An Alternative to Decoherence by Environment and the Appearance of a Classical World

Manfred Requardt

Institut für Theoretische Physik
Universität Göttingen
Friedrich-Hund-Platz 1
37077 Göttingen Germany
(E-mail: requardt@theorie.physik.uni-goettingen.de)

Abstract

We provide an alternative approach to the decoherence-by-environment paradigm in the field of the quantum measurement process and the appearance of a classical world. In contrast to the decoherence approach we argue that the transition from pure states to mixtures and the appearance of macro objects (and macroscopic properties) can be understood without invoking the measurement-like influence of the environment on the pointer-states of the measuring instrument. We show that every generic many-body system contains within the class of microscopic quantum observables a subalgebra of macro observables, the spectrum of which comprises the macroscopic properties of the many-body system. Our analysis is based (among other things) on two ingenious papers by v.Neumann and v.Kampen.

1 Introduction

In recent years *environment induced decoherence* has apparently aquired the status of a new paradigm in the context of the *quantum measurement process*, the *appearance of a classical world*, or the nature of *superselection sectors* (see, just to mention a few representative sources, [1],[2],[3],[4]). It is even sometimes erroneously claimed, that it solves the quantum measurement problem (cf. [5] and the reply by Adler [6]).

In our view the measurement problem consists of a deeper mystery, that is, how particular measurement values appear in a single measurement, and a problem which is of a somewhat lesser calibre, i.e. the transition of a superposition of states into a corresponding mixture within the *ensemble picture interpretation* of quantum mechanics. As papers about this field go into the hundreds, we refer the reader to the above mentioned reviews what concerns the decoherence approach and to [7] as to the older history of the quantum measurement process. Our main concern in the following is not to write another review but develop an (in our view) coherent complementary approach to the above mentioned problems which is based on a deep (but seldomly cited) paper by v.Neumann, a later equally important paper by v.Kampen and prior work of the author (see [8],[9],[10],[11]).

Remark: One should note that practically nothing of the content of [8] can be found in his famous book about the foundations of quantum mechanics!

One of the reasons why [8] has been largely neglected in the context of the quantum measurement process is possibly that it is written in German and that it deals mainly rather with the ergodic problem. Furthermore, in the fifties (for reasons difficult to understand) it has been unjustly criticized as being 'empty' etc. (a quite ridiculous remark in our view). As to the reception history see the recent analysis by Lebowitz et al ([12]).

To describe the difference of our appoach compared to the decoherence appoach in a nutshell one may say: It goes without saying that no (macroscopic) object is completely isolated, i.e. is in a sense an open system. This does however not! imply that in an idealized but perhaps nevertheless reasonable description of nature, we are not allowed to either neglect these effects or incorporate them in some averaged statistical manner (as e.g. in the *random phase approximation* in statistical mechanics, cf. [13]). The decoherence by environment philosophy claims that the entanglement with the environment is *the* crucial property while we will argue that e.g. the appearance of *macroscopic objects* can be alternatively understood in a more intrinsic manner without invoking the (in our view) rather *contingent* influence of some environment.

Remark: We note that a similar dichotomy exists for example in statistical mechanics and in particular in ergodic theory. That is, can ergodicity or statistical behavior only be understood by invoking some disorder assumption (coming from outside) or can it also be understood within *closed many-body systems*.

But before we embark on the development of an alternative approach to the quantum measurement process and the concept of '*classicality*' in the quantum context, we want to give a very brief description of the ideas of the decoherence-by-environment framework as formulated in e.g. [14].

