (1) is given by [6, 17]:

$$U_{\mu,\mu'}(z) = e^{i\frac{\pi}{2}(\mu'-\mu)} e^{i\frac{Bz}{2}(\mu'+\mu)} J_{\mu'-\mu}\left(4C/B\, sin\left(Bz/2\right)\right), \tag{2}$$

where $J_\mu(x)$ is the μ^{th} Bessel function of the first kind. Since any input state can be expressed with the creation operators $a_\mu^\dagger$ and the vacuum state $|0\rangle$, the evolution of nonclassical states along the lattice can be calculated using Eq. (2). The probability to locate at site μ a photon that is injected into the lattice at site $\mu' = 0$ is given by the photon density $n_\mu = \langle a_\mu^\dagger a_\mu\rangle = |U_{\mu,\mu'=0}|^2$ and is depicted in Fig. 1a. The photon exhibits BOs: it

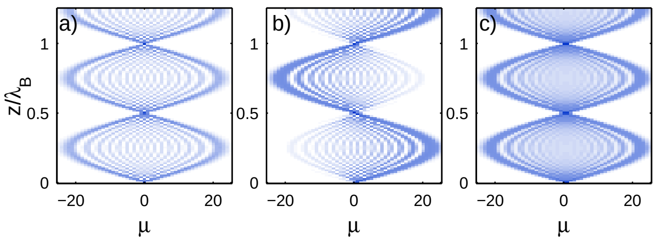

Figure 1: (a) (Color online) The photon density $n_\mu(z) = \langle a_\mu^\dagger a_\mu \rangle$ for a single photon initiated at the waveguide $\mu' = 0$. The photon is localized in two branches. Each branch consists of a main lobe which covers approximately 4 waveguides and oscillates around the input waveguide with a period λ_B. (b) The photon density $n_\mu(z)$ for a NOON state input with N=1 coupled to waveguides $\mu' = 0$ and $\nu' = 1$. The photon is located at a single branch which oscillates around the input waveguide with a period λ_B. (c) The photon density $n_\mu(z)$ for a NOON input state with N=2 coupled to waveguides $\mu' = 0$ and $\nu' = 1$ The contribution of photons coming from the $\mu' = 0$ input and the $\mu' = 1$ input add up incoherently, showing double branch oscillations.

spreads across the lattice by coupling from one waveguide to its neighbors in a pattern characterized by two strong branches. Each branch consists of a main lobe which covers approximately 4 waveguides, and oscillates around the input site with a period $\lambda_B = 4\pi/B$. We note that such a double-branch pattern is not a special feature of single photons. Any state of light which is coupled to a single waveguide in the lattice will exhibit exactly the same photon density. However, when the light is coupled to more than one waveguide, the propagation of the photons becomes state-dependent. Rai et al. have shown that a single photon which is coupled into the lattice in a superposition of several waveguides exhibits BOs like a coherent state [11]. Figure 1b shows the photon density for a superposition of a single photon coupled to two neighboring waveguides. The two paths the photon can take, starting either from waveguide $\mu' = 0$ or from waveguide $\nu' = 1$, contribute coherently to the photon density $n_\mu = |U_{\mu,\mu'=0} + U_{\mu,\nu'=1}|^2$. Due to this interference the photon oscillates in a single branch, exactly like a coherent beam. In contrast, when a NOON state with $N > 1$ is coupled to the lattice, the photon density is identical to the photon density obtained by two incoherent beams $n_\mu = \frac{N}{2}|U_{\mu,\mu'=0}(z)|^2 + \frac{N}{2}|U_{\mu,\nu'=1}(z)|^2$ (Fig. 1c).

Non-classical features of quantum states of light, and specifically of NOON states, are probed by the correlations between the photons. We focus on the multiple detection probability for detecting p photons in waveguide μ and q photons in waveguide ν $\Gamma^{(p,q)}_{\mu,\nu} = \langle a_\mu^{\dagger^p} a_\nu^{\dagger^q} a_\nu^q a_\mu^p \rangle$ [18]. For a NOON state coupled to waveguides $\mu'$and ν', the multiple detection probability is:

