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Comment on" Entanglement of two interacting bosons in a two-dimensional isotropic harmonic trap"[Physics Letters A 373 (2009) 3833-3837]

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Abstract

The correct form of the Schmidt decomposition of the stationary wave functions for a system of two interacting particles trapped in a two-dimensional harmonic potential is given.

Keywords: Schmidt decomposition

We note that in Ref.[1] the expansion of the two-particle wave function (Eq.11) is mistakenly interpreted as the Schmidt decomposition. The mode functions appearing in it cannot be considered as the Schmidt orbitals due to their incorrect normalization in radial direction in two-dimensional (2D) space. It should be stressed that this mistake does not affect the validity of the results presented in Ref. [1] since Eq.11 has not been used to their determination. However, because of the utility of the model in many areas of physics it is important to provide the correct Schmidt form decomposition that we derive below.

The system of two interacting particles trapped in a 2D isotropic harmonic potential, irrespectively of the interaction potential between the particles, possess the stationary wavefunctions ($m_r = M_c = 0$)[1] that depend only on ρ and ϱ, where $|\vec{\varrho}| = |(\vec{\rho}_1 + \vec{\rho}_2)/2| = \varrho(\rho_1, \rho_2, cos(\varphi_2 - \varphi_1))$ and $|\vec{\rho}| = |\vec{\rho}_2 - \vec{\rho}_1| = \rho(\rho_1, \rho_2, cos(\varphi_2 - \varphi_1))$. Being the function of ρ and ϱ only, Ψ is symmetric under permutation of the particles $((\rho_1, \varphi_1) \longleftrightarrow (\rho_2, \varphi_2))$ and may be assumed to be real for simplicity. An application the method of Ref. [2] to this case results in

$$\Psi(\rho, \varrho) = \sum_{m=-\infty \ldots \infty} \frac{A_m(\rho_1, \rho_2)}{\sqrt{\rho_1 \rho_2}} e^{im\varphi_1} e^{-im\varphi_2}, \tag{1}$$

where to ensure correct normalization in radial directions we have introduced $\sqrt{\rho_1 \rho_2}$ which is the crucial difference from Eq. 11 of Ref. [1]. The function $A_m(\rho_1, \rho_2)$ is given by the following integral

$$A_m(\rho_1, \rho_2) = \sqrt{\rho_1 \rho_2} \int_0^{2\pi} \int_0^{2\pi} \Psi(\rho, \varrho) e^{im(\varphi_2 - \varphi_1)} d\varphi_1 d\varphi_2 =$$

[1]The Hamiltonian of two interacting particles in the isotropic harmonic trap is separated into center of mass (c.m.) and relative (rel) motion. The stationary states of this system may be chosen as $\Psi_{n,m_r,N,M_c}(\vec{\rho}_1, \vec{\rho}_2) = \psi_n^{rel.}(\rho) e^{im_r \varphi_{rel.}} \psi_N^{c.m.}(\varrho) e^{iM_c \varphi_{c.m.}}$, where the functions ψ_n^{rel} and $\psi_N^{c.m.}$ are solutions of the radial Schrödinger equations rel. and c.m., respectively.

$$\sqrt{\rho_1\rho_2}\int_0^{2\pi}\int_0^{2\pi}\Psi(\rho,\varrho)cos(m(\varphi_2-\varphi_1))d\varphi_1 d\varphi_2, \tag{2}$$

where the simplification has been achieved by elementary symmetry considerations. Being real and symmetric, the function $A_m(\rho_1,\rho_2)$ has the following Schmidt form

$$A_m(\rho_1,\rho_2)=\sum_{s=0}\kappa_{s,m}\chi_s^{(m)}(\rho_1)\chi_s^{(m)}(\rho_2), \tag{3}$$

where the coefficients $\kappa_{s,m}$ and the orbitals $\chi_s^{(m)}(\rho)$ satisfy the integral equation

$$\int_0^\infty A_m(\rho_1,\rho_2)\chi_s^{(m)}(\rho_2)d\rho_2=\kappa_{s,m}\chi_s^{(m)}(\rho_1).$$

The family $\{\chi_s^{(m)}(\rho)\}_{s=0}$ forms a complete and orthogonal set ($\int_0^\infty\chi_s^{(m)}\chi_{s'}^{(m)}d\rho = C\delta_{ss'}$). Using the expansion (3) the final form of the decomposition of the wave function Ψ now reads

$$\Psi(\rho,\varrho)=\sum_{\substack{m=-\infty\ldots\infty\\ s=0}}\kappa_{s,m}v_{s,m}(\rho_1,\varphi_1)v^*_{s,m}(\rho_2,\varphi_2), \tag{4}$$

where

$$v_{s,m}(\rho,\varphi)=\frac{\chi_s^{(m)}(\rho)}{\sqrt{\rho}}e^{im\varphi}. \tag{5}$$

Since the orbitals (5) satisfy the condition of orthogonality in 2D space

$$\int_0^\infty\int_0^{2\pi}\rho v^*_{s,m}v_{s',m'}d\rho d\varphi=2\pi\delta_{m,m'}\int_0^\infty\chi_s^{(m)}\chi_{s'}^{(m)}d\rho=2\pi C\delta_{mm'}\delta_{ss'},$$

we recognize them as the Schmidt modes (natural orbitals). One can point out that they are the eigenfunctions of the angular momentum operator $\hat{L}_z$.

In conclusion, we have presented in details a procedure of obtaining the Schmidt decomposition of the two-particle wave function that is a function of distances ρ and ϱ only.

References

[1] B. Sun, M. Pindzola Physics Letters A 373 (2009) 3833-3837

[2] Jia Wang, C. K. Law, and M.-C. Chu, Phys. Rev. A 72, 022346 (2005)