2 The Decoherence by Environment Concept

In a nutshell, the idea is quite simple. In the ordinary presentation of the quantum measurement process (in the v.Neumann spirit) we start from the following chain of equations. Let Φ_0, Φ_i be the (pure) states of the measuring apparatus, or rather of a subsystem (typically some macro system). Note however that in the decoherence approach they are frequently called *pointer states*, the true nature of which is often not openly specified. Let ψ_i be the eigenstates of the quantum observable, A, to be measured. We then have (with frequently $\psi_i\Phi_i$ etc. as shorthand for $\psi_i \otimes \Phi_i$ etc.)

$$\psi_i\Phi_0 \to \psi_i\Phi_i \tag{1}$$

The superposition principle of quantum mechanics then yields:

$$\left(\sum_i c_i\psi_i\right)\Phi_0 \to \sum_i c_i\psi_i\Phi_i \tag{2}$$

Then follows the argument that the rhs of the last equation cannot be identified (even in the case of macro objects) with the corresponding mixture

$$\sum_i |c_i|^2 P_{\psi_i\Phi_i} \tag{3}$$

$P_{\psi_i\Phi_i}$ being the projector on the state $\psi_i\Phi_i$ irrespectively of the fact that usually $(\psi_i\Phi_i|\psi_j\Phi_j) = \delta_{ij}$ is assumed. The reason is that in case some of the $|c_i|$ happen to be equal, we observe a so-called *basis-ambiguity* (a mathematically coherent treatment can be found in [15],[16],[17]).

It is argued in e.g. [14] and elsewhere that in that cases there do exist other decompositions of the state vector $\sum_i c_i \psi_i \Phi_i$ with respect to different bases, which in the decoherence philosophy can then be associated with different observables, so that a unique association of macroscopic *pointer states* and microscopic states (at first glance) does not seem possible.

Remark: We give a critical analysis of this point of view in the following sections.

It is said that this stage of the measuring process is only a *premeasurement* in so far as the state $\sum_i c_i \psi_i \Phi_i$ is still a pure quantum state being *observably* different from a mixture! As a typical example the Stern-Gerlach experiment is frequently invoked where the two split beams can, in principle, be reunited again into a pure state. This (thought) experiment is frequently attributed to Wigner (see e.g. [14]). But it can already be found in the book by Ludwig ([18]) and in an even earlier interesting paper by Jordan ([19]).

The measurement process is, according to this philosophy, closed by an appropriate entanglement of the above state with the so-called *environment*. If ε_i are (in the ideal case) orthogonal states of the *environment*, it is claimed that we finally have

$$\left(\sum_i c_i \psi_i\right) \Phi_0 \varepsilon_0 \to \sum_i c_i \psi_i \Phi_i \varepsilon_i \tag{4}$$

which solves the basis ambiguity (see [15],[16],[17]).

As a typical example illustrating the basis ambiguity, the following situation is frequently invoked. The singulett state of two spin-one-half particles

$$1/\sqrt{2}\,((\uparrow)(\downarrow) - (\downarrow)(\uparrow)) \tag{5}$$

can be represented in e.g. the eigen basis of the x-component of the spin, i.e.

$$\psi_1' = ((\uparrow) + (\downarrow))\,/\sqrt{2} \quad , \quad \psi_2' = ((\uparrow) - (\downarrow))\,/\sqrt{2} \tag{6}$$

as

$$-\,1/\sqrt{2}\left(\psi_1' \psi_2' - \psi_2' \psi_1'\right) \tag{7}$$

Remark: Note that the exact compensation of the other cross terms come about because of the common prefactor $1/\sqrt{2}$, i.e. the necessary and sufficient condition for a basis ambiguity mentioned above.

In the decoherence philosophy according to e.g. Zurek the second tensor product component may then be associated with some pointer that is (part of) a measurement instrument. It is then argued that in the three-orthogonal Schmidt-decomposition , $\sum_i c_i \psi_i \Phi_i \varepsilon_i$, which is a delocalized state due to the structure of the environment states, ε_i, one can locally regard the measurement outcome as a mixture, $\sum_i |c_i|^2 P_{\psi_i \Phi_i}$, (by tracing over the environment) while the global state is still a pure vector state with the information spread into the environment. That is, the crucial point is that one remains globally in the regime of unreduced quantum states!