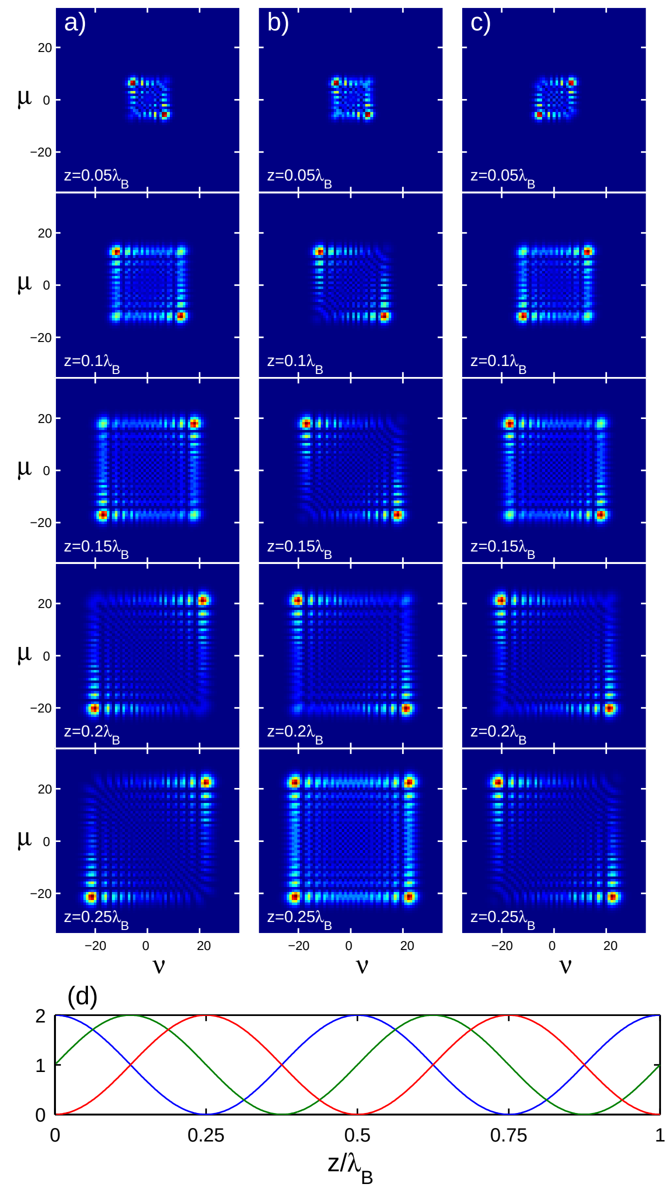

Figure 2: (Color online) Bloch oscillations of NOON states with $N = 2$ coupled to two adjacent waveguides $|\psi\rangle = \frac{1}{\sqrt{2}}(|2\rangle_0 |0\rangle_1 + e^{i\varphi} |0\rangle_0 |2\rangle_1)$. (a) The multiple detection probability $\Gamma^{(1,1)}_{\mu,\nu}$ at several propagation distances, for $\varphi = 0$. At the beginning of the propagation the two photons exhibit antibunching and are located at the two different branches of the oscillations. As the photons approach the turning point ($z = \frac{\lambda_B}{4}$), they bunch and are found with the highest probability in the same branch. (b) Same as (a) for $\varphi = \frac{\pi}{2}$. The photons show bunching/antibunching cycle, but in this case start the oscillation partially bunched. (c) Same as (a) and (b), for $\varphi = \pi$. Here the photons start the bunching/antibunching cycle bunched. (d) The normalized coincidence rate $\gamma^{(1,1)}(z)$ as a function of the lattice length for $\varphi = 0$ (blue line), $\varphi = \frac{\pi}{2}$ (green line), and $\varphi = \pi$ (red line). The coincidence rate is calculated between the positions of the central waveguide in each branch, showing oscillations with a period $\lambda_B/2$. See [25] for a movie visualizing the propagation of (a).

$$\Gamma^{(p)}_{\mu,\nu} = \binom{N}{p,q}\Big| J_{\mu'-\mu}(\zeta)^p J_{\mu'-\nu}(\zeta)^q + e^{i\theta(z)} J_{\nu'-\mu}(\zeta)^p J_{\nu'-\nu}(\zeta)^q \Big|^2 \quad (3)$$

Where $\binom{N}{p,q} = \frac{N!}{p!q!(N-p-q)!}$, $\zeta = 4C/B\, sin\,(Bz/2)$, and the phase $\theta(z)$ is given by:

$$\theta(z) = \varphi + \frac{1}{2}(\pi + Bz)(\nu' - \mu')N\,. \quad (4)$$

Eq. (3) shows that two terms contribute to the multiple detection probability: the detected photons arrive either from the input waveguide μ' or from waveguide ν'. Since the photons are indistinguishable, these two paths interfere. The phase of the interference term increases linearly with $N\,z$, an indication for oscillations with a period that scales like $1/N$ (see below).