In our view, at least two of the conceptual ingredients are problematical. First, in case of e.g. the Stern-Gerlach experiment, which serves as kind of a paradigm, the (silver) atoms, carrying the spin degree of freedom, are frequently regarded as pointers, or at least a similar device. In our view, and in the original quantum measurement literature (see e.g. [20]), one would rather call such a subsystem a *quantum probe* in the context of *quantum non-demolition* measurements. A similar role is played by the photon in the *quantum microscope*. In general it may be subsumed under the catchword of *shift of the cut* between the micro and the macro world in the measurement process. To put it briefly, we have the impression that important subsystems like photo plates, magnets, and the like, which we would prefer to regard as essential parts of the measurement instrument are now simply called environment in the decoherence approach.

Second, it is claimed that the *pointer basis* (and ultimately the correct functioning as a measurement instrument) is established via the interaction of the pointer states with the environment. We must say that we are extremely sceptical, if this point of view is really correct and we will substantiate our scepticism below. We rather think that pointer basis and functioning as a measurement instrument are a priori fixed by the concrete setup of the instrument according to some pre-theory of measurement, typically incorporating pieces of classical and quantum physics. This we can at least learn from the analysis of concrete measurement situations ([20]) and the work of the founding fathers of quantum theory (cf. the beautiful discussion between Einstein and Heisenberg as described in [21]). A typical ingredient is usually some sub-system being in a meta-stable state (photo plate, Wilson chamber, spark chamber etc.).

Third, the ordinary environment is usually of a very contingent character and it is at least debatable to attribute pure quantum states to it, and, a fortiori, states which are assumed to play a role relative to the pointer states similar to measuring instruments relative to the micro objects. We would like to emphasize that the interaction of a measuring instrument with a micro object is a very special one while the interaction of a pointer with the environment is usually of the ordinary statistical type.

On the other hand, the influence of the environment has played an important role already in the classical literature about the quantum measurement process (cf. e,g, the lucid analysis of Heisenberg in his contribution to the Bohr-Festschrift [22]). He clearly states that an apparatus, not interacting with the exterior world, is a quantum system and cannot be used as a measuring instrument. It is, in his words, in a potential, i.e. a quantum state. It becomes a macro system via its contact with the environment (thus aquiring factual properties). Furthermore, the illustrations of concrete measuring instruments in the contribution to the Bohr-Einstein debate in [7], with their solid clamps and bolts clearly show that a strong contact with the environment is important.

As a last point, the influence of the environment is also incorporated in statistical mechanics. Starting from a global pure state (system plus environment) it is shown in e.g. [13] how one arrives via the *random phase approximation* at a statistical state of the system. That is, it is not the influence of the environment which is denied by us but rather the ubiquity of the invoked *measurement-like* effect on the pointer states and its role for the appearance of a classical world.

3 Macro Observables from Quantum Theory

In this section we descibe in a, as we think, coherent way how *macro observables* and macroscopic properties do emerge *within* the framework of quantum theory. The description is based on the highly original papers by v.Neumann and v.Kampen ([8],[9]), some related work of Ludwig ([23],[24]) and prior work of the author ([10],[11]).

Most of [23],[24],[10],[11] is written in the many-body-language approach to the measurement process with relations to phase transitions and super selection sectors. Papers written in a similar spirit are e.g. from the italian school (see for example [25]). A central problem discussed in these papers was the treatment of macroscopic systems as quantum systems, a problen which also troubled Legett (see e.g. [26]). We think, a transplantation of the above ideas of v.Neumann and v.Kampen into this measurement context will clarify this longstanding open question. That is, we will show in the following how the macroscopic regime is embedded as a subtheory in general quantum physics.

In this section, for short, we will mainly discuss the ideas of v.Kampen. We start from a many-body wave function

$$\Psi(q) = \sum a_n \psi_n(q) \, e^{-iE_n t/\hbar} \quad , \quad q = (q_1, \ldots, q_f) \tag{8}$$

with $\psi_n(q)$ the eigenfunctions of the microscopic Hamiltonian, H. We need not discuss the distribution of spectral values of H in any detail. We know that for $f \gg 1$ they are irregularly distributed in dense clusters and are also typically (highly) degenerated. The crucial idea is the existence of what v.Neumann and v.Kampen call *macroscopic observables* (a possible construction is given in e.g. [8]), other constructions are given in [24] or [10],[11]; we come back to this point below, in particular in the last section.

In the following we mainly use the notation of v.Kampen.

Observation 3.1 *There exist (almost) commuting observables in the representation space of the many-body system, denoted by $E, A, B, \ldots$ (E representing the macroscopic, i.e. coarse-grained energy operator) and a complete, orthonormal set of (approximate) common eigenvectors, Φ_{Ji}, with the property*

$$A \circ \Phi_{Ji} = A_J \cdot \Phi_{Ji} + O(\Delta A) \tag{9}$$

where ΔA is the measurement uncertainty of the macro observable A. It is always assumed that ΔA is macroscopically small but large compared to the quantum mechanical uncertainty δA. The approximate common eigenvectors come in groups, indexed by J with i labelling the vectors belonging to the group J.

The above equation is assumed to hold for all macro observables. The subspace, belonging to J is called a *phase cell*. It is assumed that the eigen values A_J are macroscopically discernible, i.e. they describe different macroscopic behavior. That is, quantum states belonging to the same phase cell have the *same* macroscopic properties.

Remark: In order that an observable qualify as a macro observable, some properties have to be fulfilled (cf. e.g. [9]).

Typically a macro observable is the sum over few-body micro variables (cf. [24] or [11], see also the last section) like e.g.

$$A = c_f^{-1} \sum_{partitions} a(q_{i_1}, \ldots, q_{i_n}) \quad , \quad f \gg n \tag{10}$$

with the sum extending over all partitions of $(1, \ldots, f)$ into n-element subsets and c_f is of the order f. It can be shown that such observables fulfill the above assumptions.

One can now represent an arbitrary state vector $\Psi(q)$ as a sum over this new basis, i.e.

$$\Psi(q) = \sum b_{Ji} \Phi_{Ji}(q) \tag{11}$$

In a next step we will introduce *coarse* observables $\overline{E}, \overline{A}, \overline{B}, \ldots$ with the property

$$\overline{A}\,\Phi_{Ji} = A_J\,\Phi_{Ji} \quad , \quad \overline{A} = \sum_{Ji} A_J \cdot P_{Ji} \tag{12}$$

i.e., the Φ_{Ji} are now exact common eigenvectors of the commuting set $\{\overline{E}, \overline{A}, \overline{B}, \ldots\}$.

Remark: Note that the existence of such observables is guaranteed by the explicit construction via the above spectral representation.

The expectation of e.g. $\overline{A}$ in the state $\Psi(q)$ is

$$\langle\Psi|\overline{A}|\Psi\rangle = \sum_J A_J \left(\sum_i |b_{Ji}|^2\right) =: \sum_J A_J\, w_J \tag{13}$$

with w_J the probability that the (macro) system is found in the phase cell J.

Observation 3.2 *The w_J fix the macroscopic properties of the state $\Psi(q)$.*

As to the technical details of the construction of such a set of macro observables see the above cited papers. We give only one technical property.

Observation 3.3 *With $A = c_f^{-1} \sum_k a_k$, $B = c_{f'}^{-1} \sum_{k'} a_{k'}$ $a_k, b_{k'}$ microscopic few-body observables, we have*

$$[A, B] = c_f^{-1} \cdot (c'_f)^{-1} \sum_{kk'} [a_k, b_{k'}] \approx 0 \quad \textit{for} \quad f \gg 1 \tag{14}$$

Proof: Note that by assumption most of the $[a_k, b_{k'}] \approx 0$, that is, the set of terms, $[a_k, b_{k'}]$, being essentially different from zero is of cardinality $O(f)$ and that $c_f = O(f)$.

A fortiori, a macro observable (almost) comutes with all micro observables in the large f-limit.