In Fig. 2 we depict $\Gamma^{(1,1)}_{\mu,\nu}$, the probability to detect one photon at waveguide μ and another photon at waveguide ν, for a NOON state with $N = 2$. The left column shows the evolution of the probability for a NOON state with a phase $\varphi = 0$. At the beginning of the propagation ($Bz \ll \frac{\pi}{2}$), the photons follow the same path as in a periodic array of identical waveguides [9, 10]. At this stage the off-diagonal terms of the probability matrix $\Gamma^{(1,1)}_{\mu,\nu}$are much stronger than the diagonal term, indicating that the photons exhibit antibunching: each photon takes a different branch of the oscillation. However, during the expansion period of the BO, as the photons approach the turning point, the symmetry of the two-photon probability matrix changes significantly. The diagonal terms of the matrix become more pronounced, i.e. there is a higher probability to find the two photons in the same branch of oscillations. At the turning point $z = \frac{\lambda_B}{4}$, the photons bunch: the off-diagonal terms of the probability matrix vanish, indicating that the photons are never found simultaneously at the two different branches. Remarkably, even though the photons start the propagation in spatially separated branches, at the turning point they bunch to one of the branches, with equal probability. Beyond this point the photon density contracts back towards the origin waveguides. During this contraction the pairs again switch to an antibunched state. We note that the bunching/antibunching transition happens when the two branches of the BO are spatially separated, whereas the bunching/antibunching transition predicted in binary lattices occurs only when the photons are in the same waveguide [12]. The cycle in the symmetry of the probability matrix is observed for any initial phase of the NOON state phase φ, as demonstrated Fig 2b and 2c. The phase φ sets how bunched/antibunched the photon is at the $z = 0$, but the period of the cycle is phase-independent (see Eq. (4)).

The bunching/antibunching cycle described above can be realized experimentally in arrays of evanescently coupled waveguides. A straight forward approach is to measure the correlation between the photons in lattices with identical parameters but with different propagation lengths, thus probing different stages of the oscillations. For each propagation length, the outputs of two the waveguides at the center of each oscillating branch (henceforth waveguides x and y) can be imaged on two photon-number resolving detectors. The rate of events of p photons detected at waveguide x and q photons at waveguide y is proportional to $\Gamma^{(p,q)}_{x,y}$. This should be compared to the rate of the same events when a delay is introduced between the photons that are injected to waveguide μ' and those injected to waveguide ν'. Similar to the Hong-Ou-Mandel (HOM) experiment, such a delay introduces distinguishability between the photons [19], and corresponds to replacing the NOON state with an initial mixed state of N photons in either one of the two input waveguides. The ratio of multiple detection events for the NOON and mixed states is given by

$$\gamma^{(p,q)} = \frac{\Gamma^{(q,p)}_{x,y}}{\binom{N}{p,q}\left(|J^p_{\mu'-x}J^q_{\mu'-y}|^2 + |J^p_{\nu'-x}J^q_{\nu'-y}|^2\right)}. \quad (5)$$

Figure 2(d) shows $\gamma^{(1,1)}$ as a function of the lattice length, for NOON states with $N = 2$ and $\varphi = 0, \pi/2, \pi$. When $\gamma^{(1,1)} = 0$, the photons are bunched and are never found in the two different branches of the BO; scanning the delay between the input ports of the lattice will yield a HOM dip. When $\gamma^{(1,1)} = 2$, the photon are antibunched, and a delay scan will result in a HOM peak [20]. Figure 2d clearly shows that bunching/antibunching oscillations have a period of $\lambda_B/2$.

We next study input states which exhibit correlation oscillations with shorter periods. Eq. (4) suggests that the period of the oscillations in the correlation properties depends on the spacing between the input waveguides and on the number of photons in the NOON state. Figure 3 shows several examples of correlation oscillations with periods shorter than $\lambda_B/2$. In figure 3a we show the propagation for a NOON state with $N = 2$, where the input sites are separated by one waveguide. In this case the photons exhibit a bunching/antibunching affect with a different spatial symmetry [9]. The correlation map oscillates between a state in which the peaks are highest at the corners of the correlations matrix, to a case in which the highest probability is between the corners. The oscillation period is $\lambda_B/4$, twice the period observed for a NOON state input with adjacent waveguides. Finally we calculate $\Gamma^{(N/2,N/2)}_{\mu,\nu}$for NOON states with $N = 6$ (Fig. 3b) and $N = 10$ (Fig. 3c), with adjacent input waveguides. The oscillation period is indeed λ_B/N, as predicted by Eq. (4). Within one oscillation of the single photon density, the N-photon distribution switches N times from all the photons in the same branch to photons divided equally between the two branches.