Conclusion 3.4 *Within the framework of true (many-body) quantum mechanics we found a subset of observables $E, A, B, \ldots$ which behave almost macroscopic, while the coarse observables $\{\overline{E}, \overline{A}, \overline{B}, \ldots\}$ exactly commute and have the common set of eigenvectors Φ_{Ji} which come in groups indexed by J. The macroscopic eigenvalues $E_J, A_J, B_J, \ldots$ are macroscopically discernible for $J \neq J'$.*

In the above approach we assumed (for convenience) that the spectrum of the observables under discussion is discrete. In case we have an observable with continuous spectrum the approach only needs a few technical modifications. On the one hand, we can form observables with discrete spectrum from observables with continuous spectrum by appropriate coarse-graining. In a next step we can e.g. via rescaling construct macro observables with (almost) continuous spectrum. As example take certain position observables of some (macroscopic) subsystems (cf. for example the last section).

4 The Quantum Mechanical Measurement Process in the Light of the preceeding Analysis

Our notion of macro systems and macro observables emerges as a second level subtheory from the underlying quantum level. I.e., in the space of microscopic observables we (rigorously) construct a subspace of macro observables, $\mathcal{A}_M$, the members of which almost commute while the corresponding coarse-grained observables, $\overline{\mathcal{A}}_M$, exactly commute by construction. The measurement devices or the pointers are assumed to be essentially macroscopic, that is, pointer states or pointer observables are assumed to belong to this class.

Observation 4.1 (Superposition Principle) *With Ψ_1, Ψ_2 many-body quantum states of a measurement instrument or of some macroscopic part of it (pointer), which are assumed to have unique macroscopic properties, i.e. belonging to single but different phase cells, that is*

$$\Psi_l(q) = \sum_i b^l_{Ji} \Phi_{Ji}(q) \quad , \quad l = 1, 2, \quad J \quad \textit{fixed} \tag{15}$$

we have

$$\Psi := \Psi_1 + \Psi_2 = \sum_i b^1_{Ji} \Phi_{Ji}(q) + \sum_{i'} b^2_{J'i'} \Phi_{J'i'}(q) \tag{16}$$

and

$$\big(\Psi|\overline{A}_M|\Psi\big) = \left(\sum_i |b^1_{Ji}|^2\right) \cdot A_J + \left(\sum_{i'} |b^2_{J'i'}|^2\right) \cdot A_{J'} = \big(\Psi_1|\overline{A}_M|\Psi_1\big) + \big(\Psi_2|\overline{A}_M|\Psi_2\big) \tag{17}$$

That is, within the realm of the smaller algebra $\overline{\mathcal{A}}_M$ states like Ψ_l or Ψ behave as mixtures and not as pure states.

Note that in our approach the system is treated as a true quantum many-body system in the microscopic regime and at the same time as a macro system with respect to the smaller algebra $\mathcal{A}_M$. This answers (in our view) also some longstanding questions as to a possible threshold where quantum properties go over (in a presumed phase-transition-like manner) into macro properties. According to our analysis there is no such threshold. It is rather the many-body behavior as such which enables the selection of a subalgebra $\mathcal{A}_M$.

Observation 4.2 (Schroedinger's Cat) *The above result concerns superpositions of macro states being observably different, a catchword being Schroedinger's Cat. In many discussions the wrong picture is invoked as if a superposition of dead and alive is something like a macroscopically blurred state. This impression is incorrect! What can be macroscopically observed is given by the class of macroscopic observables. But as we have shown, these observables annihilate the respective interference terms. Such interference terms could only be observed in some super cosmos with the help of observables which connect macroscopically many degrees of freedom at a time (cf. the last section).*

In a next step we want to show that the basis ambiguity problem becomes obsolete in our context.

Observation 4.3 *As all elements of $\mathcal{A}_M$ quasi-commute or rigorously commute in $\overline{\mathcal{A}}_M$, there do not exist the so-called complementary observables.*

This has the following effect. In e.g. the Stern-Gerlach experiment we can of course repeat the analysis of Zurek and write:

$$1/\sqrt{2}\,((\uparrow)(\downarrow) - (\downarrow)(\uparrow)) = -1/\sqrt{2}\left(\psi_1'\psi_2' - \psi_2'\psi_1'\right) \tag{18}$$

(cf. section 2) and associate again the second terms in the tensor product with some pointer states. I.e., the superposition principle is taken for granted. However, there do not! exist a *macro observable*, B_M, so that the new states, Ψ_i', are its eigenstates.

Observation 4.4 *With Φ_J eigenstates of the coarse macro observable $\overline{A}$, i.e. belonging to some phase cells $\mathcal{C}_J$,*

$$\overline{A} \circ \Phi_J = A_J \cdot \Phi_J \tag{19}$$

there does not exist a coarse observable $\overline{B}$ with e.g.

$$\overline{B} \circ (\Phi_1 + \Phi_2) = B_3 \cdot (\Phi_1 + \Phi_2) \tag{20}$$

while we have

$$\overline{B} \circ \Phi_J = B_J \cdot \Phi_J \tag{21}$$

and $B_1 \neq B_2$ being assumed (macroscopically distinct properties). The state $\Phi_1 + \Phi_2$ rather represents a mixture with respect to $\overline{\mathcal{A}}_M$.

Conclusion 4.5 *The basis ambiguity does not exist for $\overline{\mathcal{A}}_M$. We can of course represent some many-body state with respect to another basis but the macroscopic properties remain the same! They are encoded in the a priori fixed decomposition*

$$\Psi = \sum b_{Ji} \cdot \Phi_{Ji} \quad , \quad \overline{A}\Phi_{Ji} = A_J \Phi_{Ji} \tag{22}$$

In physical terms we can explain this result with the help of the Stern-Gerlach experiment, following Bohr's dictum that the quantum mechanical measurement of two complementary observables as e.g. σ_z and σ_x need two different! and mutually exclusive experimental setups. That is, in order to measure the z-component one has to split the beam along the z-axis. This implies that the magnets have to be oriented accordingly. The same procedure with respect to the x-direction implies the respective orientation of the magnets parallel to the x-axis. That is, we have to apply a macroscopic rotation of the magnets.

Observation 4.6 *This rotation cannot be described by means of a superposition of states of the magnets being oriented in the z-direction as e.g. in microscopic quantum mechanics.*

Conclusion 4.7 *We can infer that the macroscopic pointer states are not determined via interaction (by decoherence) with the environment. They are obviously fixed a priori by the concrete experimental setup as described in the above example.*

5 The Analysis of a Concrete Measurement Situation

We now want to give a concrete example illuminating the approach, described above. It was already essentially given in [10],[11]. We assume that the pointer of our measurement instrument is a macroscopic subsystem consisting of N ($N \gg 1$) quantum particles (e.g. a solid state system or an avalanche in a Geiger-counter), being capable of performing approximately a *coherent motion*, depending on the micro state of the quantum system to be measured.

I.e., in a concrete individual measurement event, the pointer as a whole starts to move with a macroscopic momentum

$$\left(\Phi_i(t)|\widehat{P}|\Phi_i(t)\right) \approx N\cdot <p>_i \tag{23}$$

with $\widehat{P} = \sum_{i=1}^{N} \hat{p}_i$ the quantum mechanical total momentum observable, $\Phi_i(t)$ a collective state of the pointer (induced by the contact with the micro object) and

$$<p>_i := \left(\Phi_i(t)|N^{-1}\cdot\widehat{P}|\Phi_i(t)\right) \tag{24}$$

the (approximately constant) *mean-momentum* per (quantum-) particle of the pointer. We assume that different measurement results imply $<p>_i \neq <p>_j$ with the $<p>_i$ being in correspondence with microscopic values q_i of some quantum observable to be measured.