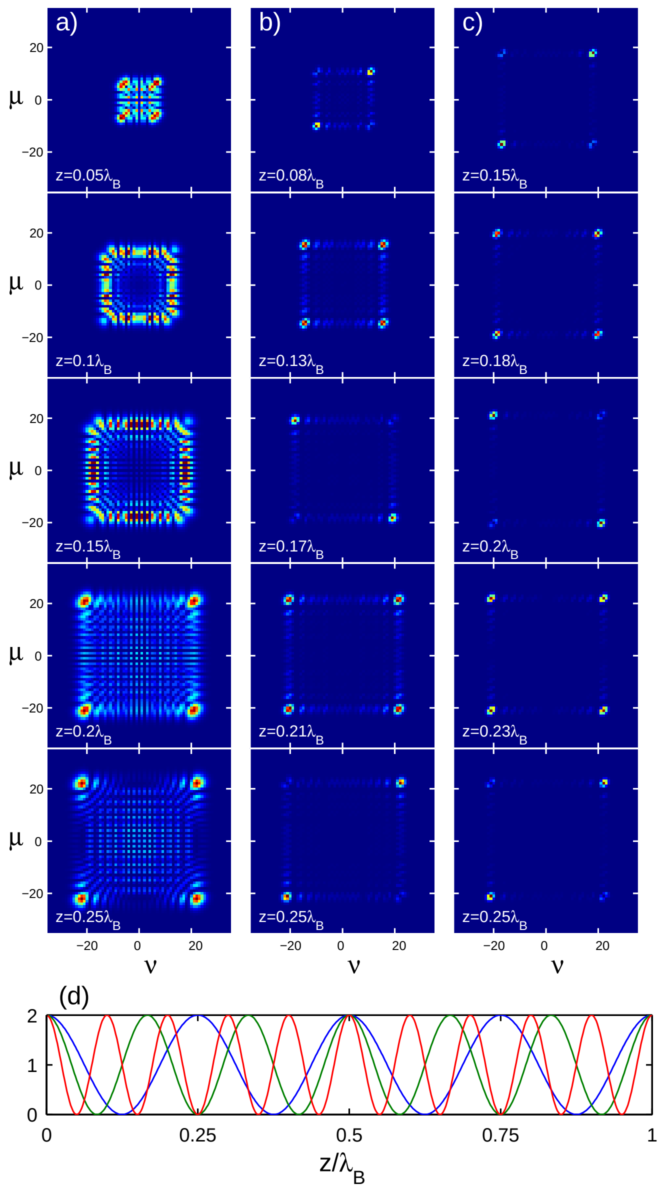

Figure 3: (Color online) Bloch oscillations of NOON states with sub $\lambda_B/2$ correlation-oscillation periods. (a) The multiple detection probability $\Gamma^{(1,1)}_{\mu,\nu}$ at several propagation distances for the input state $|\psi\rangle = \frac{1}{\sqrt{2}}(|2\rangle_{-1}|0\rangle_1 + |0\rangle_{-1}|2\rangle_1)$. The photons exhibit bunching/antibunching oscillations (see text) with a period $\lambda_B/4$. (b),(c) The multiple detection probability $\Gamma^{(N/2,N/2)}_{\mu,\nu}$ for a NOON state with $N = 6$ (b) and $N = 10$ (c), injected to adjacent waveguides $|\psi\rangle = \frac{1}{\sqrt{2}}(|N\rangle_0|0\rangle_1 + |0\rangle_0|N\rangle_1)$. The oscillations of the correlation matrix are much faster, hence the probability matrix is calculated for five lattice lengths close to the turning point $z = \frac{\lambda_B}{4}$. (d) The normalized coincidence rate $\gamma^{(N/2,N/2)}(z)$ as a function of the lattice length for the above three cases. The period of the oscillations are $\lambda_B/4$ (a, blue line) $\lambda_B/6$ (b, green) and $\lambda_B/10$ (c, red line). See [25] for a movie visualizing the propagation of (a) and (b).

In conclusion, we studied the propagation of photonic NOON states in waveguide lattices which exhibit Bloch oscillations. We found that while the photon density oscillates in the Bloch frequency, the multiple detection probability oscillates at higher frequencies. These oscillations indicate that the photons show a transition from a bunched to antibunched states, with a period that scales as $1/N$. To experimentally observe these high order oscillations, we propose to perform a Hong-Ou-Mandel measurement between two waveguides at the two branches of oscillations using photon-number resolving detectors. By scanning the delay between the photons entering the two input waveguide, oscillations between a HOM dip and peak will be observed as a function of the propagation length. Recent progress in waveguide lattice fabrication [21, 22], photon number resolving detectors [23] and photonic NOON state sources [24], make such measurements in reach.

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[25] See EPAPS Document No. [**********] for a movie visualizing BOs of NOON states. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.