The *center-of-mass* observable of the pointer

$$\widehat{R}_{CM} := \sum_i m_i \hat{r}_i / \sum_i m_i \tag{25}$$

then behaves as (with $M := \sum_i m_i$)

$$\langle\widehat{R}_{CM}\rangle_i(t) := \left(\Phi_i(t)|\widehat{R}_{CM}|\Phi_i(t)\right) \approx const. + t\cdot <p>_i \cdot N/M \tag{26}$$

Observation 5.1 *i) For $N \gg 1$ and $<p>_i \neq <p>_j$ the states $\Phi_i(t), \Phi_j(t)$ become (almost) orthogonal for macroscopic t.*
ii) In our simple model the values $\{<p>_i\}$ label different phase cells (or sectors) with (almost) sharp eigen values of the macro observables $N^{-1}\cdot\sum_i \hat{p}_i$ or $\widehat{R}_{CM}$.
iii) An arbitrary microscopic state vector of our pointer system is a superposition of the above sector states, i.e.

$$\Psi = \sum_{Ji} b_{Ji}\Phi_{Ji} \tag{27}$$

with i labelling the different vectors belonging to the same phase cell descibed by $<p>_J$.

6 Interference among Macro States

In this section we want to analyse the possibility of the observation of interference effects among macroscopically distinct macro states. We adressed this problem already in [10] and [11]. We have shown in the preceeding analysis that this cannot be done in the regime of macro observables, $\mathcal{A}_M$ or $\overline{\mathcal{A}}_M$. If one goes into the technical details one observes that one (crucial) property, in order to qualify as a macro observable, is the following

Observation 6.1 *With N the number of microscopic constituents of a macroscopic (many-body) system ($N \gg 1$), we see that typical microscopic quantum mechanical observables are so-called few-body observables. I.e.*

$$\hat{a}(x_{i_1}, \ldots, x_{i_n}) \tag{28}$$

denotes a microscopic n-particle observable, correlating $n \ll N$ microscopic constituents at a time. A typical many-body observable which qualify as a macroscopic observable can then be written as

$$\widehat{A} := \sum_{Per} \hat{a}(x_{i_1}, \ldots, x_{i_n}) \tag{29}$$

where the sum extends over all possible clusters of n micro objects out of the N constituents of the many-body system. Furthermore, a prefactor of the order N_{-1} frequently occurs in front of the sum.

If we try to observe now possible off-diagonal elements of $\widehat{A}$, that is, between different macro states, we get approximately, making certain simplifying assumptions

Observation 6.2 *The degree of overlap between different macro states with respect to the macro observable $\widehat{A}$ is approximately*

$$|\left(\Phi_i|\widehat{A}|\Phi_j\right)| \approx N!/n!(N-n)! \cdot \tau^{(N-n)} \tag{30}$$

with τ a small number ($\ll 1$) which denotes the individual overlap of the wave function relative to the same microscopic constituents in the different macro states, which do not! belong to the cluster, coupled in a contribution coming from e.g. $\hat{a}(x_{i_1}, \ldots, x_{i_n})$.

Conclusion 6.3 *Interference between macroscopically different macro states could only be observed, if we were able to construct observables which do correlate $n \approx N \gg 1$ microscopic constituents at a time. The observables we are usually using in physics have however $n \ll N$. A situation where $n \approx N$ holds, is called by Ludwig in [24] a super-macro-cosmos.*

7 Conclusion

We have shown that one can rigorously construct a subalgebra of commuting macro observables within the set of quantum observables of a generic many-body system. The common (almost) eigen values of this set of macro observables are then the macroscopic properties of the many-body system. Furthermore, for the subalgebra of macro observables the basis ambiguity is lost (no complementarity!) and there is hence no need for a (measurement-like) decoherence-by-environment mechanism to fix the so-called pointer basis. The pointer basis is in our approach already apriori fixed by the design of the measurement instrument.

References

[1] W.H.Zurek:"Decoherence, einselection, and the quantum origins of the classical", RMP 75(2003)715

[2] W.H.Zurek: "Decoherence and the Transition from Quantum to Classical-Revisited", Los Alamos Science, Number 27 (2002)

[3] E.Joos,H.D.Zeh,C.Kiefer,D.Giulini,J.Kupsch,I.-O. Stamatescu: "Decoherence and the Appearance of a Classical World in Quantum Theory", sec.ed., Springer, Berlin 2003

[4] M.Schlosshauer: "Decoherence, the measurement problem, and interpretations of quantum mechanics", RMP 76(2004)1267

[5] P.W.Anderson: "Science:'Dappled World' or a 'Seamless Web'?", Stud.Hist.Phil.Mod.Phys. 32(2001)487

[6] S.L.Adler: "Why decoherence has not solved the measurement problem: a response to P.W.Anderson", Stud.Hist.Phil.Mod.Phys. 34(2003)135

[7] J.A.Wheeler,W.H.Zurek: "Quantum Theory and Measurement", Princt.Univ.Pr., Princeton 1983

[8] J.v.Neumann: "Beweis des Ergodensatzes und des H-Theorems in der Neuen Mechanik", Z.Phys. 57(1929)30

[9] N.G.v.Kampen: "Grundlagen der Statistischen Mechanik der Irreversiblen Prozesse", Fortschr.Phys. 4(1956)405

[10] M.Requardt: "The Quantum Mechanical Measurement Process as a Scattering Phenomenon inducing a Collective Coherent Motion", Z.Naturforsch. 39a(1984)1147

[11] M.Requardt: "Why there don't exist Superposed Cricket balls. An Approach to Quantum Decoherence within the Framework of the Statistical Mechanics of Phase Transitions", Preprint (unpublished), Goettingen 1993

[12] S.Goldstein,J.L.Lebowitz,R.Tumulka,N.Zhangi: "Commentary Accompanying the English Translation of J.v.Neumanns's 1929 Article", arXiv:1003.2129

[13] K.Huang: "Statistical Mechanics", Wiley, N.Y.1963

[14] W.H.Zurek: "Environment-induced superselection rules", P.R. D26(1982)1862

[15] A.Elby,J.Bub: "Triorthogonal uniqueness theorems", P.R. A(Brief Reports) 49(1994)4213

[16] A.Peres: "Higher order Schmidt Decompositions", P.L. A202(1995)16

[17] M.Requardt: "Higher-Order-Schmidt-Representations and their Relevance for the Basis-Ambiguity", preprint, Goettingen 2010

[18] G.Ludwig: "Die Grundlagen der Quantenmechanik", Springer, Berlin 1954

[19] P.Jordan: "On the Process of measurement in Quantum Mechanics", Phil.Sci. 16(1949)269

[20] V.B.Braginsky,F.Ya.Khalili: "Quantum measurement", Cambridge Univ.Pr., Cambridge 1992

[21] W.Heisenberg: "Der Teil und das Ganze", Piper, Muenchen 1973

[22] W.Heisenberg: "The Development of the Interpretation of the Quantum Theory", in "Niels Bohr and the Development of Physics", ed. W.Pauli,L.Rosenfeld.V.Weisskopf, Pergamon Pr., London 1955

[23] G.Ludwig: "Der Messprozess", Z.Phys. 135(1953)483

[24] G.Ludwig: "Geloeste und ungeloeste Probleme des Messprozesses in der Quantenmechanik", in "W.Heisenberg und die Physik unserer Zeit", ed. F.Bopp, Vieweg, Braunschweig 1961

[25] A.Danieri,A.Loinger,G.M.Prosperi: "Quantum Theory of Measurement and Ergodic Condition", Nucl.Phys. 32(1962)297

[26] A.J.Legett: "Macroscopic Quantum Systems and the Quantum Theory of Measurement", Suppl.Progr.Theor.Phys. 69(1980)80, "Testing the Limits of Quantum Mechanics", J.Phys.Cond.Matt. 14(2002)12415